

QNo 1 i)

$\frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \dots$ is in H.P

3, 5, 7, \dots is in A.P

$$a_1 = 3, d = 5 - 3 = 2, n = 9$$

$$\text{Since } a_n = a_1 + (n-1)d$$

$$\begin{aligned} \Rightarrow a_9 &= 3 + (9-1)(2) \\ &= 3 + (8)(2) = 3 + 16 \\ &= 19 \end{aligned}$$

so 9th term of A.P is 19

hence 9th term of H.P is $\frac{1}{19}$

ii) Do yourself

QNo 2 i)

$\frac{1}{2}, \frac{1}{5}, \frac{1}{8}, \dots$ is in H.P

2, 5, 8, \dots is in A.P

$$a_1 = 2, d = 5 - 2 = 3, n = 12$$

$$\text{Since } a_n = a_1 + (n-1)d$$

$$\begin{aligned} \Rightarrow a_{12} &= 2 + (12-1)(3) \\ &= 2 + (11)(3) = 2 + 33 \\ &= 35 \end{aligned}$$

so 12 term of A.P is 35

hence 12 term of H.P is $\frac{1}{35}$

ii) Do yourself

QNo 3 i)

Let H_1, H_2, H_3, H_4, H_5 are five H.Ms between $-\frac{2}{5}$ and $\frac{2}{13}$

then

$-\frac{2}{5}, H_1, H_2, H_3, H_4, H_5, \frac{2}{13}$ are in H.P

So $-\frac{5}{2}, \frac{1}{H_1}, \frac{1}{H_2}, \frac{1}{H_3}, \frac{1}{H_4}, \frac{1}{H_5}, \frac{13}{2}$ are in A.P

$$\begin{aligned} a_1 &= -\frac{5}{2}, a_7 = \frac{13}{2} \\ \Rightarrow a_1 + 6d &= \frac{13}{2} \\ \Rightarrow -\frac{5}{2} + 6d &= \frac{13}{2} \\ \Rightarrow 6d &= \frac{13}{2} + \frac{5}{2} = 9 \end{aligned}$$

$$\Rightarrow d = \frac{9}{6} = \frac{3}{2}$$

Now

$$\frac{1}{H_1} = a_2 = a_1 + d = -\frac{5}{2} + \frac{3}{2} = -1$$

$$\Rightarrow H_1 = -1$$

$$\begin{aligned} \frac{1}{H_2} = a_3 &= a_1 + 2d = -\frac{5}{2} + 2\left(\frac{3}{2}\right) \\ &= -\frac{5}{2} + 3 = \frac{1}{2} \end{aligned}$$

$$\Rightarrow H_2 = 2$$

$$\begin{aligned} \frac{1}{H_3} = a_4 &= a_1 + 3d = -\frac{5}{2} + 3\left(\frac{3}{2}\right) \\ &= -\frac{5}{2} + \frac{9}{2} = 2 \end{aligned}$$

$$\Rightarrow H_3 = \frac{1}{2}$$

$$\begin{aligned} \frac{1}{H_4} = a_5 &= a_1 + 4d = -\frac{5}{2} + 4\left(\frac{3}{2}\right) \\ &= -\frac{5}{2} + 6 = \frac{7}{2} \end{aligned}$$

$$\Rightarrow H_4 = \frac{2}{7}$$

$$\begin{aligned} \frac{1}{H_5} = a_6 &= a_1 + 5d = -\frac{5}{2} + 5\left(\frac{3}{2}\right) \\ &= -\frac{5}{2} + \frac{15}{2} = 5 \end{aligned}$$

$$\Rightarrow H_5 = \frac{1}{5}$$

Hence $-1, 2, \frac{1}{2}, \frac{2}{7}, \frac{1}{5}$ are

five H.Ms between $-\frac{2}{5}$ and $\frac{2}{13}$

ii) Do yourself as (i)

QNo 4 Hint (i) (ii) & (iii)

Consider H_1, H_2, H_3, H_4 are four H.Ms between $\frac{1}{3}$ and $\frac{1}{23}$

