Chapter No. 06

Sequences and Series

Presented by: Prof. Nusni Natasha

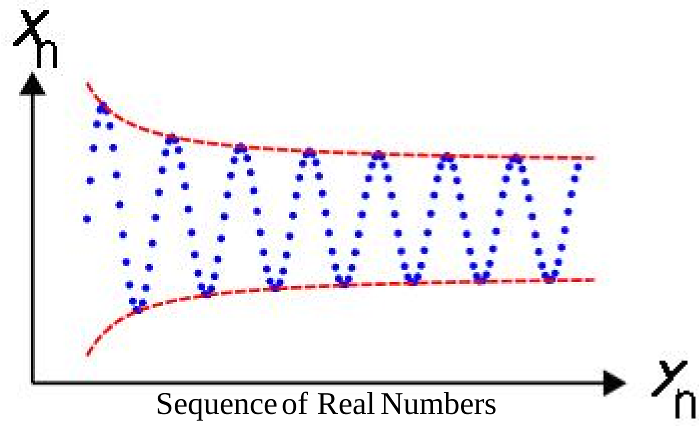
Q#.1: Define infinite sequence?

Solution (1) A sequence having an unlimited numbers of terms is called an infinite sequence,

OR

A sequence which has no last term is called an infinite sequence.

e. g 2,4,8,16,… [exempli gratia (Latin word "for example")]



Q#.2: Find the next two terms of the sequence -1,2,12,40…?

Solution (2) Given sequence is -1,2,12,40,…

$a_1=-1*2^0=-1*1=-1$,

$a_2=1*2^1= 1*2= 2$,

$a_3=3*2^2=3*4=12$,

$a_4=5*2^3=5*8=40$,

$a_5=7*2^4=7*16=112$,

$a_6=9*2^5=9*32=288$,

Q#.3: Find the indicated term of sequence 1,-3,5,-7,9,-11,…a_8?

Solution (3) Given sequence is 1,-3,5,-7,9,-11,…a_8

-3,-5,-7,-9,-11,a_8 1,5,9,a_7

Common difference=d=-7+3=-4 ; common difference=d=5-1=4

-3,-7,-11,-15, ; 1,5,9,13,

Q#.4: If $\frac{1}{a},\frac{1}{b}$ and $\frac{1}{c}$ are in A.P, show that the common difference is $\frac{a-c}{2ac}$.

Solution (4) since 1/a, 1/b, 1/c, are in A.P,

Therefore d=1/b-1/a……. (i)

And d=1/c-1/b……. (ii)

By adding both equations, we have

$$2d=\frac{1}{c}-\frac{1}{a}=\frac{(a-c)}{ac}$$
$$d=\frac{(a-c)}{2ac}$$

Hence proved

Q#.5: If the 5th term of A.P. is 13 and 17th term is 49, find a_n and a_{13}.

Solution (5) since $a_5=13$ and $a_{17}=49$

we know that $a_5=a_1+4d$

$13=a_1+4d$…. (i)

Also $a_{17}=a_1+16d$

$\Rightarrow$ $49=a_1+4d+12d$ $\Rightarrow$ $49=13+12d$ using (i)

$\Rightarrow$ $49-13=12d$ $\Rightarrow$ $36=12d$ $3=d$

Using in eq(i)

$\Rightarrow$ $13=a_1+4(3)$,

$\Rightarrow$ $13=a_1+12$

$\Rightarrow$ $1=a_1$,

Thus $a_{13}=a_1+12d$

$=1+12(3)$

$=1+36=37$

$a_n=1+(n-1)3=3n-2$

Q#.6: If 1/a ,1/b and 1/c are in A.P, Show that b=2ac/(a+c).

Solution (6) Since $\frac{1}{a},\frac{1}{b}\,and\,\frac{1}{c}$ are in A.P So

d=$\frac{1}{b}-\frac{1}{a}$ --------- (i) And d= $\frac{1}{c}-\frac{1}{b}$ ---------- (ii) By comparing (i) & (ii)

$$\frac{1}{b}-\frac{1}{a}=\frac{1}{c}-\frac{1}{b}$$

$\frac{1}{b}+\frac{1}{b}=\frac{1}{a}+\frac{1}{c}$

$\frac{2}{b}=\frac{a+c}{ac}$

$\frac{2ac}{a+c}=b$ Hence Proved

Q#.7: If $a_{n-2}=3n-11$, find the n th term of the sequence.

