Here, $a_1 = 4A$, $r = \frac{16A}{4A} = 4$, $n = n$

$\because a_n = a_1 r^{n-1}$

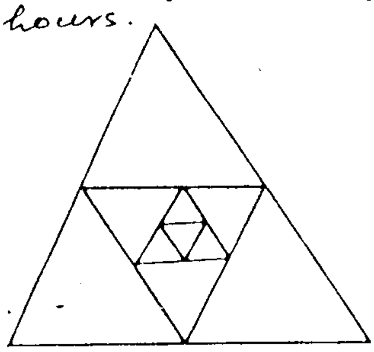
$\rightarrow a_n = 4A(4)^{n-1}$

$a_n = 4 \cdot 4^{n-1} \cdot A \rightarrow a_n = 4^n A$

or $a_n = (2^2)^n A \rightarrow a_n = 2^{2n} \cdot A$.

Which is required no. of bacterias in n hours.

Q.6



Perimeter of 1st equilateral triangle $= \frac{3}{2}$

Perimeter of 2nd equilateral triangle $= \frac{1}{2}(\frac{3}{2}) = \frac{3}{4}$

Perimeter of 3rd equilateral triangle $= \frac{1}{2}(\frac{3}{4}) = \frac{3}{8}$

The sequence of Perimeters of the nested equilateral triangles is $\frac{3}{2}, \frac{3}{4}, \frac{3}{8}, \text{---------}$

To find its sum we have

$\frac{3}{2} + \frac{3}{4} + \frac{3}{8} + \text{--------}$

Here, $a_1 = \frac{3}{2}$, $r = \frac{3/4}{3/2} = \frac{1}{2} < 1$

$\because S_\infty = \frac{a_1}{1-r}$

$S_\infty = \frac{3/2}{1-1/2} = \frac{3/2}{1/2} = \frac{3}{2} \times \frac{2}{1} = 3$

$S_\infty = 3$. Hence total perimeter of all the triangles $= 3$.

HARMONIC SEQUENCE (H.P)

A sequence of numbers whose reciprocals form an A.P is called Harmonic sequence or Harmonic Progression (H.P).

For example,

$\frac{1}{3}, \frac{1}{6}, \frac{1}{9}, \text{------}$ is in H.P.

where $3, 6, 9, \text{-------}$ is in A.P.

Generally, we represent A.P as

$a_1, a_1+d, a+2d, \text{---------}$

Similarly we represent H.P. as

$\frac{1}{a_1}, \frac{1}{a_1+d}, \frac{1}{a_1+2d}, \text{--------}$

Example.1

Given that

$\frac{1}{2}, \frac{1}{5}, \frac{1}{8}, \text{-------}$ is an H.P

$a_n = ?$ $\quad a_8 = ?$

Since $\frac{1}{2}, \frac{1}{5}, \frac{1}{8}, \text{-----}$ is in H.P

So, $2, 5, 8, \text{------}$ is in A.P.

Here $a_1 = 2$, $d = 5 - 2 = 3$

$\because a_n = a_1 + (n-1)d$

$a_n = 2 + (n-1)3 = 2 + 3n - 3$

$a_n = 3n - 1$

Put $n = 8 \rightarrow a_8 = 3(8) - 1 = 23$

Hence in H.P

$a_n = \frac{1}{3n-1}$ and $a_8 = \frac{1}{23}$.

Example.2

Given that in H.P $\quad a_4 = \frac{2}{13}$, $a_7 = \frac{2}{25}$

Now in A.P.

$a_4 = \frac{13}{2}$, $a_7 = \frac{25}{2}$

$\because a_4 = a_1 + 3d \rightarrow a_1 + 3d = \frac{13}{2} \rightarrow (1)$

$\because a_7 = a_1 + 6d \rightarrow a_1 + 6d = \frac{25}{2} \rightarrow (2)$

Subtracting eq (1) from eq (2) we get

$3d = \frac{25}{2} - \frac{13}{2}$

$3d = \frac{12}{2} \rightarrow 3d = 6$ or $d = 2$

Put $d = 2$ in (1).

$a_1 + 3(2) = \frac{13}{2}$

$a_1 = \frac{13}{2} - 6 = \frac{13-12}{2} = \frac{1}{2}$

As $a_1 = \frac{1}{2}$, $d = 2$ Then A.P is

$a_1, a_1+d, a_1+2d, \text{--------}$

$\frac{1}{2}, (\frac{1}{2}+2), (\frac{1}{2}+2(2)), \text{------}$

$\frac{1}{2}, 5/2, 9/2, \text{---------}$

Hence required H.P will be

$\frac{2}{1}, \frac{2}{5}, \frac{2}{9}, \text{-----------}$ Ans.

HARMONIC MEAN(H.M)

A number H is said to be the harmonic mean (H.M) between two numbers a and b if

a, H, b are in H.P

$\Rightarrow \frac{1}{a}, \frac{1}{H}, \frac{1}{b}$ are in A.P.