Now Do yourself as QNo 3

QNo 5

$a_7 = \frac{1}{3}$ in H.P

so $a_7 = 3$ in A.P

$$\Rightarrow a_1 + 6d = 3 \quad \text{--- (i)}$$

Also $a_{10} = \frac{5}{21}$ in H.P

so $a_{10} = \frac{21}{5}$ in A.P

$$\Rightarrow a_1 + 9d = \frac{21}{5} \quad \text{--- (ii)}$$

Subtracting (i) and (ii)

$$a_1 + 6d = 3$$

$$a_1 + 9d = \frac{21}{5}$$

$$-3d = -\frac{6}{5} \Rightarrow 3d = \frac{6}{5}$$

$$\Rightarrow d = \left(\frac{6}{5}\right)\left(\frac{1}{3}\right) \Rightarrow \boxed{d = \frac{2}{5}}$$

putting value of d in eq (i)

$$a_1 + 6\left(\frac{2}{5}\right) = 3$$

$$\Rightarrow a_1 + \frac{12}{5} = 3 \Rightarrow a_1 = 3 - \frac{12}{5}$$

$$\Rightarrow \boxed{a_1 = \frac{3}{5}}$$

Now $a_{14} = a_1 + 13d$ in A.P

$$\Rightarrow a_{14} = \frac{3}{5} + 13\left(\frac{2}{5}\right)$$

$$= \frac{3}{5} + \frac{26}{5} = \frac{29}{5}$$

So $a_{14} = \frac{29}{5}$ in H.P.

Thus 14th term of H.P is $\frac{29}{5}$

Q.No.6 $a_1 = -\frac{1}{3}$ in H.P

So $a_1 = -3$ in A.P

Also $a_5 = \frac{1}{5}$ in H.P

So $a_5 = 5$ in A.P

$$\Rightarrow a_1 + 4d = 5$$

put $a_1 = -3$ in above

$$-3 + 4d = 5 \Rightarrow 4d = 5 + 3$$

$$\Rightarrow 4d = 8 \Rightarrow \boxed{d = 2}$$

Now $a_9 = a_1 + 8d$ in A.P

$$\Rightarrow a_9 = -3 + 8(2) = -3 + 16 = 13$$

So $a_9 = \frac{1}{13}$ is in H.P

Thus 9th term of H.P is $\frac{1}{13}$

Q.No.7 here

$$H.M = 5, a = 2, b = b$$

$$\text{Since } H.M = \frac{2ab}{a+b}$$

$$\Rightarrow 5 = \frac{2(2)b}{2+b}$$

$$\Rightarrow 5(2+b) = 4b$$

$$\Rightarrow 10 + 5b = 4b$$

$$\Rightarrow 5b - 4b = -10$$

$$\Rightarrow b = -10 \text{ Answer}$$

Q.No.8

Since $\frac{1}{k}, \frac{1}{2k+1}, \frac{1}{4k-1}$ are in H.P

So $k, 2k+1, 4k-1$ are in A.P.

$$\text{So } d = 2k+1 - k = 4k-1 - 2k-1$$

$$\Rightarrow k+1 = 2k-2$$

$$\Rightarrow k - 2k = -2 - 1$$

$$\Rightarrow -k = -3 \Rightarrow \boxed{k = 3} \text{ Ans}$$

Q.No.9

Since H.M between a & b = $\frac{2ab}{a+b}$ (i)

but we have given

$$H.M = \frac{a^{n+1} + b^{n+1}}{a^n + b^n} \text{ (ii)}$$

Comparing (i) and (ii)

$$\frac{2ab}{a+b} = \frac{a^{n+1} + b^{n+1}}{a^n + b^n}$$

$$\Rightarrow 2ab(a^n + b^n) = (a^{n+1} + b^{n+1})(a+b)$$

$$\Rightarrow 2a^{n+1}b + 2ab^{n+1} = a^{n+2} + ab^{n+1}$$

$$+ a^{n+1}b + b^{n+2}$$

$$\Rightarrow 2a^{n+1}b + 2ab^{n+1} - a^{n+2} - a^{n+1}b - ab^{n+1} - b^{n+2}$$

$$= a^{n+2} + b^{n+2}$$

$$\Rightarrow a^{n+1}b + ab^{n+1} = a^{n+2} + b^{n+2}$$

$$\Rightarrow a^{n+1}b - a^{n+2} = b^{n+2} - ab^{n+1}$$

$$\Rightarrow a^{n+1}(b-a) = b^{n+1}(b-a)$$

$$\Rightarrow a^{n+1} = b^{n+1}$$

$$\Rightarrow \frac{a^{n+1}}{b^{n+1}} = 1$$

$$\Rightarrow \left(\frac{a}{b}\right)^{n+1} = \left(\frac{a}{b}\right)^0 \because \left(\frac{a}{b}\right)^1 = 1$$

$$\Rightarrow n+1 = 0 \Rightarrow \boxed{n = -1} \text{ Answer}$$

Q No 10 Since a^2, b^2, c^2 are in A.P

$$\text{therefore } d = b^2 - a^2 = c^2 - b^2$$

$$\Rightarrow (b-a)(b+a) = (c-b)(c+b) \text{ --- (i)}$$

Now to prove

$a+b, c+a, b+c$ are in H.P.