Solution (7) $\because a_{n-2}=3n-11$ Put n=3

$a_{3-2}=3(3)-11,$ $a_1=9-11=-2$

Put n = 4 $a_2=3\times 4-11=1$

Put n = 5 $a_3=3\times 5-11=4$, d= 1+2=3, $a_n=a_1+(n-1)d=-2+(n-1)3$

= -2 + 3n -3 =3n-5

Q#.8: Show that the sum of n A.Ms between a and b is equal to n times their A.M.

Solution (8) Let $A_1, A_2, A_3 .....A_n$ are

n A.Ms b/w $a \& b$

$a, A_1, A_2 ....A_n, b$ are in A.P, $A_1+A_2+A_3+....+A_n=\frac{n}{2}[A_1+A_n]$ $\because A_n+d=b$

$=\frac{n}{2}[a+d+b-d]$, $=\frac{n}{2}(a+b)$, $=n\left(\frac{a+b}{2}\right)$ Hence Proved

Q#.9: Sum the series; $\frac{1}{1+\sqrt{x}}+\frac{1}{1-x}+\frac{1}{1-\sqrt{x}}+.....$ to n terms.

Solution (9) $\because \frac{1}{1+\sqrt{x}}+\frac{1}{1-x}+\frac{1}{1-\sqrt{x}}$

d$=\frac{1}{1-x}-\frac{1}{1+\sqrt{x}}$, $=\frac{1}{(1)^2-(\sqrt{x})^2}-\frac{1}{1+\sqrt{x}}$ $=\frac{1}{(1-\sqrt{x})(1+\sqrt{x})}-\frac{1}{1+\sqrt{x}}$

$=\frac{1-1+\sqrt{x}}{(1\_\sqrt{x})(1+\sqrt{x})}$, $=\frac{\sqrt{x}}{(1-x)}$ $S_n=\frac{n}{2}(2a_1+(n-1)d)$

$=\frac{n}{2}\left(2\left(\frac{1}{1+\sqrt{x}}\right)+(n-1)\frac{\sqrt{x}}{1-x}\right)$ $=$ $\frac{n}{2}\left(\frac{2}{1+\sqrt{x}}+\frac{(n-1)\sqrt{x}}{1-n}\right)$

Q#.10: Sum the series;

$3+5-7+9+11-13+15+17-19+......$ to $3n$ terms.

Solution (10) Given that

3+5-7+9+11-13+15+17-19+………………… to 3n terms

$\Rightarrow$ (3+5-7) + (9+11-13) + (15+17-19) + ……….. + 3n terms

$\Rightarrow$ 1+7+13+…………+ n terms

d= 7-1=6

$S_n = \frac{n}{2}(2a_1 + (n-1)d)$, $\frac{n}{2}(2(1) + (n-1)6)$, $\frac{n}{2}(2+6n-6)$

$= \frac{n}{2}(6n-4)$ $S_n = n(3n-2)$

Q#.11: Find the sum of 20 terms of the series whose r th term is $3r+1$.

Solution (11) Given n=20; $a_r = 3r+1$

r=1; a_1=4, a_2=7, r=3; a_3=10

The series is

4+7+10+……+20th term, d=7-4=3, $S_n = \frac{n}{2}(2a_1 + (n-1)d)$

$= \frac{20}{2}(2(4) + (20-1)3) = \frac{20}{2}(8+60-3) = \frac{20}{2}(65) = \frac{1300}{2} = 650$

Q#.12: If a,b,c,d are in G.P, prove that $a^2-b^2, b^2-c^2, c^2-d^2$ are in G.P.

Solution (12) $\because a^2, b^2, c^2$ *are in* A.P

$\because b^2 - a^2 = c^2 - b^2 ....(i)$

To Prove $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ *are in A.P*

$\frac{1}{c+a} - \frac{1}{b+c} = \frac{1}{a+b} - \frac{1}{c+a}$,

$\frac{b+c-c-a}{(b+c)(c+a)} = \frac{c+a-a-b}{(a+b)(c+a)}$

$\frac{b-a}{(b+c)(c+a)} = \frac{c-b}{(a+b)(c+a)}$,

$\frac{b-a}{b+c} = \frac{c-b}{a+b}$

$b^{2-}a^2 = c^2 - b^2$ Hence Proved

Q#.13: A man repay his loan of Rs. 1120 by paying Rs. 15 in the first installment and then increases the payment by Rs. 10 every month. How long will it take to clear his loan.

