

## Binomial Theorem

If  $a$  and  $x$  are two real number and  $n$  is a positive integer then

$$(a+x)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}x^1 + \binom{n}{2}a^{n-2}x^2 + \dots + \binom{n}{n-1}ax^{n-1} + \binom{n}{n}x^n$$

### Proof

We will use mathematical induction to prove this so let  $S(n)$  be the given statement.

Put  $n=1$

$$S(1): (a+x)^1 = \binom{1}{0}a^1 + \binom{1}{1}a^{1-1}x^1 = (1)a + (1)(1)x \Rightarrow a+x = a+x$$

$S(1)$  is true so condition I is satisfied.

Now suppose that  $S(n)$  is true for  $n=k$ .

$$S(k): (a+x)^k = \binom{k}{0}a^k + \binom{k}{1}a^{k-1}x^1 + \binom{k}{2}a^{k-2}x^2 + \dots + \binom{k}{k-1}ax^{k-1} + \binom{k}{k}x^k \dots (i)$$

The statement for  $n=k+1$

$$S(k+1): (a+x)^{k+1} = \binom{k+1}{0}a^{k+1} + \binom{k+1}{1}a^{k+1-1}x^1 + \binom{k+1}{2}a^{k+1-2}x^2 + \dots$$

$$+ \binom{k+1}{k+1-1}ax^{k+1-1} + \binom{k+1}{k+1}x^{k+1}$$

$$\Rightarrow (a+x)^{k+1} = \binom{k+1}{0}a^{k+1} + \binom{k+1}{1}a^kx^1 + \binom{k+1}{2}a^{k-1}x^2 + \dots$$

$$+ \binom{k+1}{k}ax^k + \binom{k+1}{k+1}x^{k+1}$$

Multiplying both sides of equation (i) by  $(a+x)$

$$(a+x)^k(a+x) = \left( \binom{k}{0}a^k + \binom{k}{1}a^{k-1}x^1 + \binom{k}{2}a^{k-2}x^2 + \dots + \binom{k}{k-1}ax^{k-1} + \binom{k}{k}x^k \right) (a+x)$$

$$\Rightarrow (a+x)^{k+1} = \left( \binom{k}{0}a^k + \binom{k}{1}a^{k-1}x^1 + \binom{k}{2}a^{k-2}x^2 + \dots + \binom{k}{k-1}ax^{k-1} + \binom{k}{k}x^k \right) (a)$$

$$+ \left( \binom{k}{0}a^k + \binom{k}{1}a^{k-1}x^1 + \binom{k}{2}a^{k-2}x^2 + \dots + \binom{k}{k-1}ax^{k-1} + \binom{k}{k}x^k \right) (x)$$

$$\Rightarrow (a+x)^{k+1} = \binom{k}{0}a^{k+1} + \binom{k}{1}a^kx^1 + \binom{k}{2}a^{k-1}x^2 + \dots + \binom{k}{k-1}a^2x^{k-1} + \binom{k}{k}ax^k$$

$$+ \binom{k}{0}a^kx + \binom{k}{1}a^{k-1}x^2 + \binom{k}{2}a^{k-2}x^3 + \dots + \binom{k}{k-1}ax^k + \binom{k}{k}x^{k+1}$$

$$\Rightarrow (a+x)^{k+1} = \binom{k}{0}a^{k+1} + \left( \binom{k}{1} + \binom{k}{0} \right) a^kx^1 + \left( \binom{k}{2} + \binom{k}{1} \right) a^{k-1}x^2 + \dots$$

$$+\left(\binom{k}{k} + \binom{k}{k-1}\right)ax^k + \binom{k}{k}x^{k+1}$$

$$\text{Since } \binom{n}{0} = \binom{n+1}{0}, \quad \binom{n}{r} + \binom{n}{r-1} = \binom{n+1}{r} \quad \text{and} \quad \binom{n}{n} = \binom{n+1}{n+1}$$

$$\Rightarrow (a+x)^{k+1} = \binom{k+1}{0}a^{k+1} + \binom{k+1}{1}a^kx + \binom{k+1}{2}a^{k-1}x^2 + \dots + \binom{k+1}{k}ax^k + \binom{k+1}{k+1}x^{k+1}$$

Thus  $S(k+1)$  is true when  $S(k)$  is true so condition II is satisfied and  $S(n)$  is true for all positive integral value of  $n$ .

### Question # 1

Using binomial theorem, expand the following:

$$(i) (a+2b)^5 \quad (ii) \left(\frac{x}{2} - \frac{2}{x^2}\right)^6 \quad (iii) \left(3a - \frac{x}{3a}\right)^4$$

$$(iv) \left(2a - \frac{x}{a}\right)^7 \quad (v) \left(\frac{x}{2y} - \frac{2y}{x}\right)^8 \quad (vi) \left(\sqrt{\frac{a}{x}} - \sqrt{\frac{x}{a}}\right)^6$$

### Solution

(i)

$$\begin{aligned} (a+2b)^5 &= \binom{5}{0}a^5 + \binom{5}{1}a^{5-1}(2b)^1 + \binom{5}{2}a^{5-2}(2b)^2 + \binom{5}{3}a^{5-3}(2b)^3 + \binom{5}{4}a^{5-4}(2b)^4 + \binom{5}{5}a^{5-5}(2b)^5 \\ &= (1)a^5 + (5)a^4(2b) + (10)a^3(4b^2) + (10)a^2(8b^3) + (5)a^1(16b^4) + (1)a^0(32b^5) \\ &= a^5 + 10a^4b + 40a^3b^2 + 80a^2b^3 + 80ab^4 + 32b^5 \quad \because a^0 = 1 \end{aligned}$$