In this case,

$d = \frac{1}{H} - \frac{1}{a}$ also $d = \frac{1}{b} - \frac{1}{H}$

$\Rightarrow \frac{1}{H} - \frac{1}{a} = \frac{1}{b} - \frac{1}{H}$

$\Rightarrow \frac{1}{H} + \frac{1}{H} = \frac{1}{a} + \frac{1}{b}$

$\frac{1+1}{H} = \frac{b+a}{ab}$

$\frac{2}{H} = \frac{a+b}{ab}$

$\Rightarrow (a+b)H = 2ab$

$\Rightarrow H = \frac{2ab}{a+b}$

Example.3

Let H_1, H_2, H_3 be H.M's between $\frac{1}{5}$ and $\frac{1}{17}$, Then

$\frac{1}{5}, H_1, H_2, H_3, \frac{1}{17}$ are in H.P

$\Rightarrow 5, \frac{1}{H_1}, \frac{1}{H_2}, \frac{1}{H_3}, 17$ are in A.P

$a_1 = 5, \ a_n = 17, \ n = 5$

$a_n = a_1 + (n-1)d$

$17 = 5 + (5-1)d$

$17 - 5 = 5d - d \Rightarrow 4d = 12 \Rightarrow d = 3$

$\frac{1}{H_1} = a_1 + d = 5 + 3 = 8$

$\Rightarrow H_1 = \frac{1}{8}$

$\frac{1}{H_2} = \frac{1}{H_1} + d = 8 + 3 = 11$

$\Rightarrow H_2 = \frac{1}{11}$

$\frac{1}{H_3} = \frac{1}{H_2} + d = 11 + 3 = 14$

$\Rightarrow H_3 = \frac{1}{14}$ Hence required H.M's are $\frac{1}{8}, \frac{1}{11}, \frac{1}{14}, \text{------}$

Example.4

Let, $H_1, H_2, H_3, \text{---------} H_n$ be n H.M's between a and b. Then

$a, H_1, H_2, H_3, \text{--------}, H_n, b$ are in H.P

$\Rightarrow \frac{1}{a}, \frac{1}{H_1}, \frac{1}{H_2}, \frac{1}{H_3}, \text{------}, \frac{1}{H_n}, \frac{1}{b}$ are in A.P.

$\because a_n = a_1 + (n-1)d$

$\Rightarrow \frac{1}{b} = \frac{1}{a} + (n+2-1)d$

$\frac{1}{b} - \frac{1}{a} = (n+1)d$

$\frac{a-b}{ab} = (n+1)d$

$d = \frac{a-b}{ab(n+1)}$

$\frac{1}{H_1} = \frac{1}{a} + d = \frac{1}{a} + \frac{a-b}{ab(n+1)}$

$= \frac{b(n+1)+a-b}{ab(n+1)} = \frac{bn+b+a-b}{ab(n+1)}$

$= \frac{bn+a}{ab(n+1)}$

$\Rightarrow H_1 = \frac{ab(n+1)}{bn+a}$

$\frac{1}{H_2} = \frac{1}{H_1} + d$

$= \frac{nb+a}{ab(n+1)} + \frac{a-b}{ab(n+1)}$

$= \frac{nb+a+a-b}{ab(n+1)}$

$= \frac{b(n-1)+2a}{ab(n+1)}$

$\Rightarrow H_2 = \frac{ab(n+1)}{(n-1)b+2a}$

Similarly,

$H_3 = \frac{ab(n+1)}{(n-2)b+3a}$

$H_n = \frac{ab(n+1)}{[n-(n-1)]b+na}$

$H_n = \frac{ab(n+1)}{(n-n+1)b+na}$

$= \frac{ab(n+1)}{b+na}$

Hence n H.M's between a and b are

$\frac{ab(n+1)}{nb+a}, \frac{ab(n+1)}{(n-1)b+2a}, \frac{ab(n+1)}{(n-2)b+3a}, \text{------} \text{-----}, \frac{ab(n+1)}{b+na}$

RELATION BETWEEN A.M,G.M AND H.M

Q. Prove that A, G, H are in G.P

PROOF:- We know that for any two numbers a and b.

$A = \frac{a+b}{2}, \ G = \pm\sqrt{ab}, \ H = \frac{2ab}{a+b}$

Now, $AH = \left(\frac{a+b}{2}\right)\left(\frac{2ab}{a+b}\right) = ab$

also, $G^2 = (\pm\sqrt{ab})^2 = ab$

$\Rightarrow G^2 = AH$

$\Rightarrow G.G = AH$

$\Rightarrow \frac{G}{A} = \frac{H}{G}$

$\Rightarrow$ A, G, H are in G.P

Q. Prove that $A > G > H$ if a, b are any two distinct +ve real numbers and $G = \sqrt{ab}$

Proof:- We know that,

$A = \frac{a+b}{2}$, $G = \pm\sqrt{ab}$, $H = \frac{2ab}{a+b}$

Let $A > G$

$\Rightarrow \frac{a+b}{2} > \pm\sqrt{ab}$

Squaring both sides

$\left(\frac{a+b}{2}\right)^2 > (\pm\sqrt{ab})^2$

$\frac{(a+b)^2}{4} > ab$

$a^2+b^2+2ab > 4ab$

$a^2+b^2+2ab-4ab > 0$

$a^2+b^2-2ab > 0$

$\Rightarrow (a-b)^2 > 0$

Which is always true,

so $A > G \longrightarrow (1)$

Again let $G > H$

$\Rightarrow \pm\sqrt{ab} > \frac{2ab}{a+b}$

Squaring both sides.

$ab > \frac{4a^2b^2}{(a+b)^2}$

$(a+b)^2 > \frac{4a^2b^2}{ab}$

$a^2+b^2+2ab > 4ab$

$a^2+b^2+2ab-4ab > 0$

$a^2+b^2-2ab > 0$

$\Rightarrow (a-b)^2 > 0$

Which is always true.