Solution (13) Give that

1st Installment $a_1 = 15$

Monthly increase in payment d=10

Let n be the time he will take clear his loan S_n=1120

$$\Rightarrow \frac{n}{2}\{2a_1+(n-1)d\}=1120$$

$$\Rightarrow \frac{n}{2}\{2(150)+(n-1)10\}=1120$$

$$\Rightarrow \frac{n}{2}\times 10\{3+(n-1)10\}=1120$$

$$\Rightarrow\ 224=n(n+2)$$

$$\Rightarrow n^2+2n-224=0$$

$$\Rightarrow n=\frac{-2\pm\sqrt{4+896}}{2(1)}$$

$$=\frac{-2\pm 30}{2}$$

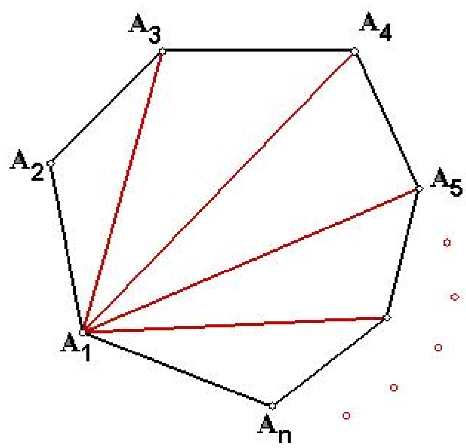
= (-2+30)/2 =14

n= $\frac{-2-30}{2}$ implies n = -16(It is not possible)

So n = 14, that is the time he will take to clear his loan is 14 month

Q#.14: The sum of interior angles of polygons having sides 3,4,5,…etc. form an A.P. find the sum of the interior angles for a 16 sided polygone.

Solution (14) Since 3,4,5,………………,16 sided



®,2®,3®.......a_{14} $a_1=$ ® $d=$ 2® − ® = ®

$\because\ an=a_1+(n-1)d$

For n = 14

$a_{14}=a_1+(14-1)d$

= ® + 13®

= 14®

Alternative: If n-sided polygon then sum of interior angle is = $(n-2)$®

For n=16

a_{14}= $(16-2)$® = 14®

Q#.15: A student saves Rs. 12 at the end to the first week and goes on increasing his saving Rs. 4 weekly. After how many weeks will be able to save Rs. 2100?

Solution (15) Given that

$a_1=12;d=4;S_n=2100$ n=? (n=number of weeks)

$\because S_n=\frac{n}{2}(2a_1+(n-1)d)$,

$2100 = \frac{n}{2}\left(2(12)+(n-1)4\right),$

$4200 = n(21+4n-4)$

$4200 = 20n+4n^2$

$4n^2+20n-4200=0$

$4(n+5n-1050)=0$

$\because 4 \neq 0,$

$\therefore n^2+5n-1050=0$

$n = \frac{-5 \pm \sqrt{(5)^2-4(1)(-1050)}}{2(1)} = \frac{-5 \pm 65}{2}$

Either $n = \frac{-5-65}{2}$ or $n = \frac{-5+15}{2}$

n=-35 (It is not possible) ; n=30 So n=30 Required answer

Q#.16: Find the 11th term of the sequence, 1+i,2,4/1+i

Solution (16) Since $a = 1+i$ *and* $r = \frac{2}{1+i}$

$a_{11} = a_1 r^{10}$

$= (1+i)\left(\frac{2}{1+i}\right)^{10}$ $= (1+i)\left(\frac{2}{1+i} \times \frac{1-i}{1-i}\right)^{10}$ $= (1+i)\left(\frac{2(1-i)}{2}\right)^{10}$

$= (1+i)(1-i)^{10}$ $= (1+i)\left[(1-i)^2\right]^5$, $= (1+i)(1-2i+i^2)^5$,
$= (1+i)(1-2i-1)^5$

$= (1+i)(-2i)^5$ $= (1+i)(-2)^5\ i^5$ $= (1+i)(-32)\ i^2, i^2, i$

$= -32(1+i)(-1,(-1))$

$= -32i(1+i)$ $= -32(i+1)$ $= 32(1-i)$

Q#.17: If 1/a,1/b and 1/c are in G.P. Show that the common ratio is $\pm\sqrt{\frac{a}{c}}$.