$$\begin{aligned} (ii) \left(\frac{x}{2} - \frac{2}{x^2}\right)^6 &= \binom{6}{0}\left(\frac{x}{2}\right)^6 + \binom{6}{1}\left(\frac{x}{2}\right)^{6-1}\left(-\frac{2}{x^2}\right)^1 + \binom{6}{2}\left(\frac{x}{2}\right)^{6-2}\left(-\frac{2}{x^2}\right)^2 + \binom{6}{3}\left(\frac{x}{2}\right)^{6-3}\left(-\frac{2}{x^2}\right)^3 \\ &\quad + \binom{6}{4}\left(\frac{x}{2}\right)^{6-4}\left(-\frac{2}{x^2}\right)^4 + \binom{6}{5}\left(\frac{x}{2}\right)^{6-5}\left(-\frac{2}{x^2}\right)^5 + \binom{6}{6}\left(\frac{x}{2}\right)^{6-6}\left(-\frac{2}{x^2}\right)^6 \\ &= (1)\left(\frac{x}{2}\right)^6 - (6)\left(\frac{x}{2}\right)^5\left(\frac{2}{x^2}\right) + (15)\left(\frac{x}{2}\right)^4\left(\frac{2}{x^2}\right)^2 - (20)\left(\frac{x}{2}\right)^3\left(\frac{2}{x^2}\right)^3 \\ &\quad + (15)\left(\frac{x}{2}\right)^2\left(\frac{2}{x^2}\right)^4 - (6)\left(\frac{x}{2}\right)^1\left(\frac{2}{x^2}\right)^5 + (1)(1)\left(\frac{2}{x^2}\right)^6 \\ &= \left(\frac{x^6}{64}\right) - 6\left(\frac{x^5}{32}\right)\left(\frac{2}{x^2}\right) + 15\left(\frac{x^4}{16}\right)\left(\frac{4}{x^4}\right) - 20\left(\frac{x^3}{8}\right)\left(\frac{8}{x^6}\right) \\ &\quad + 15\left(\frac{x^2}{4}\right)\left(\frac{16}{x^8}\right) - 6\left(\frac{x}{2}\right)\left(\frac{32}{x^{10}}\right) + \left(\frac{64}{x^{12}}\right) \\ &= \frac{x^6}{64} - \frac{3x^3}{8} + \frac{15}{4} - \frac{20}{x^3} + \frac{60}{x^6} - \frac{96}{x^9} + \frac{64}{x^{12}} \end{aligned}$$

(iii) *Do yourself*(iv) *Do yourself*(v) *Do yourself*

$$\begin{aligned}
\text{(vi)} \quad \left(\sqrt{\frac{a}{x}} - \sqrt{\frac{x}{a}}\right)^6 &= \binom{6}{0} \left(\sqrt{\frac{a}{x}}\right)^6 + \binom{6}{1} \left(\sqrt{\frac{a}{x}}\right)^{6-1} \left(-\sqrt{\frac{x}{a}}\right)^1 + \binom{6}{2} \left(\sqrt{\frac{a}{x}}\right)^{6-2} \left(-\sqrt{\frac{x}{a}}\right)^2 + \binom{6}{3} \left(\sqrt{\frac{a}{x}}\right)^{6-3} \\
&\quad \left(-\sqrt{\frac{x}{a}}\right)^3 + \binom{6}{4} \left(\sqrt{\frac{a}{x}}\right)^{6-4} \left(-\sqrt{\frac{x}{a}}\right)^4 + \binom{6}{5} \left(\sqrt{\frac{a}{x}}\right)^{6-5} \left(-\sqrt{\frac{x}{a}}\right)^5 + \binom{6}{6} \left(\sqrt{\frac{a}{x}}\right)^{6-6} \left(-\sqrt{\frac{x}{a}}\right)^6 \\
&= (1) \left(\sqrt{\frac{a}{x}}\right)^6 - (6) \left(\sqrt{\frac{a}{x}}\right)^5 \left(\sqrt{\frac{x}{a}}\right)^1 + (15) \left(\sqrt{\frac{a}{x}}\right)^4 \left(\sqrt{\frac{x}{a}}\right)^2 - (20) \left(\sqrt{\frac{a}{x}}\right)^3 \left(\sqrt{\frac{x}{a}}\right)^3 - \\
&\quad \left(\sqrt{\frac{x}{a}}\right)^3 + (15) \left(\sqrt{\frac{a}{x}}\right)^2 \left(\sqrt{\frac{x}{a}}\right)^4 - (6) \left(\sqrt{\frac{a}{x}}\right)^1 \left(\sqrt{\frac{x}{a}}\right)^5 + (1) \left(\sqrt{\frac{a}{x}}\right)^0 \left(\sqrt{\frac{x}{a}}\right)^6 \quad = \\
&\quad \left(\sqrt{\frac{a}{x}}\right)^6 - 6 \left(\sqrt{\frac{a}{x}}\right)^5 \left(\sqrt{\frac{a}{x}}\right)^{-1} + 15 \left(\sqrt{\frac{a}{x}}\right)^4 \left(\sqrt{\frac{a}{x}}\right)^{-2} - 20 \left(\sqrt{\frac{a}{x}}\right)^3 \left(\sqrt{\frac{a}{x}}\right)^{-3} \\
&\quad + 15 \left(\sqrt{\frac{x}{a}}\right)^{-2} \left(\sqrt{\frac{x}{a}}\right)^4 - 6 \left(\sqrt{\frac{x}{a}}\right)^{-1} \left(\sqrt{\frac{x}{a}}\right)^5 + 1(1) \left(\sqrt{\frac{x}{a}}\right)^6 \\
&= \left(\sqrt{\frac{a}{x}}\right)^6 - 6 \left(\sqrt{\frac{a}{x}}\right)^{5-1} + 15 \left(\sqrt{\frac{a}{x}}\right)^{4-2} - 20 \left(\sqrt{\frac{a}{x}}\right)^{3-3} + 15 \left(\sqrt{\frac{x}{a}}\right)^{-2+4} - 6 \left(\sqrt{\frac{x}{a}}\right)^{-1+5} + 1 \left(\sqrt{\frac{x}{a}}\right)^6 \\
&= \left(\sqrt{\frac{a}{x}}\right)^6 - 6 \left(\sqrt{\frac{a}{x}}\right)^4 + 15 \left(\sqrt{\frac{a}{x}}\right)^2 - 20 \left(\sqrt{\frac{a}{x}}\right)^0 + 15 \left(\sqrt{\frac{x}{a}}\right)^2 - 6 \left(\sqrt{\frac{x}{a}}\right)^4 + \left(\sqrt{\frac{x}{a}}\right)^6 \\
&= \left(\left(\frac{a}{x}\right)^{\frac{1}{2}}\right)^6 - 6 \left(\left(\frac{a}{x}\right)^{\frac{1}{2}}\right)^4 + 15 \left(\left(\frac{a}{x}\right)^{\frac{1}{2}}\right)^2 - 20(1) + 15 \left(\left(\frac{x}{a}\right)^{\frac{1}{2}}\right)^2 - 6 \left(\left(\frac{x}{a}\right)^{\frac{1}{2}}\right)^4 + \left(\left(\frac{x}{a}\right)^{\frac{1}{2}}\right)^6 \\
&= \left(\frac{a}{x}\right)^3 - 6 \left(\frac{a}{x}\right)^2 + 15 \left(\frac{a}{x}\right) - 20 + 15 \left(\frac{x}{a}\right) - 6 \left(\frac{x}{a}\right)^2 + \left(\frac{x}{a}\right)^3 \\
&= \frac{a^3}{x^3} - 6 \frac{a^2}{x^2} + 15 \frac{a}{x} - 20 + 15 \frac{x}{a} - 6 \frac{x^2}{a^2} + \frac{x^3}{a^3}
\end{aligned}$$

