

**Question # 1**

$$\frac{9x-7}{(x^2+1)(x+3)}$$

**Solution**

$$\frac{9x-7}{(x^2+1)(x+3)}$$

Resolving it into partial fraction.

$$\frac{9x-7}{(x^2+1)(x+3)} = \frac{Ax+B}{x^2+1} + \frac{C}{x+3}$$

Multiplying both sides by  $(x^2+1)(x+3)$ .

$$9x-7 = (Ax+B)(x+3) + C(x^2+1) \dots (i)$$

Put  $x+3=0 \Rightarrow x=-3$  in equation (i).

$$9(-3)-7 = (A(-3)+B)(0) + C((-3)^2+1) \Rightarrow -27-7 = 0 + C(9+1)$$

$$\Rightarrow -34 = 10C \Rightarrow C = -\frac{34}{10} \Rightarrow \boxed{C = -\frac{17}{5}}$$

Now equation (i) can be written as

$$9x-7 = A(x^2+3x) + B(x+3) + C(x^2+1)$$

Comparing the coefficients of  $x^2$ ,  $x$  and  $x^0$ .

$$0 = A + C \dots (ii)$$

$$9 = 3A + B \dots (iii)$$

$$-7 = +3B + C \dots (iv)$$

Putting value of  $C$  in equation (ii)

$$0 = A - \frac{17}{5} \Rightarrow \boxed{A = \frac{17}{5}}$$

Now putting value of  $A$  in equation (iii)

$$9 = 3\left(\frac{17}{5}\right) + B \Rightarrow 9 = \frac{51}{5} + B$$

$$\Rightarrow 9 - \frac{51}{5} = B \Rightarrow \boxed{B = -\frac{6}{5}}$$

Hence

$$\begin{aligned} \frac{9x-7}{(x^2+1)(x+3)} &= \frac{\frac{17}{5}x - \frac{6}{5}}{x^2+1} + \frac{-\frac{17}{5}}{x+3} \\ &= \frac{17x-6}{5(x^2+1)} - \frac{17}{5(x+3)} = \frac{17x-6}{5(x^2+1)} - \frac{17}{5(x+3)} \quad \text{Answer} \end{aligned}$$

**Question # 2**  $\frac{1}{(x^2 + 1)(x + 1)}$

**Solution**  $\frac{1}{(x^2 + 1)(x + 1)}$

Now Consider

$$\frac{1}{(x^2 + 1)(x + 1)} = \frac{Ax + B}{x^2 + 1} + \frac{C}{x + 1}$$

Multiplying both sides by  $(x^2 + 1)(x + 1)$ .

$$1 = (Ax + B)(x + 1) + C(x^2 + 1) \dots \dots (i)$$

Put  $x + 1 = 0 \Rightarrow x = -1$  in equation (i)

$$1 = 0 + C((-1)^2 + 1) \Rightarrow 1 = 2C \Rightarrow \boxed{C = \frac{1}{2}}$$

Now eq. (i) can be written as

$$1 = A(x^2 + x) + B(x + 1) + C(x^2 + 1)$$

Comparing the coefficients of  $x^2$ ,  $x$  and  $x^0$ .

$$0 = A + C \dots (ii)$$

$$0 = A + B \dots (iii)$$

$$1 = A + C \dots (iv)$$

Putting value of  $C$  in equation (ii)

$$0 = A + \frac{1}{2} \Rightarrow \boxed{A = -\frac{1}{2}}$$

Putting value of  $A$  in equation (iii)

$$0 = -\frac{1}{2} + B \Rightarrow \boxed{B = \frac{1}{2}}$$

Hence 
$$\frac{1}{(x^2 + 1)(x + 1)} = \frac{-\frac{1}{2}x + \frac{1}{2}}{x^2 + 1} + \frac{\frac{1}{2}}{x + 1} = \frac{-x + 1}{2(x^2 + 1)} + \frac{1}{2(x + 1)}$$

$$= \frac{-x + 1}{2(x^2 + 1)} + \frac{1}{2(x + 1)} = \frac{1 - x}{2(x^2 + 1)} + \frac{1}{2(x + 1)}$$

*Answer*

**Question # 3**  $\frac{3x + 7}{(x^2 + 4)(x + 3)}$

**Solution**  $\frac{3x + 7}{(x^2 + 4)(x + 3)}$

Resolving it into partial fraction.

