

Q_{No 1} $1, \frac{1}{3}, \frac{1}{9}, \dots$
 $a_1 = 1, r = \frac{a_2}{a_1} = \frac{\frac{1}{3}}{1} = \frac{1}{3}, n = 15$

Now $S_n = \frac{a_1(1-r^n)}{1-r}$
 $= \frac{1(1-(\frac{1}{3})^{15})}{1-\frac{1}{3}} = \frac{1-\frac{1}{3^{15}}}{\frac{2}{3}}$
 $= \frac{3}{2} \left[\frac{3^{15}-1}{3^{15}} \right] = \frac{3}{2} \left[\frac{14348907-1}{14348907} \right]$
 $= \frac{3}{2} \left[\frac{14348906}{14348907} \right]$
 $= \frac{7174453}{4782969}$ \div ing by 6.
Ans

Q_{No 2} i)
 $0.2 + 0.22 + 0.222 + \dots$
 $= 2(0.1 + 0.11 + 0.111 + \dots \text{ to } n \text{ terms})$
 $= \frac{2}{9}(0.9 + 0.99 + 0.999 + \dots \text{ to } n \text{ terms})$
 $= \frac{2}{9}[(1-0.1) + (1-0.01) + (1-0.001) + \dots \text{ to } n \text{ terms}]$
 $= \frac{2}{9}[(1+1+1+\dots \text{ to } n \text{ terms}) - (0.1 + 0.01 + 0.001 + \dots \text{ to } n \text{ terms})]$
 $= \frac{2}{9} \left[n - \left(\frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \dots \text{ to } n \text{ terms} \right) \right]$
 $a_1 = \frac{1}{10}, r = \frac{1}{10}, n = n$
 $= \frac{2}{9} \left[n - \frac{1 - (\frac{1}{10})^n}{1 - \frac{1}{10}} \right]$
 $= \frac{2}{9} \left[n - \frac{1 - \frac{1}{10^n}}{\frac{9}{10}} \right]$
 $= \frac{2}{9} \left[n - \frac{1}{9} \left(1 - \frac{1}{10^n} \right) \right]$ Answer

ii) $3 + 33 + 333 + \dots$ to n terms
 $= \frac{1}{3}(9 + 99 + 999 + \dots \text{ to } n \text{ terms})$
 $= \frac{1}{3}[(10-1) + (100-1) + (1000-1) + \dots \text{ to } n \text{ terms}]$
 $= \frac{1}{3}[(10+100+1000+\dots \text{ to } n \text{ terms}) - (1+1+1+\dots \text{ to } n \text{ terms})]$

$$= \frac{1}{3} [(10+100+1000+\dots \text{ to } n \text{ terms}) - n]$$

$$a_1=10, r=10, n=n$$

$$= \frac{1}{3} \left[\frac{10(10^n-1)}{10-1} - n \right]$$

$$= \frac{1}{3} \left[\frac{10(10^n-1)}{9} - n \right]$$
 Answer
OR

$$= \frac{1}{3} \left[\frac{10}{9} (10^n-1) - n \right]$$
 Answer

Q_{No 3} i)

$$1 + (a+b) + (a^2+ab+b^2) + \dots \text{ to } n \text{ terms}$$

\div ing and \times ing by $a-b$.

$$= \frac{a-b}{a-b} [1 + (a+b) + (a^2+ab+b^2) + \dots \text{ to } n \text{ terms}]$$

$$= \frac{1}{a-b} [(a-b) + (a-b)(a+b) + (a-b)(a^2+ab+b^2) + \dots \text{ to } n \text{ terms}]$$

$$= \frac{1}{a-b} [(a-b) + (a^2-b^2) + (a^3-b^3) + \dots \text{ to } n \text{ terms}]$$

$$= \frac{1}{a-b} [(a+a^2+a^3+\dots \text{ to } n \text{ terms}) - (b+b^2+b^3+\dots \text{ to } n \text{ terms})]$$

$$= \frac{1}{a-b} \left[\frac{a(a^n-1)}{a-1} - \frac{b(b^n-1)}{b-1} \right]$$

$$= \frac{1}{a-b} \left[\frac{a(a^n-1)(b-1) - b(b^n-1)(a-1)}{(a-1)(b-1)} \right]$$

$$= \frac{a(b-1)(a^n-1) - b(a-1)(b^n-1)}{(a-b)(a-1)(b-1)}$$
 Answer

ii)

$$r + (1+k)r^2 + (1+k+k^2)r^3 + \dots$$

\dots to n terms

\div ing and \times ing by $1-k$.