So, $G > H \longrightarrow (2)$

From (1) & (2)

$A > G > H$

Q. Prove that $A < G < H$ if a, b are any two distinct negative real numbers and $G = -\sqrt{ab}$

Proof:- Suppose $a = -p$, $b = -q$ $\forall\, q, b \in \mathbb{R}$

Let $A < G$

$\because A = \frac{a+b}{2} \Rightarrow A = \frac{-p+(-q)}{2}$

$G = -\sqrt{ab}$ (given)

$= -\sqrt{(-p)(-q)} = -\sqrt{pq}$

So, $\frac{-p-q}{2} < -\sqrt{pq}$

$\Rightarrow -\left(\frac{p+q}{2}\right) < -\sqrt{pq}$

$\Rightarrow \frac{p+q}{2} > \sqrt{pq}$

As $-2 < -1$ But $2 > 1$

$\Rightarrow p+q > 2\sqrt{pq}$

Squaring both sides

$(p+q)^2 > 4pq$

$p^2+q^2+2pq-4pq > 0$ Bad Scanned Area

$\Rightarrow (p-q)^2 > 0$

Which is always true.

Hence $A < G \longrightarrow (1)$

Again let $G < H$

$\because G = -\sqrt{ab} = -\sqrt{(-p)(-q)} = -\sqrt{pq}$

$H = \frac{2ab}{a+b} = \frac{2(-p)(-q)}{(-p)+(-q)} = \frac{2pq}{-p-q}$

So, $-\sqrt{pq} < \frac{2pq}{-p-q}$

$-\sqrt{pq} < -\left(\frac{2pq}{p+q}\right)$

$\Rightarrow \sqrt{pq} > \frac{2pq}{p+q}$

Squaring both sides

$pq > \frac{(2pq)^2}{(p+q)^2}$

$\Rightarrow (p+q)^2 > \frac{4p^2q^2}{pq}$

$\Rightarrow p^2+q^2+2pq > 4pq$

$p^2+q^2+2pq-4pq>0$

$p^2+q^2-2pq>0$

or $(p-q)^2>0$

which is always true

$G<H \longrightarrow ②$

so from ① and ②

$A<G<H.$

EXERCISE.6.10

Q.1 Given that $a_9=?$

(i) When $\frac{1}{3},\frac{1}{5},\frac{1}{7},\text{-------}$ are in H.P

$\Rightarrow$ $3,5,7,\text{--------}$ are in A.P

Here $a_1=3,\ d=5-3=2$

$\because a_n=a_1+(n-1)d$

$\Rightarrow$ $a_9=a_1+(9-1)d$

$a_9=a_1+8d$

So, $a_9=3+8(2)=19$

Hence in H.P $a_9=\frac{1}{19}$

(ii) Given that $a_9=?$

When $-\frac{1}{5},-\frac{1}{3},-1,\text{------}$ are in H.P

$\Rightarrow$ $-5,-3,-1,\text{------}$ are in A.P

Here $a_1=-5,\ d=-3-(-5)=-3+5=2$

$\because a_n=a_1+(n-1)d$

$\Rightarrow$ $a_9=a_1+(9-1)d$

$a_9=a_1+8d$

So, $a_9=-5+8(2)=11$

Hence in H.P

$a_9=\frac{1}{11}$

Q.2 (i) Given that $a_{12}=?$

When, $\frac{1}{2},\frac{1}{5},\frac{1}{8},\text{------}$ are in H.P

$\Rightarrow$ $2,5,8,\text{---------}$ are in A.P

Here $a_1=2,\ d=5-2=3$

$\because a_n=a_1+(n-1)d$

$\Rightarrow$ $a_{12}=a_1+(12-1)d$

$a_{12}=a_1+11d$

So, $a_{12}=2+11(3)=35$

Hence in H.P $a_{12}=\frac{1}{35}$

(ii) Given that $a_{12}=?$

When, $\frac{1}{3},\frac{2}{9},\frac{1}{6},\text{------}$ are in H.P

$\Rightarrow$ $3,\frac{9}{2},6,\text{----------}$ are in A.P

Here $a_1=3,\ d=\frac{9}{2}-3=\frac{9-6}{2}=\frac{3}{2}$

$\because a_n=a_1+(n-1)d$

$a_{12}=a_1+(12-1)d$

$a_{12}=a_1+11d$

So, $a_{12}=3+11(\frac{3}{2})$

$a_{12}=3+\frac{33}{2}=\frac{6+33}{2}$

$a_{12}=\frac{39}{2}$

Hence in H.P $a_{12}=\frac{2}{39}$

Q.3 (i) Let H_1,H_2,H_3,H_4 and H_5 be five H.M's between $-\frac{2}{5}$ and $\frac{2}{13}$

Then, $-\frac{2}{5},H_1,H_2,H_3,H_4,H_5,\frac{2}{13}$ are in H.P

$\Rightarrow$ $-\frac{5}{2},\frac{1}{H_1},\frac{1}{H_2},\frac{1}{H_3},\frac{1}{H_4},\frac{1}{H_5},\frac{13}{2}$ are in A.P