**Question # 2**

Calculate the following by means of binomial theorem:

(i)  $(0.97)^3$       (ii)  $(2.02)^4$       (iii)  $(9.98)^4$       (iv)  $(2.1)^5$

**Solution** (i)  $(0.97)^3 = (1 - 0.03)^3$ 

$$= \binom{3}{0} (1)^3 + \binom{3}{1} (1)^2 (-0.03) + \binom{3}{2} (1)^1 (-0.03)^2 + \binom{3}{3} (-0.03)^3$$

$$= (1)(1) + 3(1)(-0.03) + 3(1)(0.0009) + (1)(-0.000024)$$

$$= 1 - 0.09 + 0.0027 - 0.000027 = 0.912673$$

(ii)  $(2.02)^4 = (2 + 0.02)^4$       *Now do yourself.*

(iii)  $(9.98)^4 = (10 - 0.02)^4$

$$= \binom{4}{0}(10)^4 + \binom{4}{1}(10)^3(-0.02) + \binom{4}{2}(10)^2(-0.02)^2 + \binom{4}{3}(10)^1(-0.02)^3$$

$$+ \binom{4}{4}(10)^0(-0.02)^4$$

$$= (1)(10000) + 4(1000)(-0.02) + 6(100)(0.0004) + 4(10)(-0.000008)$$

$$+ (1)(1)(0.00000016)$$

$$= 10000 - 80 + 0.24 - 0.00032 + 0.00000016 = 9920.23968$$

(iv)  $(2.1)^5 = (2 + 0.1)^5$       *Now do yourself.*

**Question # 3**

Expand and simplify the following:

(i)  $(a + \sqrt{2}x)^4 + (a - \sqrt{2}x)^4$       (ii)  $(2 + \sqrt{3})^5 + (2 - \sqrt{3})^5$

(iii)  $(2 + i)^5 - (2 - i)^5$       (iv)  $(x + \sqrt{x^2 - 1})^3 + (x - \sqrt{x^2 - 1})^3$

**Solution** (i)  $(a + \sqrt{2}x)^4 + (a - \sqrt{2}x)^4$

We take

$$(a + \sqrt{2}x)^4$$

$$= \binom{4}{0}a^4 + \binom{4}{1}a^3(\sqrt{2}x) + \binom{4}{2}a^2(\sqrt{2}x)^2 + \binom{4}{3}a(\sqrt{2}x)^3 + \binom{4}{4}a^0(\sqrt{2}x)^4$$

$$= (1)a^4 + (4)a^3(\sqrt{2}x) + (6)a^2(2x^2) + (4)a(2\sqrt{2}x^3) + (1)(1)(4x^4)$$

$$\Rightarrow (a + \sqrt{2}x)^4 = a^4 + 4\sqrt{2}a^3x + 12a^2x^2 + 8\sqrt{2}ax^3 + 4x^4 \dots\dots\dots (i)$$

Replacing  $\sqrt{2}$  by  $-\sqrt{2}$  in eq. (i)

$$(a - \sqrt{2}x)^4 = a^4 + 4(-\sqrt{2})a^3x + 12a^2x^2 + 8(-\sqrt{2})ax^3 + 4x^4$$

$$= a^4 - 4\sqrt{2}a^3x + 12a^2x^2 - 8\sqrt{2}ax^3 + 4x^4 \dots\dots\dots (ii)$$

Adding (i) & (ii)

$$(a + \sqrt{2}x)^4 + (a - \sqrt{2}x)^4 = 2a^4 + 24a^2x^2 + 8x^4$$

(ii) *Do yourself.*

(iii) Since

$$\begin{aligned}
 (2+i)^5 &= \binom{5}{0}2^5 + \binom{5}{1}2^{5-1}i + \binom{5}{2}2^{5-2}i^2 + \binom{5}{3}2^{5-3}i^3 + \binom{5}{4}2^{5-4}i^4 + \binom{5}{5}2^{5-5}i^5 \\
 &= (1)2^5 + (5)2^4i + (10)2^3i^2 + (10)2^2i^3 + (5)2^1i^4 + (1)2^0i^5 \\
 &= 32 + 80i + 80i^2 + 40i^3 + 10i^4 + i^5 \dots\dots\dots (i)
 \end{aligned}$$

Replacing  $i$  by  $-i$  in eq. (i)

$$\begin{aligned}
 (2+i)^5 &= 32 + 80(-i) + 80(-i)^2 + 40(-i)^3 + 10(-i)^4 + (-i)^5 \\
 &= 32 - 80i + 80i^2 - 40i^3 + 10i^4 - i^5 \dots\dots\dots(ii)
 \end{aligned}$$

Subtracting (i) & (ii)

$$\begin{aligned}
 (2+i)^5 - (2-i)^5 &= 160i + 80i^3 + 2i^5 \\
 &= 160i + 80(-1) \cdot i + 2(-1)^2 \cdot i \\
 &= 160i - 80i + 2i = 82i
 \end{aligned}$$

(iv)  $\left(x + \sqrt{x^2 - 1}\right)^3 + \left(x - \sqrt{x^2 - 1}\right)^3$

Suppose  $t = \sqrt{x^2 - 1}$  then

$$\begin{aligned}
 \left(x + \sqrt{x^2 - 1}\right)^3 + \left(x - \sqrt{x^2 - 1}\right)^3 &= (x+t)^3 + (x-t)^3 \\
 &= \left((x)^3 + 3(x)^2(t) + 3(x)(t)^2 + (t)^3\right) + \left((x)^3 + 3(x)^2(-t) + 3(x)(-t)^2 + (-t)^3\right) \\
 &= x^3 + 3x^2t + 3xt^2 + t^3 + x^3 - 3x^2t + 3xt^2 - t^3 \\
 &= 2x^3 + 6xt^2 \\
 &= 2x^3 + 6x\left(\sqrt{x^2 - 1}\right)^2 \quad \because t = \sqrt{x^2 - 1} \\
 &= 2x^3 + 6x(x^2 - 1) = 2x^3 + 6x^3 - 6x = 8x^3 - 6x
 \end{aligned}$$