$$\frac{3x + 7}{(x^2 + 4)(x + 3)} = \frac{Ax + B}{x^2 + 4} + \frac{C}{x + 3}$$

$$\left[ \begin{array}{l} \text{Now do yourself, you will get} \\ A = \frac{2}{13}, B = \frac{33}{13} \text{ and } C = -\frac{2}{13} \end{array} \right]$$


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**Question # 4**  $\frac{x^2 + 15}{(x^2 + 2x + 5)(x - 1)}$

**Solution**  $\frac{x^2 + 15}{(x^2 + 2x + 5)(x - 1)}$

Resolving it into partial fraction.

$$\frac{x^2 + 15}{(x^2 + 2x + 5)(x - 1)} = \frac{Ax + B}{x^2 + 2x + 5} + \frac{C}{x - 1}$$

$$\Rightarrow x^2 + 15 = (Ax + B)(x - 1) + C(x^2 + 2x + 5) \dots (i)$$

Put  $x - 1 = 0 \Rightarrow x = 1$  in equation (i)

$$(1)^2 + 15 = (A(1) + B)(0) + C((1)^2 + 2(1) + 5)$$

$$\Rightarrow 1 + 15 = 0 + C(1 + 2 + 5)$$

$$\Rightarrow 16 = 8C \Rightarrow \frac{16}{8} = C \Rightarrow \boxed{C = 2}$$

Now equation (i) can be written as

$$x^2 + 15 = A(x^2 - x) + B(x - 1) + C(x^2 + 2x + 5)$$

Comparing the coefficients of  $x^2$ ,  $x$  and  $x^0$ .

$$1 = A + C \dots (ii)$$

$$0 = -A + B + 2C \dots (iii)$$

$$15 = -B + 5C \dots (iv)$$

Putting value of  $C$  in equation (ii).

$$1 = A + 2 \Rightarrow 1 - 2 = A$$

$$\Rightarrow \boxed{A = -1}$$

Putting value of  $A$  and  $C$  in equation (iii)

$$0 = -(-1) + B + 2(2) \Rightarrow 0 = 1 + B + 4$$

$$\Rightarrow 0 = B + 5 \Rightarrow \boxed{B = -5}$$

Hence 
$$\frac{x^2 + 15}{(x^2 + 2x + 5)(x - 1)} = \frac{(-1)x - 5}{x^2 + 2x + 5} + \frac{2}{x - 1}$$

$$= \frac{-x - 5}{x^2 + 2x + 5} + \frac{2}{x - 1} \quad \text{Answer}$$


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**Question # 5**  $\frac{x^2}{(x^2 + 4)(x + 2)}$

**Solution**  $\frac{x^2}{(x^2 + 4)(x + 2)}$

Resolving it into partial fraction.

$$\frac{x^2}{(x^2+4)(x+2)} = \frac{Ax+B}{x^2+4} + \frac{C}{x+2}$$

$$\left[ \begin{array}{l} \text{Now do yourself, you will get} \\ A = \frac{1}{2}, B = -1 \text{ and } C = -\frac{1}{2} \end{array} \right]$$


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**Question # 6**  $\frac{x^2+1}{x^3+1}$

**Solution**  $\frac{x^2+1}{x^3+1} = \frac{x^2+1}{(x+1)(x^2-x+1)} \quad \because x^3+1=(x+1)(x^2-x+1)$

Now consider

$$\frac{x^2+1}{(x+1)(x^2-x+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2-x+1} \quad \dots (A)$$

$$x^2+1 = A(x^2-x+1) + (Bx+C)(x+1)$$

$$x^2+1 = A(x^2-x+1) + (Bx^2+Bx+Cx+C)$$

$$x^2+1 = (A+B)x^2 + (B+C-A)x + (A+C)$$

By comparing the coefficients of  $x^2$ ,  $x$  and  $x^0$

$$A+B=1 \quad \dots (1)$$

$$B+C-A=0 \quad \dots (2)$$

$$A+C=1 \quad \dots (3)$$

From (3), We have

$$C=1-A$$

Put the value of  $C$  in (2).