Here $a_1=-\frac{5}{2},\ a_n=\frac{13}{2},\ n=7$

$\because a_n=a_1+(n-1)d$

$\Rightarrow$ $\frac{13}{2}=-\frac{5}{2}+(7-1)d$

$\frac{13}{2}+\frac{5}{2}=6d$

$\frac{18}{2}=6d \Rightarrow 6d=9$

$\Rightarrow d=\frac{9}{6}=\frac{3}{2}$

Now, $\frac{1}{H_1}=a_1+d=\frac{-5}{2}+\frac{3}{2}=\frac{-2}{2}=-1$

$\Rightarrow H_1=-1$

$\frac{1}{H_2}=\frac{1}{H_1}+d=-1+\frac{3}{2}=\frac{-2+3}{2}=\frac{1}{2}$

$\Rightarrow H_2=2$

$\frac{1}{H_3}=\frac{1}{H_2}+d=\frac{1}{2}+\frac{3}{2}=\frac{4}{2}=2$

$\Rightarrow H_3=\frac{1}{2}$

$\frac{1}{H_4}=\frac{1}{H_3}+d=2+\frac{3}{2}=\frac{4+3}{2}=\frac{7}{2}$

$\Rightarrow H_4=\frac{2}{7}$

$\frac{1}{H_5}=\frac{1}{H_4}+d=\frac{7}{2}+\frac{3}{2}=\frac{10}{2}=5$

$\Rightarrow H_5=\frac{1}{5}$

Hence required 5 H.M's are $-1,2,\frac{1}{2},\frac{2}{7},\frac{1}{5}$

(ii) Let H_1, H_2, H_3, H_4 and H_5 be five H.M's between $\frac{1}{4}$ and $\frac{1}{24}$

Then, $\frac{1}{4}, H_1, H_2, H_3, H_4, H_5, \frac{1}{24}$ are in H.P

$\Rightarrow 4, \frac{1}{H_1}, \frac{1}{H_2}, \frac{1}{H_3}, \frac{1}{H_4}, \frac{1}{H_5}, 24$ are in A.P

Here $a_1 = 4$, $a_n = 24$, $n = 7$

$\because a_n = a_1 + (n-1)d$

$\Rightarrow 24 = 4 + (7-1)d$

$24 - 4 = 6d \Rightarrow 6d = 20$

$\Rightarrow d = \frac{20}{6} = \frac{10}{3}$

Now $\frac{1}{H_1} = a_1 + d = 4 + \frac{10}{3} = \frac{12+10}{3}$

$\frac{1}{H_1} = \frac{22}{3} \Rightarrow H_1 = \frac{3}{22}$

$\frac{1}{H_2} = \frac{1}{H_1} + d = \frac{22}{3} + \frac{10}{3} = \frac{32}{3}$

$\Rightarrow H_2 = \frac{3}{32}$

$\frac{1}{H_3} = \frac{1}{H_2} + d = \frac{32}{3} + \frac{10}{3}$

$\frac{1}{H_3} = \frac{42}{3} = 14 \Rightarrow H_3 = \frac{1}{14}$

$\frac{1}{H_4} = \frac{1}{H_3} + d = 14 + \frac{10}{3} = \frac{42+10}{3} = \frac{52}{3}$

$\Rightarrow H_4 = \frac{3}{52}$

$\frac{1}{H_5} = \frac{1}{H_4} + d = \frac{52}{3} + \frac{10}{3} = \frac{52+10}{3}$

$\frac{1}{H_5} = \frac{62}{3} \Rightarrow H_5 = \frac{3}{62}$

Hence required five H.M's are $\frac{3}{22}, \frac{3}{32}, \frac{1}{14}, \frac{3}{52}, \frac{3}{62}$.

Q.4 (i) Let H_1, H_2, H_3, H_4 be four H.M's between $\frac{1}{3}$ and $\frac{1}{23}$

Then, $\frac{1}{3}, H_1, H_2, H_3, H_4, \frac{1}{23}$ are in H.P

$\Rightarrow 3, \frac{1}{H_1}, \frac{1}{H_2}, \frac{1}{H_3}, \frac{1}{H_4}, 23$ are in A.P

Here, $a_1 = 3$, $a_n = 23$, $n = 6$

$\because a_n = a_1 + (n-1)d$

$\Rightarrow 23 = 3 + (6-1)d$

$23 - 3 = 5d \Rightarrow 5d = 20$

$d = 4$

Now, $\frac{1}{H_1} = a_1 + d = 3 + 4 = 7$

$\Rightarrow H_1 = \frac{1}{7}$

$\frac{1}{H_2} = \frac{1}{H_1} + d = 7 + 4 = 11$

$\frac{1}{H_2} = 11 \Rightarrow H_2 = \frac{1}{11}$

$\frac{1}{H_3} = \frac{1}{H_2} + d = 11 + 4 = 15$

$\Rightarrow H_3 = \frac{1}{15}$

$\frac{1}{H_4} = \frac{1}{H_3} + d = 15 + 4 = 19$

$\Rightarrow H_4 = \frac{1}{19}$

Hence four H.M's between $\frac{1}{3}$ and $\frac{1}{23}$ are $\frac{1}{7}, \frac{1}{11}, \frac{1}{15}, \frac{1}{19}$.

(ii) Let H_1, H_2, H_3, H_4 be four H.M's between $\frac{7}{3}$ and $\frac{7}{11}$

Then $\frac{7}{3}, H_1, H_2, H_3, H_4, \frac{7}{11}$ are in H.P.