**Question # 4**

Expand the following in ascending powers of  $x$ :

- (i)  $(2 + x - x^2)^4$                       (ii)  $(1 - x + x^2)^4$                       (iii)  $(1 - x - x^2)^4$

**Solution** (i)  $(2 + x - x^2)^4$

Put  $t = 2 + x$  then

$$\begin{aligned}
 (2 + x - x^2)^4 &= (t - x^2)^4 \\
 &= \binom{4}{0}(t)^4 + \binom{4}{1}(t)^3(-x^2) + \binom{4}{2}(t)^2(-x^2)^2 + \binom{4}{3}(t)^1(-x^2)^3 + \binom{4}{4}(t)^0(-x^2)^4 \\
 &= (1)(t)^4 - (4)(t)^3(x^2) + (6)(t)^2(x^4) - (4)(t)(x^6) + (1)(1)(x^8) \\
 &= t^4 - 4t^3x^2 + 6t^2x^4 - 4tx^6 + x^8 \dots\dots\dots (i)
 \end{aligned}$$

Now

$$\begin{aligned}
 t^4 &= (2 + x)^4 = \binom{4}{0}(2)^4 + \binom{4}{1}(2)^3(x) + \binom{4}{2}(2)^2(x)^2 + \binom{4}{3}(2)^1(x)^3 + \binom{4}{4}(2)^0(x)^4 \\
 &= (1)(16) + (4)(8)(x) + (6)(4)(x^2) + (4)(2)(x^3) + (1)(1)(x^4) \\
 &= 16 + 32x + 24x^2 + 8x^3 + x^4
 \end{aligned}$$

Also

$$\begin{aligned}
 t^3 &= (2+x)^3 = (2)^3 + (3)(2)^2(x) + (3)(2)^1(x)^2 + (x)^3 \\
 &= 8 + 12x + 6x^2 + x^3 \\
 t^2 &= (2+x)^2 = 4 + 4x + x^2
 \end{aligned}$$

Putting values of  $t^4, t^3, t^2$  and  $t$  in equation (i)

$$\begin{aligned}
 (2+x-x^2)^4 &= (16+32x+24x^2+8x^3+x^4) - 4(8+12x+6x^2+x^3)x^2 \\
 &\quad + 6(4+4x+x^2)x^4 - 4(2+x)x^6 + x^8 \\
 &= 16+32x+24x^2+8x^3+x^4 - 32x^2 - 48x^3 - 24x^4 - 4x^5 \\
 &\quad + 24x^4 + 24x^5 + 6x^6 - 8x^6 + 4x^7 + x^8 \\
 &= 16+32-8x^2-40x^3+x^4+20x^5-2x^6-4x^7-x^8
 \end{aligned}$$

(ii) Suppose  $t = 1 - x$  Do yourself

(iii) Suppose  $t = 1 - x$  Do yourself

**Question # 5**

Expand the following in descending powers of  $x$ :

(i)  $(x^2 + x - 1)^3$       (ii)  $\left(x - 1 - \frac{1}{x}\right)^3$

**Solution** (i) Suppose  $t = x - 1$  Do yourself

(ii)  $\left(x - 1 - \frac{1}{x}\right)^3$

Suppose  $t = x - 1$  then

$$\begin{aligned}
 \left(t - \frac{1}{x}\right)^3 &= (t)^3 + 3(t)^2\left(-\frac{1}{x}\right) + 3(t)\left(-\frac{1}{x}\right)^2 + \left(-\frac{1}{x}\right)^3 \\
 &= t^3 - 3t^2 \cdot \frac{1}{x} + 3t \cdot \frac{1}{x^2} - \frac{1}{x^3} \dots\dots\dots (i)
 \end{aligned}$$

Now

$$\begin{aligned}
 t^3 &= (x-1)^3 = (x)^3 + 3(x)^2(-1) + 3(x)(-1)^2 + (-1)^3 \\
 &= x^3 - 3x^2 + 3x - 1
 \end{aligned}$$

$$t^2 = (x-1)^2 = x^2 - 2x + 1$$

Putting values of  $t^3, t^2$  and  $t$  in equation (i)

$$\begin{aligned}
 \left(x - 1 - \frac{1}{x}\right)^3 &= (x^3 - 3x^2 + 3x - 1) - 3(x^2 - 2x + 1) \cdot \frac{1}{x} + 3(x - 1) \cdot \frac{1}{x^2} - \frac{1}{x^3} \\
 &= x^3 - 3x^2 + 3x - 1 - 3x + 6 - 3\frac{1}{x} + 3\frac{1}{x} - 3\frac{1}{x^2} - \frac{1}{x^3} \\
 &= x^3 - 3x^2 + 5 - \frac{3}{x^2} - \frac{1}{x^3}
 \end{aligned}$$

**Question # 6**

Find the term involving:

(i)  $x^4$  in the expansion of  $(3 - 2x)^7$ (ii)  $x^{-2}$  in the expansion of

$$\left(x - \frac{2}{x^2}\right)^{13}$$

(iii)  $a^4$  in the expansion of  $\left(\frac{2}{x} - a\right)^9$ (iv)  $y^3$  in the expansion of

$$(x - \sqrt{y})^{11}$$

**Solution** (i) Since

$$T_{r+1} = \binom{n}{r} a^{n-r} x^r$$

Here  $a = 3$ ,  $x = -2x$ ,  $n = 7$  so

$$T_{r+1} = \binom{7}{r} (3)^{7-r} (-2x)^r = \binom{7}{r} (3)^{7-r} (-2)^r (x)^r$$

For term involving  $x^4$  we must have

$$x^r = x^4 \Rightarrow r = 4$$

So

$$T_{4+1} = \binom{7}{4} (3)^{7-4} (-2)^4 (x)^4$$

$$\begin{aligned} \Rightarrow T_5 &= (35)(3)^3 (-2)^4 (x)^4 = (35)(27)(16)(x)^4 \\ &= 15120x^4 \end{aligned}$$