$$-2A+B=-1 \quad \dots (4)$$

Subtract (1) from (4), We have

$$3A=2 \Rightarrow A=\frac{2}{3}$$

Put the value of  $A$  in (1) and (3), we have

$$B=\frac{1}{3}, \quad C=\frac{1}{3}$$

Put the values of  $A$ ,  $B$  and  $C$  in (A), We have

$$\frac{x^2+1}{(x+1)(x^2-x+1)} = \frac{2}{3(x+1)} + \frac{(x+1)}{3(x^2-x+1)}$$


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**Question # 7**  $\frac{x^2+2x+2}{(x^2+3)(x+1)(x-1)}$

**Solution**  $\frac{x^2+2x+2}{(x^2+3)(x+1)(x-1)}$

Consider

$$\frac{x^2 + 2x + 2}{(x^2 + 3)(x + 1)(x - 1)} = \frac{Ax + B}{x^2 + 3} + \frac{C}{x + 1} + \frac{D}{x - 1}$$

$$\Rightarrow x^2 + 2x + 2 = (Ax + B)(x + 1)(x - 1) + C(x^2 + 3)(x - 1) + D(x^2 + 3)(x + 1) \dots (i)$$

Put  $x + 1 = 0 \Rightarrow x = -1$  in equation (i)

$$(-1)^2 + 2(-1) + 2 = 0 + C((-1)^2 + 3)((-1) - 1) + 0 \Rightarrow 1 - 2 + 2 = C(4)(-2)$$

$$\Rightarrow 1 = -8C \quad \Rightarrow \quad \boxed{C = -\frac{1}{8}}$$

Now put  $x - 1 = 0 \Rightarrow x = 1$  in equation (i)

$$\Rightarrow (1)^2 + 2(1) + 2 = 0 + 0 + D((1)^2 + 3)((1) + 1) \Rightarrow 1 + 2 + 2 = D(4)(2)$$

$$\Rightarrow 5 = 8D \quad \Rightarrow \quad \boxed{D = \frac{5}{8}}$$

Equation (i) can be written as

$$\begin{aligned} x^2 + 2x + 2 &= (Ax + B)(x^2 - 1) + C(x^3 - x^2 + 3x - 3) + D(x^3 + x^2 + 3x + 3) \\ \Rightarrow x^2 + 2x + 2 &= A(x^3 - x) + B(x^2 - 1) + C(x^3 - x^2 + 3x - 3) + D(x^3 + x^2 + 3x + 3) \end{aligned}$$

Comparing the coefficients of  $x^3$ ,  $x^2$ ,  $x$  and  $x^0$ .

$$0 = A + C + D \dots (ii)$$

$$1 = B - C + D \dots (iii)$$

$$2 = -A + 3C + 3D \dots (iv)$$

$$2 = -B - 3C + 3D \dots (v)$$

Putting values of  $C$  and  $D$  in (ii)

$$0 = A - \frac{1}{8} + \frac{5}{8} \Rightarrow 0 = A + \frac{1}{2} \Rightarrow \quad \boxed{A = -\frac{1}{2}}$$

Putting values of  $C$  and  $D$  in (iii)

$$1 = B - \left(-\frac{1}{8}\right) + \frac{5}{8} \Rightarrow 1 = B + \frac{1}{8} + \frac{5}{8} \Rightarrow 1 = B + \frac{3}{4}$$

$$\Rightarrow 1 - \frac{3}{4} = B \quad \Rightarrow \quad \boxed{B = \frac{1}{4}}$$

Hence

$$\begin{aligned} \frac{x^2 + 2x + 2}{(x^2 + 3)(x + 1)(x - 1)} &= \frac{-\frac{1}{2}x + \frac{1}{4}}{x^2 + 3} + \frac{-\frac{1}{8}}{x + 1} + \frac{\frac{5}{8}}{x - 1} \\ &= \frac{-2x + 1}{4(x^2 + 3)} + \frac{-1}{8(x + 1)} + \frac{5}{8(x - 1)} \end{aligned}$$

$$= \frac{1-2x}{4(x^2+3)} - \frac{1}{8(x+1)} + \frac{5}{8(x-1)} \quad \text{Answer}$$

**Question # 8**  $\frac{1}{(x-1)^2(x^2+2)}$

**Solution**  $\frac{1}{(x-1)^2(x^2+2)}$

Resolving it into partial fraction.