we will prove

$$\frac{1}{a+b}, \frac{1}{c+a}, \frac{1}{b+c} \text{ are in A.P}$$

Now

$$d = \frac{1}{c+a} - \frac{1}{a+b}$$

$$= \frac{a+b-c-a}{(c+a)(a+b)}$$

$$= \frac{b-c}{(c+a)(a+b)} \text{ --- (ii)}$$

Also

$$d = \frac{1}{b+c} - \frac{1}{c+a}$$

$$= \frac{c+a-b-c}{(b+c)(c+a)}$$

$$= \frac{a-b}{(b+c)(c+a)}$$

from eq (i) $c+b = \frac{(b-a)(b+a)}{(c-b)}$

putting in above

$$d = \frac{a-b}{\frac{(b-a)(b+a)}{(c-b)}(c+a)}$$

$$= \frac{(a-b)(c-b)}{(b+a)(c+a)}$$

$$= \frac{-(c-b)}{(b+a)(c+a)}$$

$$= \frac{b-c}{(c+a)(a+b)} \text{ --- (iii)}$$

from (ii) & (iii)

$$d = d$$

i.e. $\frac{1}{a+b}, \frac{1}{c+a}, \frac{1}{b+c}$ are in A.P.

$$\Rightarrow a+b, c+a, b+c \text{ are in H.P.} \text{ proved}$$

Q No 11 Suppose the harmonic sequence:

$$\frac{1}{a_1}, \frac{1}{a_1+d}, \frac{1}{a_1+2d}, \dots$$

By given condition

$$\frac{1}{a_1} + \frac{1}{a_1+4d} = \frac{4}{7} \text{ --- (i)}$$

Also we have given

$$\frac{1}{a_1} = \frac{1}{2} \Rightarrow \boxed{a_1 = 2}$$

putting in (i)

$$\frac{1}{2} + \frac{1}{2+4d} = \frac{4}{7}$$

$$\Rightarrow \frac{1}{2+4d} = \frac{4}{7} - \frac{1}{2}$$

$$\Rightarrow \frac{1}{2+4d} = \frac{1}{14}$$

$$\Rightarrow 2+4d = 14$$

$$\Rightarrow 4d = 14 - 2 \Rightarrow 4d = 12$$

$$\Rightarrow d = \frac{12}{4} \Rightarrow \boxed{d = 3}$$

Now

$$\frac{1}{a_1+d} = \frac{1}{2+3} = \frac{1}{5}$$

$$\frac{1}{a_1+2d} = \frac{1}{2+2(3)} = \frac{1}{2+6} = \frac{1}{8}$$

$$\frac{1}{a_1+3d} = \frac{1}{2+3(3)} = \frac{1}{2+9} = \frac{1}{11}$$

Thus the required sequence is

$$\frac{1}{2}, \frac{1}{5}, \frac{1}{8}, \frac{1}{11}, \dots$$

Q No 12 Since $A = \frac{a+b}{2}$

$$G = \pm \sqrt{ab}$$

$$H = \frac{2ab}{a+b}$$

Now

$$G^2 = (\pm \sqrt{ab})^2 = ab \text{ --- (i)}$$

$$AH = \left(\frac{a+b}{2}\right) \left(\frac{2ab}{a+b}\right) = ab \text{ --- (ii)}$$

from (i) and (ii)

$$G^2 = AH \text{ proved}$$

Q No 13 (i)

$$a = 2i, b = 4i$$

$$A = \frac{a+b}{2} = \frac{2i+4i}{2} = \frac{6i}{2} = 3i$$

$$G = \pm \sqrt{ab} = \pm \sqrt{2i \cdot 4i}$$

R.T.O

Q.No 13 ii) $a = 2i$, $b = 4i$

$$A = \frac{a+b}{2} = \frac{2i+4i}{2} = \frac{6i}{2} = 3i$$

$$G = \pm\sqrt{ab} = \pm\sqrt{(2i)(4i)} \\ = \pm\sqrt{8i^2} = \pm 2\sqrt{2}i$$

$$H = \frac{2ab}{a+b} = \frac{2(2i)(4i)}{2i+4i} \\ = \frac{16i^2}{6i} = \frac{8}{3}i$$