(ii) Since  $T_{r+1} = \binom{n}{r} a^{n-r} x^r$ Here  $a = x$ ,  $x = -\frac{2}{x^2}$ ,  $n = 13$  so

$$\begin{aligned} T_{r+1} &= \binom{13}{r} (x)^{13-r} \left(-\frac{2}{x^2}\right)^r = \binom{13}{r} (x)^{13-r} (-2)^r (x)^{-2r} \\ &= \binom{13}{r} (x)^{13-r-2r} (-2)^r = \binom{13}{r} (x)^{13-3r} (-2)^r \end{aligned}$$

For term involving  $x^{-2}$  we must have

$$\begin{aligned} x^{13-3r} &= x^{-2} \Rightarrow 13-3r = -2 \Rightarrow -3r = -2-13 \\ &\Rightarrow -3r = -15 \Rightarrow r = 5 \end{aligned}$$

So

$$T_{5+1} = \binom{13}{5} (x)^{13-3(5)} (-2)^5$$

$$\Rightarrow T_6 = (1287)(x)^{13-15} (-32) = -41184x^{-2}$$

(iii) Since

$$T_{r+1} = \binom{n}{r} a^{n-r} x^r$$

Here  $a = \frac{2}{x}$ ,  $x = -a$ ,  $n = 9$  so

$$T_{r+1} = \binom{9}{r} \left(\frac{2}{x}\right)^{9-r} (-a)^r = \binom{9}{r} \left(\frac{2}{x}\right)^{9-r} (-1)^r (a)^r$$

For term involving  $a^4$  we must have

$$a^r = a^4 \Rightarrow r = 4$$

So  $T_{4+1} = \binom{9}{4} \left(\frac{2}{x}\right)^{9-4} (-1)^4 (a)^4$

$$\Rightarrow T_5 = (126) \left(\frac{2}{x}\right)^5 (1) a^4 = (126) \left(\frac{32}{x^5}\right) a^4 = 4032 \frac{a^4}{x^5}$$

(iv) Here  $a = x$ ,  $x = -\sqrt{y}$ ,  $n = 11$  so

Since

$$T_{r+1} = \binom{n}{r} a^{n-r} x^r$$

$$\begin{aligned} T_{r+1} &= \binom{11}{r} (x)^{11-r} (-\sqrt{y})^r = \binom{11}{r} (x)^{11-r} (-y^{\frac{1}{2}})^r \\ &= \binom{11}{r} (x)^{11-r} (-1)^r (y^{\frac{r}{2}}) \end{aligned}$$

For term involving  $y^3$  we must have

$$y^{\frac{r}{2}} = y^3 \Rightarrow \frac{r}{2} = 3 \Rightarrow r = 6$$

So  $T_{6+1} = \binom{11}{6} (x)^{11-6} (-1)^6 (y^{\frac{6}{2}})$

$$\Rightarrow T_7 = (462) (x)^5 (1) (y^3) = 462 x^5 y^3$$

### Question # 7

Find the coefficient of;

(i)  $x^5$  in the expansion of  $\left(x^2 - \frac{3}{2x}\right)^{10}$       (ii)  $x^n$  in the expansion of  $\left(x^2 - \frac{1}{x}\right)^{2n}$

**Solution** (i) Here  $a = x^2$ ,  $x = -\frac{3}{2x}$ ,  $n = 10$  so

Since

$$T_{r+1} = \binom{n}{r} a^{n-r} x^r$$

$$\begin{aligned} T_{r+1} &= \binom{10}{r} (x^2)^{10-r} \left(-\frac{3}{2x}\right)^r = \binom{10}{r} (x)^{2(10-r)} (-1)^r \frac{(3)^r}{(2)^r (x)^r} \\ &= \binom{10}{r} (x)^{20-2r} (-1)^r (3)^r (2)^{-r} (x)^{-r} = \binom{10}{r} (x)^{20-2r-r} (-1)^r (3)^r (2)^{-r} \end{aligned}$$



$$= \binom{10}{r} (x)^{20-3r} (-1)^r (3)^r (2)^{-r}$$

For term involving  $x^5$  we must have

$$\begin{aligned} x^{20-3r} = x^5 &\Rightarrow 20 - 3r = 5 \Rightarrow -3r = 5 - 20 \\ &\Rightarrow -3r = -15 \Rightarrow r = 5 \end{aligned}$$

$$\text{So } T_{5+1} = \binom{10}{5} (x)^{20-3(5)} (-1)^5 (3)^5 (2)^{-5}$$

$$\Rightarrow T_6 = 252 (x)^{20-15} (-1)^5 (3)^5 \frac{1}{2^5} = -252 (x)^5 (243) \frac{1}{32}$$

$$= -\frac{61236}{32} x^5 = -\frac{15309}{8} x^5$$

$$\text{Hence coefficient of } x^5 = -\frac{15309}{8}$$

(ii) Here  $a = x^2$ ,  $x = -\frac{1}{x}$ ,  $n = 2n$  so

Since

$$T_{r+1} = \binom{n}{r} a^{n-r} x^r$$

$$\begin{aligned} T_{r+1} &= \binom{2n}{r} (x^2)^{2n-r} \left(-\frac{1}{x}\right)^r = \binom{2n}{r} (x)^{2(2n-r)} (-1)^r \frac{1}{x^r} \\ &= \binom{2n}{r} (x)^{4n-2r} (-1)^r x^{-r} = \binom{2n}{r} (x)^{4n-2r-r} (-1)^r \\ &= \binom{2n}{r} (x)^{4n-3r} (-1)^r \end{aligned}$$

For term involving  $x^n$  we must have

$$\begin{aligned} x^{4n-3r} = x^n &\Rightarrow 4n - 3r = n \Rightarrow -3r = n - 4n \\ &\Rightarrow -3r = -3n \Rightarrow r = n \end{aligned}$$

$$\text{So } T_{n+1} = \binom{2n}{n} (x)^{4n-3n} (-1)^n$$

$$= \frac{(2n)!}{(2n-n)! \cdot n!} (x)^n (-1)^n = \frac{(2n)!}{n! \cdot n!} (x)^n (-1)^n$$

$$= (-1)^n \frac{(2n)!}{(n!)^2} x^n$$

$$\text{Hence coefficient of } x^n = (-1)^n \frac{(2n)!}{(n!)^2}$$

### Question # 8

Find 6th term in the expansion of  $\left(x^2 - \frac{3}{2x}\right)^{10}$

**Solution** Here  $a = x^2$ ,  $x = -\frac{3}{2x}$ ,  $n = 10$  and  $r + 1 = 6 \Rightarrow r = 5$  so

Since

$$T_{r+1} = \binom{n}{r} a^{n-r} x^r$$

$$T_{5+1} = \binom{10}{5} (x^2)^{10-5} \left(-\frac{3}{2x}\right)^5$$

$$\Rightarrow T_6 = 252 (x^2)^5 \left(-\frac{3^5}{(2x)^5}\right) = 252 x^{10} \left(-\frac{243}{32x^5}\right)$$

$$= -\frac{61236}{32} x^{10-5} = -\frac{15309}{8} x^5$$

### Question # 9

Find the term independent of  $x$  in the following expansions..