$\Rightarrow \frac{3}{7}, \frac{1}{H_1}, \frac{1}{H_2}, \frac{1}{H_3}, \frac{1}{H_4}, \frac{11}{7}$ are in A.P

Here $a_1 = \frac{3}{7}$, $a_n = \frac{11}{7}$, $n = 6$

$\because a_n = a_1 + (n-1)d$

$\frac{11}{7} = \frac{3}{7} + (6-1)d$

$\frac{11}{7} - \frac{3}{7} = 5d \Rightarrow 5d = \frac{11-3}{7} = \frac{8}{7}$

$\Rightarrow d = \frac{8}{35}$ Now,

$\frac{1}{H_1} = a_1 + d = \frac{3}{7} + \frac{8}{35} = \frac{3(5)+8}{35}$

$\frac{1}{H_1} = \frac{15+8}{35} = \frac{23}{35} \Rightarrow H_1 = \frac{35}{23}$

$\frac{1}{H_2} = \frac{1}{H_1} + d = \frac{23}{35} + \frac{8}{35} = \frac{31}{35} \Rightarrow H_2 = \frac{35}{31}$

$\frac{1}{H_3} = \frac{1}{H_2} + d = \frac{31}{35} + \frac{8}{35} = \frac{39}{35} \Rightarrow H_3 = \frac{35}{39}$

$\frac{1}{H_4} = \frac{1}{H_3} + d = \frac{39}{35} + \frac{8}{35} = \frac{47}{35} \Rightarrow H_4 = \frac{35}{47}$

Hence four H.M's between $7/3$ and $7/11$ are $\frac{35}{23}, \frac{35}{31}, \frac{35}{39}, \frac{35}{47}$.

(iii) Let H_1, H_2, H_3, H_4 be four H.M's between 4 and 20. Then

$4, H_1, H_2, H_3, H_4, 20$ are in H.P.

$\Rightarrow \frac{1}{4}, \frac{1}{H_1}, \frac{1}{H_2}, \frac{1}{H_3}, \frac{1}{H_4}, \frac{1}{20}$ are in A.P.

Here $a_1 = \frac{1}{4}$, $a_n = \frac{1}{20}$, $n = 6$

$\because a_n = a_1 + (n-1)d$

$\Rightarrow \frac{1}{20} = \frac{1}{4} + (6-1)d$

$5d = \frac{1}{20} - \frac{1}{4} = \frac{1-5}{20} = \frac{-4}{20} = -1/5$

$\Rightarrow d = -1/25$

Now, $\frac{1}{H_1} = a_1 + d = \frac{1}{4} + (-\frac{1}{25}) = \frac{25-4}{100}$

$\frac{1}{H_1} = \frac{21}{100} \rightarrow H_1 = \frac{100}{21}$

$\frac{1}{H_2} = \frac{1}{H_1} + d = \frac{21}{100} + (\frac{-1}{25}) = \frac{21-4}{100}$

$\frac{1}{H_2} = \frac{17}{100} \rightarrow H_2 = \frac{100}{17}$

$\frac{1}{H_3} = \frac{1}{H_2} + d = \frac{17}{100} + (-\frac{1}{25}) = \frac{17-4}{100}$

$\frac{1}{H_3} = \frac{13}{100} \rightarrow H_3 = \frac{100}{13}$

$\frac{1}{H_4} = \frac{1}{H_3} + d = \frac{13}{100} + (\frac{-1}{25}) = \frac{13-4}{100}$

$\frac{1}{H_4} = \frac{9}{100} \rightarrow H_4 = \frac{100}{9}$

Hence four H.M's between 4 & 20

are $\frac{100}{21}, \frac{100}{17}, \frac{100}{13}, \frac{100}{9}$

Q.5

Given that $a_{14} = ?$

When in H.P

$a_7 = \frac{1}{3}$, $a_{10} = \frac{5}{21}$

$\rightarrow$ In A.P $a_7 = 3$, $a_{10} = \frac{21}{5}$

$\because a_7 = a_1 + 6d \rightarrow a_1 + 6d = 3 \rightarrow (1)$

Also, $a_{10} = a_1 + 9d \rightarrow a_1 + 9d = \frac{21}{5} \rightarrow (2)$

Subtracting eq (1) from eq (2)

$3d = \frac{21}{5} - 3 = \frac{21-15}{5} = \frac{6}{5}$

$\rightarrow d = \frac{6}{5} \times \frac{1}{3} = \frac{2}{5}$

Put $d = \frac{2}{5}$ in eq (1)

$a_1 + 6(\frac{2}{5}) = 3$

$a_1 = 3 - \frac{12}{5} = \frac{15-12}{5}$

$\rightarrow a_1 = \frac{3}{5}$

$a_{14} = a_1 + 13d = \frac{3}{5} + 13(\frac{2}{5}) = \frac{3}{5} + \frac{26}{5} = \frac{29}{5}$

Hence in H.P $a_{14} = \frac{5}{29}$

Q.6

Given that $a_9 = ?$

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When in H.P [illegible]

$\rightarrow$ In A.P [illegible]

$\because a_5 = a_1 + 4d \rightarrow a_1 + 4d = 5 \rightarrow (1)$

Put $a_1 = -3$ in eq (1).

$-3 + 4d = 5 \rightarrow 4d = 5 + 3 = 8 \rightarrow d = 2$

as $a_9 = a_1 + 8d$

$\rightarrow a_9 = -3 + 8(2) = -3 + 16 = 13$

Hence in H.P

$a_9 = \frac{1}{13}$

Q.7

Here

$a = 2$, $b = b$, $H.M = 5$

$\because H.M = \frac{2ab}{a+b}$

$\rightarrow 5 = \frac{(2)(2)b}{2+b}$

$5(2+b) = 4b$

$10 + 5b = 4b$

$5b - 4b = -10 \rightarrow b = -10$

Q.8

Given that

$\frac{1}{K}, \frac{1}{2K+1}, \frac{1}{4K-1}$ are in H.P

$\rightarrow K, 2K+1, 4K-1$ are in A.P

Now, $d = 2K+1-K$ also $d = 4K-1-(2K+1)$

$d = K+1$ also $d = 4K-1-2K-1$

$d = 2K-2$

$\rightarrow K+1 = 2K-2$

$1+2 = 2K-K$

$\rightarrow K = 3$

Q.9

We know that if H is H.M between two numbers a & b.