$$= \frac{1-k}{1-k} (r + (1+k)r^2 + (1+k+k^2)r^3 + \dots \text{ to } n \text{ terms})$$

$$= \frac{1}{1-k} [r(1-k) + (1-k)(1+k)r^2 + (1-k)(1+k+k^2)r^3 + \dots \text{ to } n \text{ terms}]$$

$$= \frac{1}{1-k} [r(1-k) + (1-k^2)r^2 + (1-k^3)r^3 + \dots \text{ to } n \text{ terms}]$$

$$1-r \quad 1-k^2 \quad 1-k^3 \quad \dots$$

$$= \frac{1}{1-k} (r - rk + r^2 - r^2k + r^3 - r^3k^2 + \dots \text{to } n \text{ terms})$$

$$= \frac{1}{1-k} ((r + r^2 + r^3 + \dots \text{to } n \text{ terms}) - (rk + r^2k^2 + r^3k^3 + \dots \text{to } n \text{ terms}))$$

$$= \frac{1}{1-k} \left(\frac{r(r^n - 1)}{r - 1} - \frac{rk(r^n k^n - 1)}{rk - 1} \right)$$

$$= \frac{r}{1-k} \left(\frac{r^n - 1}{r - 1} - \frac{k(r^n k^n - 1)}{rk - 1} \right)$$

Answer

Q No 4 $2 + (1-i) + \frac{1}{2} + \dots$ to 8 terms

$a_1 = 2, r = \frac{1-i}{2}; n = 8$

Now $S_n = \frac{a_1(r^n - 1)}{r - 1}$

$$= \frac{2 \left(\left(\frac{1-i}{2} \right)^8 - 1 \right)}{\frac{1-i}{2} - 1}$$

$$= \frac{2 \left(\frac{(1-i)^8 - 1}{2^8} - 1 \right)}{\frac{1-i-2}{2}}$$

$$= 4 \left(\frac{((1-i)^2)^4 - 1}{256} - 1 \right) \frac{2}{-1-i}$$

$$= \frac{4 \left(\frac{(1-2i+i)^4 - 256}{256} - 1 \right)}{-1-i}$$

$$= \frac{4 \left(\frac{(1-2i-1)^4 - 256}{256} - 1 \right)}{-1-i}$$

$$= \frac{4 \left(\frac{(-2i)^4 - 256}{64} - 1 \right)}{-1-i}$$

$$= \frac{4 \left(\frac{(-2)^4 (i)^4 - 256}{64} - 1 \right)}{-1-i} \quad \begin{matrix} i^4 = (i^2)^2 \\ = (-1)^2 \\ = 1 \end{matrix}$$

$$= \frac{16(1) - 256}{64(-1-i)} = \frac{-240}{64(-1-i)}$$

$$= \frac{-15}{4(1+i)} = \frac{15}{4(1+i)} \cdot \frac{1-i}{1-i}$$

$$= \frac{15(1-i)}{4((1)^2 - (i)^2)} = \frac{15(1-i)}{4(2)} = \frac{15(1-i)}{8}$$

Ans

Q No 5 i) $\frac{1}{5} + \frac{1}{25} + \frac{1}{125} + \dots$

$a_1 = \frac{1}{5}, r = \frac{1}{5}$

Now $S = \frac{a_1}{1-r} = \frac{\frac{1}{5}}{1-\frac{1}{5}}$

$$= \frac{\frac{1}{5}}{\frac{4}{5}} = \frac{1}{4} \text{ Answer}$$

ii) & iii) Do yourself

iv) $2 + 1 + 0.5 + \dots$

$a_1 = 2, r = \frac{1}{2} = 0.5$

Now $S = \frac{a_1}{1-r} = \frac{2}{1-0.5}$

$$= \frac{2}{0.5} = 4 \text{ Answer}$$

v) $4 + 2\sqrt{2} + 2 + \sqrt{2} + 1 + \dots$

$a_1 = 4, r = \frac{2\sqrt{2}}{4} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