(i)  $\left(x - \frac{2}{x}\right)^{10}$       (ii)  $\left(\sqrt{x} - \frac{1}{2x^2}\right)^{10}$       (iii)  $(1 + x^2)^3 \left(1 + \frac{1}{x^2}\right)^4$

**Solution** (i) Do yourself as Q # 9 (ii)

(ii) Here  $a = \sqrt{x}$ ,  $x = \frac{1}{2x^2}$ ,  $n = 10$  so

Since

$$T_{r+1} = \binom{n}{r} a^{n-r} x^r$$

$$T_{r+1} = \binom{10}{r} (\sqrt{x})^{10-r} \left(\frac{1}{2x^2}\right)^r = \binom{10}{r} (x^{\frac{1}{2}})^{10-r} \left(\frac{1}{2^r x^{2r}}\right)$$

$$= \binom{10}{r} (x)^{\frac{1}{2}(10-r)} \frac{1}{2^r} x^{-2r} = \binom{10}{r} (x)^{5-\frac{r}{2}} \frac{1}{2^r} x^{-2r}$$

$$= \binom{10}{r} (x)^{5-\frac{r}{2}-2r} \frac{1}{2^r} = \binom{10}{r} (x)^{5-\frac{5r}{2}} \frac{1}{2^r}$$

For term independent of  $x$  we must have

$$x^{\frac{5-5r}{2}} = x^0 \Rightarrow 5 - \frac{5r}{2} = 0 \Rightarrow -\frac{5r}{2} = -5$$

$$\Rightarrow r = (-5) \left(-\frac{2}{5}\right) \Rightarrow r = 2$$

So  $T_{2+1} = \binom{10}{2} (x)^{5-\frac{5(2)}{2}} \frac{1}{2^2}$

$$\Rightarrow T_3 = 45 (x)^{5-5} \frac{1}{4} = 45 x^0 \frac{1}{4}$$

$$= 45 (1) \frac{1}{4} = \frac{45}{4}$$

$$\begin{aligned}
 \text{(iii)} \quad (1+x^2)^3 \left(1 + \frac{1}{x^2}\right)^4 &= (1+x^2)^3 \left(\frac{x^2+1}{x^2}\right)^4 \\
 &= (1+x^2)^3 \frac{(x^2+1)^4}{(x^2)^4} = (1+x^2)^3 \frac{(1+x^2)^4}{x^8} \\
 &= x^{-8} (1+x^2)^{3+4} = x^{-8} (1+x^2)^7
 \end{aligned}$$

Now  $T_{r+1} = x^{-8} \binom{n}{r} a^{n-r} x^r$

Where  $n=7, a=1, x=x^2$

$$\begin{aligned}
 T_{r+1} &= x^{-8} \binom{7}{r} (1)^{7-r} (x^2)^r = x^{-8} \binom{7}{r} (1) x^{2r} \\
 &= \binom{7}{r} x^{2r-8}
 \end{aligned}$$

For term independent of  $x$  we must have

$$x^{2r-8} = x^0 \Rightarrow 2r-8=0 \Rightarrow 2r=8 \Rightarrow r=4$$

So

$$\begin{aligned}
 T_{4+1} &= \binom{7}{4} x^{2(4)-8} \\
 \Rightarrow T_5 &= 35 x^{8-8} = 35 x^0 = 35
 \end{aligned}$$

### Question # 10

Determine the middle term in the following expansions:

$$\text{(i)} \left(\frac{1}{x} - \frac{x^2}{2}\right)^{12} \qquad \text{(ii)} \left(\frac{3}{2}x - \frac{1}{3x}\right)^{11} \qquad \text{(iii)} \left(2x - \frac{1}{2x}\right)^{2m+1}$$

**Solution** (i)  $\left(\frac{1}{x} - \frac{x^2}{2}\right)^{12}$

Since  $n=12$  is an even so middle terms is  $\frac{n+2}{2} = \frac{12+2}{2} = 7$

Therefore  $r+1=7 \Rightarrow r=7-1=6$

And  $a = \frac{1}{x}, x = -\frac{x^2}{2}$  and  $n=12$

Now

$$\begin{aligned}
 T_{r+1} &= \binom{n}{r} a^{n-r} x^r \\
 \Rightarrow T_{6+1} &= \binom{12}{6} \left(\frac{1}{x}\right)^{12-6} \left(-\frac{x^2}{2}\right)^6
 \end{aligned}$$

$$\begin{aligned}\Rightarrow T_7 &= 924 \frac{1}{x^6} \frac{x^{12}}{64} = \frac{924}{64} x^{12-6} \\ &= \frac{231}{16} x^6\end{aligned}$$

Thus the middle terms of the given expansion is  $\frac{231}{16} x^6$ .

(ii) Since  $n=11$  is odd so the middle terms are  $\frac{n+1}{2} = \frac{11+1}{2} = 6$  and

$$\frac{n+3}{2} = \frac{11+3}{2} = 7$$

So for first middle term

$$a = \frac{3}{2}x, \quad x = -\frac{1}{3x}, \quad n=11 \text{ and } r+1=6 \Rightarrow r=5$$

Now

$$T_{r+1} = \binom{n}{r} a^{n-r} x^r \quad \Rightarrow \quad T_{5+1} = \binom{11}{5} \left(\frac{3}{2}x\right)^{11-5} \left(-\frac{1}{3x}\right)^5$$

*Now simplify yourself.*

Now for second middle term

$$r+1=7 \Rightarrow r=6$$

$$\text{so } T_{6+1} = \binom{11}{6} \left(\frac{3}{2}x\right)^{11-6} \left(-\frac{1}{3x}\right)^6 \quad \text{Now simplify yourself.}$$

(iii) Since  $n=2m+1$  is odd so there are two middle terms

$$\text{First middle term} = \frac{n+1}{2} = \frac{2m+1+1}{2} = \frac{2m+2}{2} = m+1$$

$$\text{Second middle terms} = \frac{n+3}{2} = \frac{2m+1+3}{2} = \frac{2m+4}{2} = m+2$$

$$\text{Here } a = 2x, \quad x = -\frac{1}{2x} \text{ and } n = 2m+1$$

For first middle term  $r+1 = m+1 \Rightarrow r = m$ .