Then, $H = \frac{2ab}{a+b}$

So for given condition

$\frac{a^{n+1} + b^{n+1}}{a^n + b^n} = \frac{2ab}{a+b}$

$(a+b)(a^{n+1} + b^{n+1}) = 2ab(a^n + b^n)$

$a.a^{n+1} + ab^{n+1} + a^{n+1}.b + b.b^{n+1} = 2a^{n+1}b + 2ab^{n+1}$

$a.a^{n+1} + ba^{n+1} - 2a^{n+1}b = 2ab^{n+1} - ab^{n+1} - b.b^{n+1}$

$a.a^{n+1} - ba^{n+1} = ab^{n+1} - b.b^{n+1}$

$a^{n+1}(a-b) = (a-b)b^{n+1}$

$\rightarrow (a-b)(a^{n+1} - b^{n+1}) = 0$

Either $a - b = 0$ or $a^{n+1} - b^{n+1} = 0$

But $a - b \neq 0$ because $a \neq b$

so, $a^{n+1} = b^{n+1}$

$\frac{a^{n+1}}{b^{n+1}} = \frac{b^{n+1}}{b^{n+1}}$

$\rightarrow (\frac{a}{b})^{n+1} = 1 \rightarrow (\frac{a}{b})^{n+1} = (\frac{a}{b})^0$

$\rightarrow n+1 = 0$

$\rightarrow \boxed{n = -1}$ Ans

Q.10

Given that

a^2, b^2, c^2 are in A.P

$\Rightarrow b^2 - a^2 = c^2 - b^2$

$(b-a)(b+a) = (c-b)(c+b)$

$-(a-b)(a+b) = -(b-c)(b+c)$

$\Rightarrow (a-b)(a+b) = (b-c)(b+c)$

$\Rightarrow \dfrac{a-b}{b+c} = \dfrac{b-c}{a+b} \longrightarrow ①$

We are to prove

$a+b,\ c+a,\ b+c$ are in H.P

or $\dfrac{1}{a+b}, \dfrac{1}{c+a}, \dfrac{1}{b+c}$ are in A.P

$\Rightarrow \dfrac{1}{c+a} - \dfrac{1}{a+b} = \dfrac{1}{b+c} - \dfrac{1}{c+a}$

$\dfrac{a+b-(c+a)}{(c+a)(a+b)} = \dfrac{(c+a)-(b+c)}{(b+c)(c+a)}$

$\dfrac{a+b-c-a}{a+b} = \dfrac{c+a-b-c}{b+c}$

$\Rightarrow \dfrac{b-c}{a+b} = \dfrac{a-b}{b+c}$

Which is already proved

Thus, $a+b,\ c+a,\ b+c$ are in H.P

Q.11

Given that in H.P

$a_1 + a_5 = \frac{4}{7}$, $a_1 = \frac{1}{2}$

Put $a_1 = \frac{1}{2}$

$\frac{1}{2} + a_5 = \frac{4}{7} \Rightarrow a_5 = \frac{4}{7} - \frac{1}{2}$

$a_5 = \dfrac{8-7}{14} \Rightarrow a_5 = \dfrac{1}{14}$

So in H.P

$a_1 = \frac{1}{2}$, $a_5 = \frac{1}{14}$

$\Rightarrow$ In A.P $a_1 = 2$, $a_5 = 14$

$\because a_5 = a_1 + 4d$

$\Rightarrow a_1 + 4d = 14 \Rightarrow 4d = 14 - a_1$

$\Rightarrow 4d = 14 - 2 = 12 \Rightarrow d = \frac{12}{4} = 3$

Now A.P is

$a_1, a_1+d, a_1+2d, a_1+3d, \text{-------}$

$2, (2+3), (2+2(3)), \text{--------}$

$2, 5, 8, 11 \text{-------}$

Now, $\frac{1}{2}, \frac{1}{5}, \frac{1}{8}, \frac{1}{11}, \text{-------}$ is required H.P

Q.12

We know that

$A = \dfrac{a+b}{2}$, $G = \pm\sqrt{ab}$, $H = \dfrac{2ab}{a+b}$

Now $G^2 = ab \longrightarrow ①$

$AH = \left(\dfrac{a+b}{2}\right)\left(\dfrac{2ab}{a+b}\right) = ab$

or $AH = ab \longrightarrow ②$

From eq ① & eq ②.

$G^2 = AH$.

Q.13

(i) Given that

$a = -2$, $b = -6$

$\because A = \dfrac{a+b}{2} \Rightarrow A = \dfrac{-2+(-6)}{2} = \dfrac{-8}{2} = -4$

$\because G = \pm\sqrt{ab} \Rightarrow G = \pm\sqrt{(-2)(-6)} = \pm\sqrt{12}$

$\because H = \dfrac{2ab}{a+b} \Rightarrow H = \dfrac{2(-2)(-6)}{-2+(-6)} = \dfrac{24}{-8} = -3$

Now, $G^2 = (\pm\sqrt{12})^2 = 12$

and $AH = (-4)(-3) = 12$

So, $G^2 = AH$.