Now

$$G^2 = (\pm 2\sqrt{2}i)^2 = 4(2)(-1) = -8 \quad \text{--- (i)}$$

$$AH = (3i)\left(\frac{8}{3}i\right) = 8i^2 = -8 \quad \text{--- (ii)}$$

From (i) and (ii)

$$G^2 = AH$$

Q.No 14 i) $a = 2$, $b = 8$

$$A = \frac{a+b}{2} = \frac{2+8}{2} = \frac{10}{2} = 5$$

$$G = \sqrt{ab} \quad \because G > 0 \\ = \sqrt{(2)(8)} = \sqrt{16} = 4$$

$$H = \frac{2ab}{a+b} = \frac{2(2)(8)}{2+8} = \frac{32}{10} = \frac{16}{5} \\ = 3\frac{1}{5}$$

$$\text{Since } 5 > 4 > 3\frac{1}{5}$$

$$\Rightarrow A > G > H \quad \text{proved}$$

Q.No 15 ii) $a = -\frac{2}{5}$, $b = -\frac{8}{5}$

$$A = \frac{a+b}{2} = \frac{-\frac{2}{5} - \frac{8}{5}}{2} = \frac{-10/5}{2} \\ = \frac{-10}{10} = -1$$

$$G = -\sqrt{ab} \quad \because G < 0$$

$$= -\sqrt{\left(-\frac{2}{5}\right)\left(-\frac{8}{5}\right)} = -\sqrt{\frac{16}{25}} = -\frac{4}{5}$$

$$H = \frac{2ab}{a+b} = \frac{2\left(-\frac{2}{5}\right)\left(-\frac{8}{5}\right)}{-\frac{2}{5} - \frac{8}{5}} = \frac{32/25}{-10/5} \\ = \frac{32/25}{-2} = -\frac{32}{50} = -\frac{16}{25}$$

$$\text{Since } -1 < -\frac{4}{5} < -\frac{16}{25}$$

$$\Rightarrow A < G < H \quad \text{proved}$$

Q.No 16 Let a & b be two number

$$\text{Since } H \cdot M = 4$$

$$\Rightarrow \frac{2ab}{a+b} = 4$$

$$\Rightarrow 2ab = 4(a+b)$$

$$\Rightarrow ab = 2(a+b) \quad \text{--- (i)}$$

$$\text{Also } A \cdot M = \frac{9}{2}$$

$$\Rightarrow \frac{a+b}{2} = \frac{9}{2}$$

$$\Rightarrow a+b = 9 \quad \text{--- (ii)}$$

putting value of $a+b$ in (i)

$$ab = 2(9) \Rightarrow ab = 18$$

$$\Rightarrow a = \frac{18}{b} \quad \text{--- (iii)}$$

putting in (ii)

$$\frac{18}{b} + b = 9$$

$$\Rightarrow \frac{18+b^2}{b} = 9$$

$$\Rightarrow 18+b^2 = 9b$$

$$\Rightarrow b^2 - 9b + 18 = 0$$

$$\Rightarrow b^2 - 6b - 3b + 18 = 0$$

$$\Rightarrow b(b-6) - 3(b-6) = 0$$

$$\Rightarrow (b-6)(b-3) = 0$$

$$b-6 = 0 \quad \text{or} \quad b-3 = 0$$

$$b = 6 \quad \text{or} \quad b = 3$$

putting in (iii)

$$a = \frac{18}{6} = 3 \quad \Bigg| \quad a = \frac{18}{3} = 6$$

Thus 3, 6 OR 6, 3 are required numbers

Q.No. 17

Let a & b be two number.

$$\text{Since } G \cdot M = 4$$

$$\Rightarrow \sqrt{ab} = 4$$

$$\Rightarrow ab = 16 \quad \text{--- (i) on squaring}$$

Also

$$H \cdot M = \frac{16}{5}$$

$$\Rightarrow \frac{2ab}{a+b} = \frac{16}{5}$$

Do your self
The End