Solution (17) $\because \frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ *are in G.P*

$\because r = \frac{1/b}{1/a} = \frac{a}{b} \rightarrow (i)$

$r = \frac{1/c}{1/b} = \frac{b}{c} \rightarrow (ii)$

$r.r = \frac{a}{b}.\frac{b}{c}$ by Multiplying equation (i) and (ii)

$$\sqrt{r^2} = \sqrt{\frac{a}{c}}$$

$$r = \pm\sqrt{\frac{a}{c}}.$$ Hence Proved

Q#.18: If a,b,c,d are in G.P, prove that $a^2-b^2, b^2-c^2, c^2-d^2$ are in G.P.

Solution (18) $\because a,b,c,d \quad are \;\; in \;\; G.P$

$r=\dfrac{b}{a}$ $\qquad$ $r=\dfrac{c}{b}$ $\qquad$ $r=\dfrac{d}{c}$

b=ar ………(i) $\qquad$ c=br $\qquad$ d=cr

c=(ar)r (using (i)) $\qquad$ d= (ar²)r (using (ii))

$=ar^2$ .....(ii) $\qquad$ $=ar^3$

To Show $\quad$ $a^2-b^2, b^2-c^2, c^2-a^2$ are in G.P

$$\frac{b^2-c^2}{a^2-b^2}=\frac{c^2-d^2}{b^2-c^2}, \qquad \frac{(ar)^2-(ar^2)^2}{a^2-(ar)^2}=\frac{(ar^2)^2-(ar^3)^2}{(ar)^2-(ar^2)^2}$$

$$\frac{a^2r^2(1-r^2)}{a^2(1-r^2)}=\frac{a^2r^4(1-r^2)}{a^2r^2(1-r^2)}$$

$$r^2=r^2$$

So $\quad$ a^2-b^2, b^2-c^2, c^2-d^2 are in G.P

Q#.19: Find the nth term of the geometric sequence if; $\dfrac{a_5}{a_3}=\dfrac{4}{9}$ and $a_2=\dfrac{4}{9}$.

Solution (19) $\because \dfrac{a_5}{a_3}=\dfrac{4}{9}$, $\qquad \dfrac{a_1r^4}{a_1r^2}=\dfrac{4}{9}$, $\qquad r^2=\dfrac{4}{9}$, $\qquad \sqrt{r^2}=\sqrt{\dfrac{4}{9}}$

$r=\pm\dfrac{2}{3}$, $\qquad a_2=\dfrac{4}{9}$, $\qquad \Rightarrow \; a_1r=\dfrac{4}{9}, \Rightarrow a_1\left(\pm\dfrac{2}{3}\right)=\dfrac{4}{9}, \Rightarrow a_1=\dfrac{4}{9}\times\left(\pm\dfrac{3}{2}\right), \Rightarrow a_1=\pm\dfrac{2}{3}$

$$a_n=a_1r^{n-1}=\left(\pm\frac{2}{3}\right)^1\left(\pm\frac{2}{3}\right)^{n-1}=(-1)^n\left(\frac{2}{3}\right)^n$$

Q#.20: If both x and y are positive distinct real numbers, show that the geometric mean between x and y is less than their arithmetic mean.

Solution (20) Show That $\qquad$ G.M < A.M $\quad \Rightarrow$ 0<A.M-G.M

For two real numbers x & y $\qquad$ A.M-G.M $\qquad \Rightarrow \quad \dfrac{x+y}{2}-\sqrt{xy}$

$$\frac{x+y-2\sqrt{xy}}{2} \qquad =\frac{(\sqrt{x})^2+(\sqrt{y})^2-2\sqrt{x\sqrt{y}}}{2} \qquad =\frac{(\sqrt{x}-\sqrt{y})^2}{2}>0$$

$\Rightarrow$ A.M-G.M>0 $\qquad$ or $\qquad$ A.M > G.M.