(ii) Given that

$a = 2i$, $b = 4i$

$\because A = \dfrac{a+b}{2} \Rightarrow A = \dfrac{2i+4i}{2} = \dfrac{6i}{2} = 3i$

$\because G = \pm\sqrt{ab} \Rightarrow G = \pm\sqrt{(2i)(4i)} = \pm\sqrt{8i^2}$

$= \pm\sqrt{8(-1)} = \pm\sqrt{-8}$

$\because H = \dfrac{2ab}{a+b} \Rightarrow H = \dfrac{2(2i)(4i)}{2i+4i} = \dfrac{16i^2}{6i} = \dfrac{8i}{3}$

Now, $G^2 = (\pm\sqrt{-8})^2 = -8$

$AH = (3i)\left(\dfrac{8i}{3}\right) = 8i^2 = -8$

Hence, $G^2 = AH$.

(iii) Given that $G > 0$

$a = 9$, $b = 4$

$\because A = \dfrac{a+b}{2} \Rightarrow A = \dfrac{9+4}{2} = \dfrac{13}{2}$

$\because G = \sqrt{ab} \Rightarrow G = \sqrt{(9)(4)} = \sqrt{36} = 6$

$\because H = \dfrac{2ab}{a+b} = \dfrac{2(9)(4)}{9+4} = \dfrac{72}{13}$

Now, $G^2 = (6)^2 = 36$

$AH = \left(\dfrac{13}{2}\right)\left(\dfrac{72}{13}\right) = \dfrac{72}{2}$

$AH = 36$

So, $G^2 = AH$

Q.14 Given that $G>0$

(i) $a=2$, $b=8$

$\because A=\frac{a+b}{2}=\frac{2+8}{2}=\frac{10}{2}=5$

$\because G=\sqrt{ab}=\sqrt{2\times 8}=\sqrt{16}=4$

$\because H=\frac{2ab}{a+b}=\frac{2(2)(8)}{2+8}=\frac{32}{10}=\frac{16}{5}=3.2$

$\Rightarrow 5>4>3.2$

$\Rightarrow A>G>H$

(ii) Given that $G>0$

$a=\frac{2}{5}$, $b=8/5$

$\because A=\frac{a+b}{2}=\frac{1}{2}\left(\frac{2}{5}+\frac{8}{5}\right)=\frac{10}{10}=1$

$\because G=\sqrt{ab}=\sqrt{\frac{2}{5}\times\frac{8}{5}}=\sqrt{\frac{16}{25}}=\frac{4}{5}$

$\because H=\frac{2ab}{a+b}=\frac{2(\frac{2}{5})(8/5)}{\frac{2}{5}+\frac{8}{5}}=\frac{32/25}{(2+8)/5}$

$H=\frac{32/25}{10/5}=\frac{32}{25}\times\frac{5}{10}=\frac{32}{50}$

$\Rightarrow 1>4/5>\frac{32}{50}$

$\Rightarrow A>G>H$

Q.15 Given that $G<0$

(i) $a=-2$, $b=-8$

$\because A=\frac{a+b}{2}=\frac{(-2)+(-8)}{2}=\frac{-10}{2}=-5$

$\because G=-\sqrt{ab}=-\sqrt{(-2)(-8)}=-\sqrt{16}=-4$

$\because H=\frac{2ab}{a+b}=\frac{2(-2)(-8)}{-2+(-8)}=\frac{32}{-10}=-3.2$

$\Rightarrow -5<-4<-3.2$.

$\Rightarrow A<G<H$

(ii) Given that $G<0$

$a=\frac{-2}{5}$, $b=-8/5$

$\because A=\frac{a+b}{2} \Rightarrow A=\frac{1}{2}\left(\frac{-2}{5}-\frac{8}{5}\right)$

$A=\frac{1}{2}\left(\frac{-2-8}{5}\right)=\frac{1}{2}\left(\frac{-10}{5}\right)=-1$

$G=-\sqrt{ab}=-\sqrt{(\frac{-2}{5})(-8/5)}=-\sqrt{\frac{16}{25}}=-4/5$

$H=\frac{2ab}{a+b}=\frac{2(-2/5)(-8/5)}{-2/5+(-8/5)}=\frac{32/25}{-10/5}$

$H=\frac{-32}{50}=-16/25$

$\Rightarrow -1<-4/5<-16/25$

$\Rightarrow A<G<H$.

Q.16 Let a and b be numbers

Given that

$H.M=4$, $A.M=\frac{9}{2}$

$\because H.M=\frac{2ab}{a+b} \Rightarrow \frac{2ab}{a+b}=4 \rightarrow (1)$

also $A.M=\frac{a+b}{2} \Rightarrow \frac{a+b}{2}=\frac{9}{2} \rightarrow (2)$

From eq (2) $a+b=9 \rightarrow (3)$

Putting value of $a+b$ in (1)

$\frac{2ab}{9}=4 \Rightarrow 2ab=36$

$\Rightarrow ab=18$

$\because (a-b)^2=(a+b)^2-4ab$

$\Rightarrow (a-b)^2=(9)^2-4(18)=81-72=9$

$\Rightarrow a-b=\pm 3$

$a-b=3 \rightarrow (4)$, $a-b=-3 \rightarrow (5)$

Adding (3) & (4)

$2a=12$

$\Rightarrow a=6$

If $a=6$ then by (3)

$6+b=9$

$b=9-6=3$

Adding (3) & (5)

$2a=6$

$\Rightarrow a=3$

If $a=3$ then by (3)