$$\frac{1}{(x-1)^2(x^2+2)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{Cx+D}{x^2+2}$$

$$\Rightarrow 1 = A(x-1)(x^2+2) + B(x^2+2) + (Cx+D)(x-1)^2 \dots (i)$$

Put  $x-1=0 \Rightarrow x=1$  in equation (i)

$$1 = 0 + B((1)^2 + 2) + 0$$

$$\Rightarrow 1 = 3B \Rightarrow \boxed{B = \frac{1}{3}}$$

Now equation (i) can be written as

$$1 = A(x^3 - x^2 + 2x - 2) + B(x^2 + 2) + (Cx + D)(x^2 - 2x + 1)$$

$$\Rightarrow 1 = A(x^3 - x^2 + 2x - 2) + B(x^2 + 2) + C(x^3 - 2x^2 + x) + D(x^2 - 2x + 1)$$

Comparing the coefficients of  $x^3$ ,  $x^2$ ,  $x$  and  $x^0$ .

$$0 = A + C \dots (ii)$$

$$0 = -A + B - 2C + D \dots (iii)$$

$$0 = 2A + C - 2D \dots (iv)$$

$$1 = -2A + 2B + D \dots (v)$$

Multiplying eq. (iii) by 2 and adding in (iv)

$$\begin{array}{r} 0 = -2A + 2B - 4C + 2D \\ 0 = 2A \quad + \quad C - 2D \\ \hline 0 = \quad 2B - 3C \end{array}$$

Putting value of  $B$  in above

$$0 = 2\left(\frac{1}{3}\right) - 3C \Rightarrow 0 = \frac{2}{3} - 3C \Rightarrow 3C = \frac{2}{3} \Rightarrow \boxed{C = \frac{2}{9}}$$

Putting value of  $C$  in eq. (ii)

$$0 = A + \frac{2}{9} \Rightarrow \boxed{A = -\frac{2}{9}}$$

Putting value of  $A$  and  $B$  in eq. (v)

$$1 = -2\left(-\frac{2}{9}\right) + 2\left(\frac{1}{3}\right) + D \Rightarrow 1 = \frac{4}{9} + \frac{2}{3} + D$$

$$\Rightarrow 1 - \frac{4}{9} - \frac{2}{3} = D \Rightarrow \boxed{D = -\frac{1}{9}}$$

Hence

$$\begin{aligned}\frac{1}{(x-1)^2(x^2+2)} &= \frac{-\frac{2}{9}}{x-1} + \frac{\frac{1}{3}}{(x-1)^2} + \frac{\left(\frac{2}{9}\right)x + \left(-\frac{1}{9}\right)}{x^2+2} \\ &= \frac{-\frac{2}{9}}{x-1} + \frac{\frac{1}{3}}{(x-1)^2} + \frac{\frac{2x-1}{9}}{x^2+2} \\ &= \frac{-2}{9(x-1)} + \frac{1}{3(x-1)^2} + \frac{2x-1}{9(x^2+2)}\end{aligned}$$

**Question # 9**

$$\frac{x^4}{1-x^4}$$

**Solution**

$$\begin{aligned}\frac{x^4}{1-x^4} &= -1 + \frac{1}{1-x^4} = -1 + \frac{1}{(1-x^2)(1+x^2)} \\ &= -1 + \frac{1}{(1-x)(1+x)(1+x^2)}\end{aligned}$$

$$\begin{array}{r} 1-x^4 \overline{) x^4} \\ \underline{-x^4} \phantom{-1} \\ 1 \end{array}$$

Now consider

$$\frac{1}{(1-x)(1+x)(1+x^2)} = \frac{A}{1-x} + \frac{B}{1+x} + \frac{Cx+D}{1+x^2}$$

$$\left[ \begin{array}{l} \text{Now find values of } A, B, C \text{ and } D \text{ yourself.} \\ \text{You will get } A = \frac{1}{4}, B = \frac{1}{4}, C = 0 \text{ and } D = \frac{1}{2} \end{array} \right]$$