Since

$$\begin{aligned}T_{r+1} &= \binom{n}{r} a^{n-r} x^r \\ \Rightarrow T_{m+1} &= \binom{2m+1}{m} (2x)^{2m+1-m} \left(-\frac{1}{2x}\right)^m = \frac{(2m+1)!}{(2m+1-m)! \cdot m!} (2x)^{m+1} \left(-\frac{1}{2x}\right)^m \\ &= \frac{(2m+1)!}{(m+1)! \cdot m!} (2)^{m+1} (x)^{m+1} (-1)^m \left(\frac{1}{2}\right)^m \left(\frac{1}{x}\right)^m \\ &= \frac{(2m+1)!}{(m+1)! \cdot m!} (2)^{m+1} (x)^{m+1} (-1)^m (2)^{-m} (x)^{-m}\end{aligned}$$

$$\begin{aligned}
&= \frac{(2m+1)!}{(m+1)! \cdot m!} (2)^{m+1-m} (x)^{m+1-m} (-1)^m = \frac{(2m+1)!}{(m+1)! \cdot m!} (2)^1 (x)^1 (-1)^m \\
&= \frac{(2m+1)!}{(m+1)! \cdot m!} 2x(-1)^m
\end{aligned}$$

For second middle term

$$r+1 = m+2 \Rightarrow r = m+2-1 \Rightarrow r = m+1$$

$$\text{As } T_{r+1} = \binom{n}{r} a^{n-r} x^r$$

$$\Rightarrow T_{m+1+1} = \binom{2m+1}{m+1} (2x)^{(2m+1)-(m+1)} \left(-\frac{1}{2x}\right)^{m+1}$$

Now simplify yourself

### Question # 11

Find  $(2n+1)$ th term of the end in the expansion of  $\left(x - \frac{1}{2x}\right)^{3n}$

**Solution** Here  $a = x$ ,  $x = -\frac{1}{2x}$ ,

Number of term from the end =  $2n + 1$

To make it from beginning we take  $a = -\frac{1}{2x}$ ,  $x = x$  and  $r+1 = 2n+1$

$$\Rightarrow r = 2n$$

$$\text{As } T_{r+1} = \binom{n}{r} a^{n-r} x^r$$

$$\begin{aligned}
\Rightarrow T_{2n+1} &= \binom{3n}{2n} \left(-\frac{1}{2x}\right)^{3n-2n} (x)^{2n} = \frac{(3n)!}{(3n-2n)! \cdot (2n)!} \left(-\frac{1}{2x}\right)^n x^{2n} \\
&= \frac{(3n)!}{(n)! \cdot (2n)!} (-1)^n \frac{1}{2^n \cdot x^n} x^{2n} = \frac{(3n)!}{n! \cdot (2n)!} (-1)^n \frac{1}{2^n} x^{2n-n} \\
&= \frac{(-1)^n (3n)!}{2^n n! \cdot (2n)!} x^n \quad \text{Answer}
\end{aligned}$$

**Note:** If there are  $p$  term in some expansion and  $q$ th term is from the end then the term from the beginning will be  $= p - q + 1$ .

So in above you can use term from the end  $= (3n+1) - (2n+1) + 1 = n+1$

### Question # 12

Show that the middle term of  $(1+x)^{2n}$  is  $\frac{1.3.5 \dots (2n-1)}{n!} 2^n x^n$

**Solution** Since  $2n$  is even so the middle term is  $\frac{2n+2}{2} = n+1$  and

$$a=1, \quad x=x, \quad n=2n, \quad r+1=n+1 \Rightarrow r=n$$

$$\text{Now } T_{r+1} = \binom{n}{r} a^{n-r} x^r$$

$$\begin{aligned} \Rightarrow T_{n+1} &= \binom{2n}{n} (1)^{2n-n} x^n \\ \Rightarrow T_{n+1} &= \frac{(2n)!}{(2n-n)! \cdot n!} (1)^n x^n = \frac{(2n)!}{n! \cdot n!} x^n \\ &= \frac{2n(2n-1)(2n-2)(2n-3)(2n-4) \cdot \dots \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{n! \cdot n!} x^n \\ &= \frac{[2n(2n-2)(2n-4) \cdot \dots \cdot 4 \cdot 2][(2n-1)(2n-3) \cdot \dots \cdot 5 \cdot 3 \cdot 1]}{n! \cdot n!} x^n \\ &= \frac{2^n [n(n-1)(n-2) \cdot \dots \cdot 2 \cdot 1][(2n-1)(2n-3) \cdot \dots \cdot 5 \cdot 3 \cdot 1]}{n! \cdot n!} x^n \\ &= \frac{2^n n! [(2n-1)(2n-3) \cdot \dots \cdot 5 \cdot 3 \cdot 1]}{n! \cdot n!} x^n \\ &= \frac{2^n [1 \cdot 3 \cdot 5 \cdot \dots \cdot (2n-1)]}{n!} x^n \\ &= \frac{1 \cdot 3 \cdot 5 \cdot \dots \cdot (2n-1)}{n!} 2^n x^n \end{aligned}$$

**Question # 13**

Show that:

$$\binom{n}{1} + \binom{n}{3} + \binom{n}{5} + \dots + \binom{n}{n-1} = 2^{n-1}$$

**Solution** Consider

$$(1+x)^n = \binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \binom{n}{3}x^3 + \binom{n}{4}x^4 + \binom{n}{5}x^5 + \dots + \binom{n}{n-1}x^{n-1} + \binom{n}{n}x^n \tag{i}$$

Put  $x = 1$

$$\begin{aligned} (1+1)^n &= \binom{n}{0} + \binom{n}{1}(1) + \binom{n}{2}(1)^2 + \binom{n}{3}(1)^3 + \binom{n}{4}(1)^4 + \binom{n}{5}(1)^5 + \dots + \binom{n}{n-1}(1)^{n-1} + \binom{n}{n}(1)^n \\ \Rightarrow 2^n &= \binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \binom{n}{3} + \binom{n}{4} + \binom{n}{5} + \dots + \binom{n}{n-1} + \binom{n}{n} \\ \Rightarrow 2^n &= \left[ \binom{n}{0} + \binom{n}{2} + \binom{n}{4} + \dots + \binom{n}{n} \right] + \left[ \binom{n}{1} + \binom{n}{3} + \binom{n}{5} + \dots + \binom{n}{n-1} \right] \tag{ii} \end{aligned}$$