$3+b=9$

$b=9-3=6$

Hence required numbers are
6, 3 OR 3, 6

Q.17 Let a and b be numbers

Given that

$G.M=4$, $H.M=\frac{16}{5}$

$\because G.M$ (positive) $=\sqrt{ab}=4$

$\Rightarrow ab=16 \rightarrow (1)$

$\because H.M=\frac{2ab}{a+b} \Rightarrow \frac{2ab}{a+b}=\frac{16}{5}$

$\Rightarrow \frac{ab}{a+b}=8/5 \Rightarrow 5ab=8(a+b)$

$\Rightarrow a+b=5/8(ab)=5/8(16)=10$

$a+b=10 \rightarrow (2)$

$\because (a-b)^2=(a+b)^2-4ab$

$\Rightarrow (a-b)^2=(10)^2-4(16)$

$(a-b)^2=100-64=36$

$\Rightarrow a-b=\pm 6$

$a-b=6 \rightarrow (3)$, $a-b=-6 \rightarrow (4)$

Adding ② & ③

$2a = 16 \Rightarrow a = 8$

If $a=8$ Then by ②

$8 + b = 10$

$\Rightarrow b = 10 - 8$

$\Rightarrow b = 2$

Adding ② & ④

$2a = 4 \Rightarrow a = 2$

If $a = 2$ then by ②

$2 + b = 10$

$b = 10 - 2 = 8$

$\Rightarrow b = 8$

Hence required numbers are

8, 2 or 2, 8.

Q.18 Let the three consecutive terms of G.P be

a, ar, ar^2

According to given condition

$a - \frac{1}{2}, ar - \frac{4}{21}, ar^2 - \frac{1}{36}$ are in H.P

also $a(ar)(ar^2) = \frac{1}{27}$

$a^3r^3 = (\frac{1}{3})^3$

$\Rightarrow ar = \frac{1}{3} \Rightarrow r = \frac{1}{3a}$

Now putting $r = \frac{1}{3a}$ we have H.P as

$a - \frac{1}{2}, a(\frac{1}{3a}) - \frac{4}{21}, a(\frac{1}{3a})^2 - \frac{1}{36}$

$\frac{2a-1}{2}, \frac{1}{3} - \frac{4}{21}, \frac{1}{9a} - \frac{1}{36}$ are in H.P

$\frac{2a-1}{2}, \frac{7-4}{21}, \frac{4-a}{36a}$ are in H.P

$\frac{2a-1}{2}, \frac{3}{21}, \frac{4-a}{36a}$ are in H.P

$\frac{2a-1}{2}, \frac{1}{7}, \frac{4-a}{36a}$ are in H.P

$\Rightarrow \frac{2}{2a-1}, 7, \frac{36a}{4-a}$ are in A.P

$\Rightarrow 7 - \frac{2}{2a-1} = \frac{36a}{4-a} - 7$

$\Rightarrow 7 + 7 = \frac{36a}{4-a} + \frac{2}{2a-1}$

$14 = \frac{36a(2a-1) + 2(4-a)}{(4-a)(2a-1)}$

$14 = \frac{72a^2 - 36a + 8 - 2a}{8a - 4 - 2a^2 + a}$

$14 = \frac{72a^2 - 38a + 8}{-2a^2 + 9a - 4}$

$14(-2a^2 + 9a - 4) = 72a^2 - 38a + 8$

$-28a^2 + 126a - 56 = 72a^2 - 38a + 8$

$72a^2 + 28a^2 - 38a - 126a + 8 + 56 = 0$

$100a^2 - 164a + 64 = 0$

$25a^2 - 41a + 16 = 0$

$25a^2 - 25a - 16a + 16 = 0$

$25a(a-1) - 16(a-1) = 0$

$(a-1)(25a-16) = 0$

$a - 1 = 0 \quad , \quad 25a - 16 = 0$

$a = 1 \quad , \quad 25a = 16$

$\Rightarrow a = 16/25$

If $a = 1$ then $r = \frac{1}{3(1)} = \frac{1}{3}$

If $a = \frac{16}{25}$ then $r = \frac{1}{3(16/25)} = \frac{25}{48}$

If $a = 1$, $r = \frac{1}{3}$, then numbers are

a, ar, ar^2

$1, 1(\frac{1}{3}), 1(\frac{1}{3})^2$

$1, \frac{1}{3}, \frac{1}{9}$

If $a = \frac{16}{25}$, $r = \frac{25}{48}$ Then numbers are a, ar, ar^2

$\frac{16}{25}, \frac{16}{25}(\frac{25}{48}), \frac{16}{25}(\frac{25}{48})^2$

$\frac{16}{25}, \frac{1}{3}, \frac{25 \times 25 \times 16}{25 \times 48 \times 48}$

$\frac{16}{25}, \frac{1}{3}, \frac{25}{144}$ Ans.

SUMMATION NOTATION

The Greek letter Σ (sigma) is used to sum a sequence of numbers. We write the sum in sigma notation as,

$$a_1 + a_2 + a_3 + \cdots\cdots + a_n = \sum_{k=1}^{n} a_k$$

① Here Σ indicates the sum and k is called index of summation.

② The summation begins from $k=1$ and ends with $k=n$.

③ $k=1$ is called lower limit while $k=n$ is called upper limit.

PROPERTIES OF SUMMATION :-

(i) $\sum_{k=1}^{n}(a_k + b_k) = \sum_{k=1}^{n} a_k + \sum_{k=1}^{n} b_k$

(ii) $\sum_{k=1}^{n}(a_k - b_k) = \sum_{k=1}^{n} a_k - \sum_{k=1}^{n} b_k$

(iii) $\sum_{k=1}^{n} c a_k = c \sum_{k=1}^{n} a_k$