So

$$\begin{aligned}\frac{1}{(1-x)(1+x)(1+x^2)} &= \frac{\frac{1}{4}}{1-x} + \frac{\frac{1}{4}}{1+x} + \frac{(0)x + \frac{1}{2}}{1+x^2} \\ &= \frac{1}{4(1-x)} + \frac{1}{4(1+x)} + \frac{1}{2(1+x^2)}\end{aligned}$$

Hence

$$\frac{x^4}{1-x^4} = -1 + \frac{1}{4(1-x)} + \frac{1}{4(1+x)} + \frac{1}{2(1+x^2)} \quad \text{Answer}$$

**Question # 10**

$$\frac{x^2-2x+3}{x^4+x^2+1}$$

**Solution**

$$\begin{aligned}\frac{x^2-2x+3}{x^4+x^2+1} &= \frac{x^2-2x+3}{(x^2+x+1)(x^2-x+1)}\end{aligned}$$

$$\begin{aligned}\because x^4+x^2+1 &= x^4+2x^2+1-x^2 \\ &= (x^2+1)^2-x^2 \\ &= (x^2+1+x)(x^2+1-x) \\ &= (x^2+x+1)(x^2-x+1)\end{aligned}$$

Now Consider

$$\frac{x^2 - 2x + 3}{(x^2 + x + 1)(x^2 - x + 1)} = \frac{Ax + B}{x^2 + x + 1} + \frac{Cx + D}{x^2 - x + 1}$$

$$\Rightarrow x^2 - 2x + 3 = (Ax + B)(x^2 - x + 1) + (Cx + D)(x^2 + x + 1) \dots (i)$$

$$\Rightarrow x^2 - 2x + 3 = A(x^3 - x^2 + x) + B(x^2 - x + 1) + C(x^3 + x^2 + x) + D(x^2 + x + 1)$$

Comparing the coefficients of  $x^3$ ,  $x^2$ ,  $x$  and  $x^0$ .

$$0 = A + C \dots (ii)$$

$$1 = -A + B + C + D \dots (iii)$$

$$-2 = A - B + C + D \dots (iv)$$

$$3 = B + D \dots (v)$$

Subtracting (ii) and (iv)

$$\begin{array}{r} 0 = A + C \\ -2 = A - B + C + D \\ \hline 2 = B - D \end{array} \dots (vi)$$

$$\Rightarrow 2 = B - D \dots (vi)$$

Adding (v) and (vi)

$$\begin{array}{r} 3 = B + D \\ 2 = B - D \\ \hline 5 = 2B \end{array}$$

$$\Rightarrow \boxed{B = \frac{5}{2}}$$

Putting value of  $B$  in (v)

$$3 = \frac{5}{2} + D$$

$$\Rightarrow 3 - \frac{5}{2} = D \Rightarrow \boxed{D = \frac{1}{2}}$$

Putting value of  $B$  and  $D$  in (iii)

$$1 = -A + \frac{5}{2} + C + \frac{1}{2}$$

$$\Rightarrow 1 - \frac{5}{2} - \frac{1}{2} = -A + C$$

$$\Rightarrow -2 = -A + C \dots (vii)$$

Adding (ii) and (vii)

$$\begin{array}{r} 0 = A + C \\ -2 = -A + C \\ \hline -2 = 2C \end{array}$$

$$\Rightarrow \boxed{C = -1}$$

Putting value of  $C$  in equation (ii)

$$0 = A - 1$$

$$\Rightarrow \boxed{A = 1}$$



Hence

$$\begin{aligned}
 \frac{x^2 - 2x + 3}{(x^2 + x + 1)(x^2 - x + 1)} &= \frac{(1)x + \frac{5}{2}}{x^2 + x + 1} + \frac{(-1)x + \frac{1}{2}}{x^2 - x + 1} \\
 &= \frac{2x + 5}{2(x^2 + x + 1)} + \frac{-2x + 1}{2(x^2 - x + 1)} \\
 &= \frac{2x + 5}{2(x^2 + x + 1)} + \frac{1 - 2x}{2(x^2 - x + 1)} \quad \text{Answer}
 \end{aligned}$$

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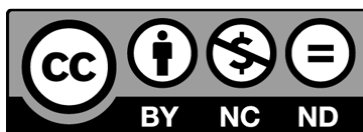
**Book:**      **Exercise 5.3**

*Text Book of Algebra and Trigonometry Class XI*  
*Punjab Textbook Board, Lahore.*

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