Now put  $x = -1$  in equation (i)

$$\begin{aligned} (1-1)^n &= \binom{n}{0} + \binom{n}{1}(-1) + \binom{n}{2}(-1)^2 + \binom{n}{3}(-1)^3 + \binom{n}{4}(-1)^4 + \binom{n}{5}(-1)^5 + \dots \\ &\quad \dots + \binom{n}{n-1}(-1)^{n-1} + \binom{n}{n}(-1)^n \end{aligned}$$

If we consider  $n$  is even then

$$\Rightarrow (0)^n = \binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \binom{n}{3} + \binom{n}{4} - \binom{n}{5} + \dots - \binom{n}{n-1} + \binom{n}{n}$$

$$\Rightarrow 0 = \left[ \binom{n}{0} + \binom{n}{2} + \binom{n}{4} + \dots + \binom{n}{n} \right] - \left[ \binom{n}{1} + \binom{n}{3} + \binom{n}{5} + \dots + \binom{n}{n-1} \right]$$

$$\Rightarrow \left[ \binom{n}{1} + \binom{n}{3} + \binom{n}{5} + \dots + \binom{n}{n-1} \right] = \left[ \binom{n}{0} + \binom{n}{2} + \binom{n}{4} + \dots + \binom{n}{n} \right]$$

Using it in equation (ii)

$$2^n = \left[ \binom{n}{1} + \binom{n}{3} + \binom{n}{5} + \dots + \binom{n}{n-1} \right] + \left[ \binom{n}{0} + \binom{n}{2} + \binom{n}{4} + \dots + \binom{n}{n} \right]$$

$$\Rightarrow 2^n = 2 \left[ \binom{n}{1} + \binom{n}{3} + \binom{n}{5} + \dots + \binom{n}{n-1} \right]$$

$$\Rightarrow \frac{2^n}{2} = \left[ \binom{n}{1} + \binom{n}{3} + \binom{n}{5} + \dots + \binom{n}{n-1} \right]$$

$$\Rightarrow 2^{n-1} = \left[ \binom{n}{1} + \binom{n}{3} + \binom{n}{5} + \dots + \binom{n}{n-1} \right]$$

$$\Rightarrow \binom{n}{1} + \binom{n}{3} + \binom{n}{5} + \dots + \binom{n}{n-1} = 2^{n-1}$$

### Question # 14

Show that:

$$\binom{n}{0} + \frac{1}{2} \binom{n}{1} + \frac{1}{3} \binom{n}{2} + \frac{1}{4} \binom{n}{3} + \dots + \frac{1}{n+1} \binom{n}{n} = \frac{2^{n+1} - 1}{n+1}$$

**Solution**

$$\begin{aligned} \text{L.H.S} &= \binom{n}{0} + \frac{1}{2} \binom{n}{1} + \frac{1}{3} \binom{n}{2} + \frac{1}{4} \binom{n}{3} + \dots + \frac{1}{n+1} \binom{n}{n} \\ &= \left[ \binom{n}{0} + \frac{1}{2} \left( \frac{n!}{(n-1)! \cdot 1!} \right) + \frac{1}{3} \left( \frac{n!}{(n-2)! \cdot 2!} \right) + \frac{1}{4} \left( \frac{n!}{(n-3)! \cdot 3!} \right) + \dots + \frac{1}{n+1} \binom{n}{n} \right] \\ &= \frac{n+1}{n+1} \left[ 1 + \frac{1}{2} \left( \frac{n!}{(n-1)! \cdot 1!} \right) + \frac{1}{3} \left( \frac{n!}{(n-2)! \cdot 2!} \right) + \frac{1}{4} \left( \frac{n!}{(n-3)! \cdot 3!} \right) + \dots + \frac{1}{n+1} \cdot 1 \right] \\ &= \frac{1}{n+1} \left[ (n+1) + \frac{1}{2} \left( \frac{(n+1)n!}{(n-1)! \cdot 1!} \right) + \frac{1}{3} \left( \frac{(n+1)n!}{(n-2)! \cdot 2!} \right) + \frac{1}{4} \left( \frac{(n+1)n!}{(n-3)! \cdot 3!} \right) + \dots + \frac{n+1}{n+1} \cdot 1 \right] \\ &= \frac{1}{n+1} \left[ (n+1) + \left( \frac{(n+1)!}{(n-1)! \cdot 2 \cdot 1!} \right) + \left( \frac{(n+1)!}{(n-2)! \cdot 3 \cdot 2!} \right) + \left( \frac{(n+1)!}{(n-3)! \cdot 4 \cdot 3!} \right) + \dots + 1 \right] \\ &= \frac{1}{n+1} \left[ (n+1) + \left( \frac{(n+1)!}{(n+1-2)! \cdot 2!} \right) + \left( \frac{(n+1)!}{(n+1-3)! \cdot 3!} \right) + \left( \frac{(n+1)!}{(n+1-4)! \cdot 4!} \right) + \dots + 1 \right] \\ &= \frac{1}{n+1} \left[ \binom{n+1}{1} + \binom{n+1}{2} + \binom{n+1}{3} + \binom{n+1}{4} + \dots + \binom{n+1}{n+1} \right] \\ &= \frac{1}{n+1} \left[ -1 + 1 + \binom{n+1}{1} + \binom{n+1}{2} + \binom{n+1}{3} + \binom{n+1}{4} + \dots + \binom{n+1}{n+1} \right] \end{aligned}$$

$$\begin{aligned}
&= \frac{1}{n+1} \left[ -1 + \binom{n+1}{0} + \binom{n+1}{1} + \binom{n+1}{2} + \binom{n+1}{3} + \binom{n+1}{4} + \dots + \binom{n+1}{n+1} \right] \\
&= \frac{1}{n+1} \left[ -1 + 2^{n+1} \right] \\
&= \frac{2^{n+1} - 1}{n+1} = \text{R.H.S}
\end{aligned}$$

**Remember**

$$\binom{n+1}{0} = 1, \quad \binom{n+1}{1} = n+1 \quad \text{and} \quad \binom{n+1}{n+1} = 1$$

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**Book:**            **Exercise 8.2**  
*Text Book of Algebra and Trigonometry Class XI*  
*Punjab Textbook Board, Lahore.*

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