Now $S = \frac{a_1}{1-r} = \frac{4}{1-\frac{1}{\sqrt{2}}}$

$$= \frac{4}{\frac{\sqrt{2}-1}{\sqrt{2}}} = \frac{4\sqrt{2}}{\sqrt{2}-1}$$

Answer OR

$$= \frac{4\sqrt{2} \cdot \sqrt{2} + 1}{\sqrt{2}-1 \cdot \sqrt{2} + 1}$$

$$= \frac{4((\sqrt{2})^2 + \sqrt{2})}{(\sqrt{2})^2 - (1)^2} = \frac{4(2 + \sqrt{2})}{2 - 1}$$

$$= 4(2 + \sqrt{2}) \text{ Answer}$$

vi) Do yourself

Q No 6 i) $1.3\bar{4}$

$$= 1.343434 \dots$$

$$= 1 + 0.343434 \dots$$

$$= 1 + (0.34 + 0.0034 + 0.000034 + \dots)$$

$a_1 = 0.34, r = \frac{0.0034}{0.34} = 0.01$

$$= \frac{0.34}{1-0.01} + \frac{0.34}{0.99} = \frac{34}{99}$$

$$= \frac{99 + 34}{99} = \frac{133}{99} \text{ Answer}$$

ii) $0.2\dot{5}9$
 $= 0.259259259\dots$
 $= 0.259 + 0.000259 + 0.000000259 + \dots$
 $a_1 = 0.259, r = \frac{0.000259}{0.259} = 0.001$
 $= \frac{0.259}{1-0.001} = \frac{0.259}{0.999}$
 $= \frac{259}{999}$ Answer

i) $1.1\dot{4}7$
 $= 1.1474747\dots$
 $= 1.1 + 0.0474747\dots$
 $= 1.1 + (0.047 + 0.00047 + 0.0000047 + \dots)$
 $a_1 = 0.047, r = \frac{0.00047}{0.047} = 0.01$
 $= 1.1 + \frac{0.047}{1-0.01} = 1.1 + \frac{0.047}{0.99}$
 $= \frac{11}{10} + \frac{47/1000}{99/100} = \frac{11}{10} + \frac{47 \cdot 100}{1000 \cdot 99}$
 $= \frac{11}{10} + \frac{47}{990} = \frac{1089 + 47}{990}$
 $= \frac{1136}{990}$ Answer

Qno7 $r + (1+k)r^2 + (1+k+k^2)r^3 + \dots$
 ÷ing & xing by $1-k$
 $= \frac{1-k}{1-k} (r + (1+k)r^2 + (1+k+k^2)r^3 + \dots)$
 $= \frac{1}{1-k} ((1-k)r + (1-k)(1+k)r^2 + (1-k)(1+k+k^2)r^3 + \dots)$
 $= \frac{1}{1-k} ((1-k)r + (1-k^2)r^2 + (1-k^3)r^3 + \dots)$
 $= \frac{1}{1-k} (r - rk + r^2 - r^2k^2 + r^3 - r^3k^3 + \dots)$
 $= \frac{1}{1-k} ((r + r^2 + r^3 + \dots) - (rk + r^2k^2 + r^3k^3 + \dots))$
 $= \frac{1}{1-k} \left(\frac{r}{1-r} - \frac{rk}{1-rk} \right) \because S = \frac{a_1}{1-r}$

$$= \frac{1}{1-k} \left(\frac{r(1-rk) - rk(1-r)}{(1-r)(1-rk)} \right)$$

$$= \frac{r - r^2k - rk + r^2k}{(1-k)(1-r)(1-rk)}$$

$$= \frac{r(1-k)}{(1-k)(1-r)(1-rk)}$$

$$= \frac{r}{(1-r)(1-rk)}$$
 Answer

Qno8 $x = \frac{x}{2} + \frac{1}{4}x^2 + \frac{1}{8}x^3 + \dots$
 $a_1 = \frac{x}{2}, r = \frac{\frac{1}{4}x^2}{\frac{x}{2}} = \frac{\frac{1}{4}x^2}{\frac{x}{2}} = \frac{x}{2}$

So This is an infinite Geometric series also $0 < x < 2 \Rightarrow 0 < \frac{x}{2} < 1$
 i.e $r = \frac{x}{2} < 1$ so solution exists.
 $\Rightarrow y = \frac{a_1}{1-r} = \frac{x/2}{1-x/2}$
 $\Rightarrow y = \frac{x/2}{2-x} \Rightarrow y = \frac{x}{2-x}$
 $\Rightarrow y(2-x) = x \Rightarrow 2y - xy = x$
 $\Rightarrow 2y = x + xy \Rightarrow 2y = x(1+y)$
 $\Rightarrow \frac{2y}{1+y} = x$ i.e $x = \frac{2y}{1+y}$ Answer