Q#.21: Find the sum of n terms of the geometric series if $a_n = (-3)(2/5)^n$.

Solution (21) $\because a_n = (-3)\left(\frac{2}{5}\right)^n = (-3)\left(\frac{2}{5}\right)\left(\frac{2}{5}\right)^{n-1} = \left(-\frac{6}{5}\right)\left(\frac{2}{5}\right)^{n-1}$

$a_n = \left(-\frac{6}{5}\right)\left(\frac{2}{5}\right)^{n-1}$ $\quad \because a_n = a_1 r^{n-1}$ $\quad \left. a_1 = -\frac{6}{5} \right\}$ by comparing

$r = \frac{2}{5} < 1$

Q#.22: Write down the condition for convergence of infinite geometric series?

OR

Under what condition an infinite geometric is convergent or divergent?

Solution (22) An infinite geometric series is convergent if $|r| < 1$

r=common ratio

An infinite geometric series is divergent if |r|>1

Q#.23: Derive a formula for sum of infinite geometric series if |r|<1.

Solution (23) We know that $S_n = \frac{a_1(1-r^n)}{1-r}$ if $|r| < 1$

Then $r^n \to 0$ when $n \to \infty$ So we conclude that

$Sn \to \frac{a_1}{1-r}$ when $n \to \infty$

Thus S=lim $S_n = \frac{a_1}{1-r}$, where S is the sum of $n \to \infty$

infinite geometric series having $|r| < 1$

Q#.24: If a=1-x+x²-x³+… x<1 b=1+x+x²+x³+… x<1

Then show that 2ab = a + b

Solution (24) $a = \frac{1}{1-(-x)} = \frac{1}{1+x}$

$\Rightarrow 1 + x = \frac{1}{a}$........$(i)$ And

$b = \frac{1}{1-x}\left(\because S_\infty = \frac{a_1}{1-r}\right)$, and r = x

$\Rightarrow 1 - x = \frac{1}{b}$........$(ii)$

Adding (i) and (ii)

$\frac{1}{a} + \frac{1}{b} = 2 + x + 1 - x \quad \Rightarrow \quad \frac{a+b}{ab} = 2,$

a+b=2ab

Q#.25: Sum to n terms the series; $0.2+0.22+0.222+........$

Solution (25) $S_n = 0.2 + 0.22 + 0.222 + \ldots\ldots\ldots$ to n terms

$=2\{0.1+0.11+0.111+......to\ n\ terms\}$ $=\frac{2}{9}\{0.9+0.99+0.999+.....\text{to n terms}\}$

$=\frac{2}{9}\{(1-0.1)+(1-0.01)+(1-0.001)+......\text{to n terms}\}$

$=\frac{2}{9}\{(1+1+1+1+.....\text{to n terms})-(0.1+0.01+0.001+.....n\,terms)\}$

$$=\frac{2}{9}\left\{n-\frac{\frac{1}{10}\left\{1-\left(\frac{1}{10}\right)^n\right\}}{1-\frac{1}{10}}\right\}=\frac{2}{9}\left\{n-\frac{1}{9}\left(1-\frac{1}{10^n}\right)\right\}$$

Q#.26: Find the sum up to infinite terms of geometric series;

$4+2\sqrt{2}+2+\sqrt{2}+1+.....$

Solution (26) Let S be the sum of infinite series

$4+2\sqrt{2}+2+\sqrt{2}+1+..........$

Here $a_1=4$ and $r=\frac{2\sqrt{2}}{4}=\frac{\sqrt{2}}{2}=\frac{\sqrt{2}}{\sqrt{2}\sqrt{2}}=\frac{1}{\sqrt{2}}$ So $S=\frac{4}{1-\frac{1}{\sqrt{2}}}$ using $S_\infty=\frac{a_1}{1-r}$

$=\frac{4}{\frac{\sqrt{2}-1}{\sqrt{2}}}$ $=\frac{\sqrt{2}}{\sqrt{2}-1}\times\frac{\sqrt{2}+1}{\sqrt{2}+1}$ $=\frac{4(2+\sqrt{2})}{2-1}=4(2+\sqrt{2})$

Q#.27: Find vulgar fractions equivalent to the $1.1\dot{4}\dot{7}$

Solution (27) $1.1\dot{4}\dot{7}$=1.474747 ……….

= 1.1+0.47+0.0047+0.000047+………

=1.1+(0.047+0.0047+0.000047+…….), $=1.1+\left(\frac{a}{1-r}\right)$

$=1.1+\left(\frac{0.047}{1-0.01}\right)$ $\because r=\frac{0.047}{0.47}$ $=1.1+\frac{0.47}{0.99}$ =0.01

$=\frac{1089+47}{990}=\frac{1136}{990}$

Q#.28: If y=1+2x+4x2+8x3+…

i) Show that x= (y-1)/2y.

ii) Find the interval in which the series is convergent.

Solution (28) $(i)\ \because\ y = 1 + 2x + 4x^2 + 8x^3 + ...$

$y = S_\infty,\quad y = \dfrac{a}{1-r}$ $\qquad \because r = \dfrac{2x}{1} = 2x\ ;\quad y = \dfrac{1}{1-2x}$ $\qquad y(1-2x) = 1$

$y - 2yx = 1 \qquad y - 1 = 2xy \qquad x = \dfrac{y-1}{2y}$

ii) For convergent series

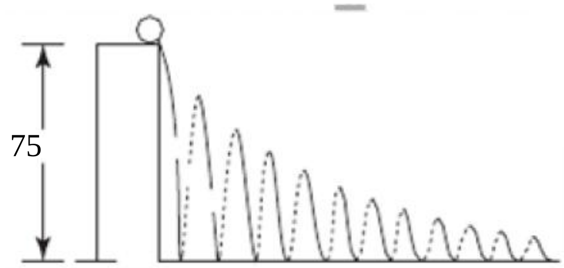
$-1<r<1$

$-1 < 2x < 1$ Dividing by ‘2’

$-1/2 < x < 1/2$ is required interval of convergent

Q#.29: What distance will a ball travel before coming to rest if it is dropped from a height of 75m and after each fall it rebound $\frac{2}{5}$ of the distance it fell?

Solution (29)



$75 \times \dfrac{2}{5} = 30 = 30 \times 2 = 60$

$30 \times \dfrac{2}{5} = 12 = 12 \times 2 = 24$

$12 \times \dfrac{2}{5} = 4.8 = 4.8 \times 2 = 9.6$

So sequence is $75 + 60 + 24 + 9.6 + ...$

$= 75 + (S_\infty) \qquad = 75 + \dfrac{a}{1-r}$

$r = \dfrac{24}{60} = \dfrac{2}{5} \qquad = 75 + \dfrac{60}{1 - \frac{2}{5}}$

$= 75 + \dfrac{300}{3} = 175m$

Q#.30: Prove that G^2=A*H.

Solution (30) $\because G = \sqrt{ab}$ **and** $A = \dfrac{a+b}{2}$, $H = \dfrac{2ab}{a+b}$

So $G^2 = A \times H$

$\left(\sqrt{ab}\right)^2 = \left(\dfrac{a+b}{2}\right) \times \left(\dfrac{2ab}{a+b}\right)$

ab = ab $\qquad$ Hence $\quad G^2 = A \times H$

Q#.31: Prove that A > G > H if a, b>0 (G>0).

Solution (31) We first show that

A > G

$\Rightarrow \dfrac{a+b}{2} > \sqrt{ab} \qquad \Rightarrow a+b > 2\sqrt{ab} \qquad \Rightarrow a+b-2\sqrt{ab} > 0$

$\Rightarrow \left(\sqrt{a}\right)^2 + \left(\sqrt{b}\right)^2 - 2\sqrt{a}\sqrt{b} > 0 \qquad \Rightarrow \left(\sqrt{a}+\sqrt{b}\right)^2 > 0$ (true)

So A > G …………. (i)

Now G > H $\qquad \sqrt{ab} > \dfrac{2ab}{a+b} \qquad a+b > 2\sqrt{ab} \qquad a+b-2\sqrt{a}\sqrt{b} > 0$

$\left(\sqrt{a}-\sqrt{b}\right)^2 > 0 \quad (true)$ So G > H …………(ii)

from (i) & (ii) A > G > H

Q#.32: If A<G<H if a,b<0 (G<0)

Solution (32) We first show that

A < G If $\dfrac{a+b}{2} < -\sqrt{ab}$

Let a= -m and b=-n where m and n are positive real numbers, then

$\dfrac{-m-n}{2} < -\sqrt{(-m)(-n)}$ Or $-\dfrac{m+n}{2} < -\sqrt{mn}$

$\Rightarrow \left(\sqrt{m}-\sqrt{n}\right)^2 > 0$ Which is true i.e A < G similarly we can prove that

A < G

So A < G < H

Q#.33: If 5 is the harmonic mean between 2 and b, Find b.