Qno9 Do yourself

Hint $0 < x < \frac{3}{2} \Rightarrow 0 < \frac{2}{3}x < 1$

Qno10

Distance travel in 1st fall = 27m
 " " " 2nd fall = $\frac{2}{3} \times 27 = 18$ m
 " " " 3rd fall = $\frac{2}{3} \times 18 = 12$ m
 So sequence of fall is
 $27 + 18 + 12 + \dots$
 This is infinite geometric sequence
 if S_1 denotes distance travel by ball
 in fall then $a_1 = 27, r = \frac{18}{27} = \frac{2}{3} < 1$
 $\Rightarrow S_1 = \frac{a_1}{1-r} = \frac{27}{1-2/3} = \frac{27}{1/3} = 81$ m

Now

Distance travel in 1st rebound = $\frac{2}{3} \times 27 = 18$

" " " 2nd rebound = $\frac{2}{3} \times 18 = 12$

" " " 3rd rebound = $\frac{2}{3} \times 12 = 8$

So sequence of rebound is

$18 + 12 + 8 + \dots$

which is infinite Geometric series

If S_2 denotes distance travel by ball in rebound then $a_1 = 18, r = \frac{2}{3} < 1$

$S_2 = \frac{a_1}{1-r} = \frac{18}{1-\frac{2}{3}} = \frac{18}{\frac{1}{3}} = 54$

Now

total distance = $S_1 + S_2 = 81 + 54 = 135m$

QNo.11 Same as QNo 10

QNo.12 $y = 1 + 2x + 4x^2 + 8x^3 + \dots$

i) $a_1 = 1, r = \frac{2x}{1} = \frac{4x^2}{2x} = 2x$

So $y = \frac{a_1}{1-r} = \frac{1}{1-2x}$

$\Rightarrow y = \frac{1}{1-2x} \Rightarrow y(1-2x) = 1$

$\Rightarrow y - 2xy = 1 \Rightarrow -2xy = 1 - y$

$\Rightarrow 2xy = y - 1 \Rightarrow x = \frac{y-1}{2x}$ *prova*

ii) Now series is convergent if

$|r| < 1$

$\Rightarrow |2x| < 1 \Rightarrow \pm 2x < 1$

$\Rightarrow \pm x < \frac{1}{2}$

$\Rightarrow x < \frac{1}{2}$ and $-x < \frac{1}{2}$

$\Rightarrow x < \frac{1}{2}$ and $x > -\frac{1}{2}$

$\Rightarrow -\frac{1}{2} < x < \frac{1}{2}$

hence series convergent if

$x \in (-\frac{1}{2}, \frac{1}{2})$ or $-\frac{1}{2} < x < \frac{1}{2}$

QNo.13 Do yourself as QNo.12

QNo.14

Let the infinite geometric series

$a_1 + a_1r + a_1r^2 + \dots$

then

$S = \frac{a_1}{1-r}$

$\Rightarrow q = \frac{a_1}{1-r} \because S = q$ (given)

$\Rightarrow q(1-r) = a_1$ (i)

Now square of its terms

$a_1^2 + a_1^2r^2 + a_1^2r^4 + \dots$

then

$S = \frac{a_1^2}{1-r^2} \because a_1 = a_1, r = r^2$

$\Rightarrow \frac{81}{5} = \frac{a_1^2}{1-r^2} \because S = \frac{81}{5}$ (given)

$\Rightarrow \frac{81}{5}(1-r^2) = a_1^2$

$\Rightarrow \frac{81}{5}(1-r)(1+r) = [q(1-r)]^2$ from (i)

$\Rightarrow \frac{81}{5}(\frac{1+r}{1-r})(1+r) = 81(1-r)^2$

$\Rightarrow \frac{1}{5}(1+r) = (1-r)$

$\Rightarrow (1+r) = 5 - 5r$

$\Rightarrow r + 5r = 5 - 1$

$\Rightarrow 6r = 4 \Rightarrow r = \frac{4}{6} = \frac{2}{3}$

putting in (i)

$q(1 - \frac{2}{3}) = a_1 \Rightarrow q(\frac{1}{3}) = a_1$

$\Rightarrow a_1 = 3$

Now

$a_1r = (3)(\frac{2}{3}) = 2$

$a_1r^2 = (3)(\frac{2}{3})^2 = (3)(\frac{4}{9}) = \frac{4}{3}$

$a_1r^3 = (3)(\frac{2}{3})^3 = (3)(\frac{8}{27}) = \frac{8}{9}$

Thus

$3 + 2 + \frac{4}{3} + \frac{8}{9} + \dots$

is the required series

END