Solution (33) Since H.M=5 & a=2

$\dfrac{2ab}{a+b} = 5 \qquad \dfrac{2(2)b}{2+b} = 5 \qquad 4b = 5(2+b) \qquad 4b = 5(2+b)$

$4b = 10 + 5b \qquad -10 = 5b - 4b$ -10=b

Q#.34: If the numbers 1/k,1/2k+1 and 1/4k-1 are in harmonic sequence, find k.

Solution (34) Since $\dfrac{1}{k}, \dfrac{1}{2k+1}, \dfrac{1}{4k+1}, \ldots\ldots\ldots\ldots \to H.P$

$K\ ,2k+1, 4k-1, \ldots\ldots\ldots \to A.P \qquad d = 2k+1-k = k+1$

& $d = 4k+1-2k-1 = 2k-2$ $\because$ Numbers are in A.P

$\because\ d = d \qquad K+1 = 2K-2 \qquad 1+2 = 2k-k$ 3=K

Q#.35: Given n th term of the series, find the sum to $2n$ term; $3n^2+2n+1$

Solution (35) Let S_n denotes the sum of 2n terms of the series

using $S_n=\sum_{k=1}^{n}T_k$ $=\sum_{k=1}^{n}\left(3k^2+2k+1\right)$

$=3\sum_{k=1}^{n}k^2+2\sum_{k=1}^{n}k+\sum_{k=1}^{n}1$ $=\frac{3\ n(2n+1)(n+1)}{6}+2\frac{n(n+1)}{2}+n$

$=\frac{n(2n+1)(n+1}{2}+n(n+1)+n$ Replacing 'n' by '2n'

$S_{2n}=\frac{2n(2(2n)+1(2n+1)}{2}+2n(2n+1)+2n$

$=n(4n+1)(2n+1)+2n(2n+1)+2n$ $=n\{8n^2+4n+2n+1+4n+2+2\}$

$=n\{8n^2+10n+5\}$

Q#.36: Given n th term of the series, find the sum to $2n$ term; n^3+2n+3

Solution (36) Let S_n denote the sum of n terms of the series, using

$S_n=\sum_{k=1}^{n}T_k$ $=\sum_{k=1}^{n}\left(k^3+2k+3\right)$ $=\sum_{k=1}^{n}k^3+2\sum_{k=1}^{n}k+3\sum_{k=1}^{n}1$

$=\left[\frac{n(n+1}{2}\right]^2+2.\frac{n(n+1)}{2}+3n$ $=n\left\{n\frac{(n+1)^2}{4}+(n+1)+3\right\}$

$=n\left\{\frac{n^3+2n^2+n}{4}+n+4\right\}$ Put n=2n

$=2n\left\{\frac{(2n)^3+2(2n)^2+2n}{4}+2n+4\right\}$ $=2n\left\{\frac{8n3+8n2+2n+8n+16}{4}\right\}$

$=\frac{n}{2}\{8n^3+8n^2+10n+16\}$ $=n\{4n^3+4n^2+5n+8\}$

Q#.37: Prove that $\sum_{k=1}^{n}k^2=\frac{n(n+1)(2n+1)}{6}$

Solution (37) We Know that

$k^2-(k-1)^2=2k-1$ …………..(A)

Taking summation on both sides of eq(A)

$\sum_{k=1}^{n}\left[k^2-(k-1)^2\right]=\sum_{k=1}^{n}(2k-1)$

i.e $n^2=2\sum_{k=1}^{n}k-n$ $\because\sum_{k=1}^{n}1=n$ $2\sum_{k=1}^{n}k=n^2+n$

$\sum_{k=1}^{n}k=\frac{n(n+1)}{2}$