

Resolving the following into partial fractions:

Question # 1

$$\frac{1}{x^2-1}$$

Solution

$$\frac{1}{x^2-1} = \frac{1}{(x-1)(x+1)}$$

Now suppose

$$\frac{1}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1}$$

Multiplying both sides by $(x-1)(x+1)$ we get

$$1 = A(x+1) + B(x-1) \dots\dots\dots (i)$$

Put $x-1=0 \Rightarrow x=1$ in equation (i)

$$1 = A(1+1) + B(0) \Rightarrow 1 = 2A + 0 \Rightarrow \boxed{A = \frac{1}{2}}$$

Now put $x+1=0 \Rightarrow x=-1$ in equation (i)

$$1 = A(0) + B(-1-1) \Rightarrow 1 = 0 - 2B \Rightarrow \boxed{B = -\frac{1}{2}}$$

Hence

$$\begin{aligned} \frac{1}{(x-1)(x+1)} &= \frac{A}{x-1} + \frac{B}{x+1} \\ &= \frac{1}{2(x-1)} - \frac{1}{2(x+1)} \quad \text{Answer} \end{aligned}$$

Question # 2

$$\frac{x^2+1}{(x+1)(x-1)}$$

$$x^2 - 1 \begin{array}{l} \sqrt{x^2 + 1} \\ x^2 - 1 \\ \hline + \\ \hline 2 \end{array}$$

Solution

$$\begin{aligned} \frac{x^2+1}{(x+1)(x-1)} &= \frac{x^2+1}{x^2-1} \\ &= 2 + \frac{1}{x^2-1} = 2 + \frac{1}{(x+1)(x-1)} \end{aligned}$$

Now consider

$$\frac{1}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1}$$

Multiplying both sides by $(x+1)(x-1)$

$$2 = A(x-1) + B(x+1) \dots\dots\dots(i)$$

Put $x+1=0 \Rightarrow x=-1$ in equation (i)

$$2 = A(-1-1) + B(0) \Rightarrow 2 = -2A + 0 \Rightarrow \boxed{A = -1}$$

Now put $x-1=0 \Rightarrow x=1$ in equation (i)

$$2 = A(0) + B(1+1) \Rightarrow 2 = 0 + 2B \Rightarrow \boxed{B = 1}$$

So
$$\frac{2}{(x+1)(x-1)} = \frac{-1}{x+1} + \frac{1}{x-1}$$

Hence

$$\begin{aligned} \frac{x^2+1}{(x+1)(x-1)} &= 2 + \frac{-1}{(x+1)} + \frac{1}{(x-1)} \\ &= 2 - \frac{1}{(x+1)} + \frac{1}{(x-1)} \end{aligned} \quad \text{Answer}$$

Question # 3

$$\frac{2x+1}{(x-1)(x+2)(x+3)}$$

Solution

$$\frac{2x+1}{(x-1)(x+2)(x+3)} = \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{x+3}$$

Multiplying both side by $(x-1)(x+2)(x+3)$

$$2x+1 = A(x+2)(x+3) + B(x-1)(x+3) + C(x-1)(x+2) \dots\dots\dots(i)$$

Put $x-1=0 \Rightarrow x=1$ in equation (i)

$$2(1)+1 = A(1+2)(1+3) + B(0) + C(0)$$

$$3 = A(3)(4) + 0 + 0 \Rightarrow 3 = 12A \Rightarrow \frac{3}{12} = A \Rightarrow \boxed{A = \frac{1}{4}}$$

Now put $x+2=0 \Rightarrow x=-2$ in equation (i)

$$2(-2)+1 = A(0) + B(-2-1)(-2+3) + C(0)$$

$$-4+1 = 0 + B(-3)(1) + 0 \Rightarrow -3 = -3B \Rightarrow \boxed{B = 1}$$

Now put $x+3=0 \Rightarrow x=-3$ in equation (i)

$$2(-3)+1 = A(0) + B(0) + C(-3-1)(-3+2)$$

$$-6+1 = 0 + 0 + C(-4)(-1) \Rightarrow -5 = 4C \Rightarrow \boxed{C = -\frac{5}{4}}$$

So

$$\begin{aligned} \frac{2x+1}{(x-1)(x+2)(x+3)} &= \frac{1/4}{x-1} + \frac{1}{x+2} + \frac{-5/4}{x+3} \\ &= \frac{1}{4(x-1)} + \frac{1}{x+2} - \frac{5}{4(x+3)} \end{aligned} \quad \text{Answer}$$

Question # 4

$$\frac{3x^2 - 4x - 5}{(x-2)(x^2 + 7x + 10)}$$

$$\begin{aligned} \therefore x^2 + 7x + 10 &= x^2 + 5x + 2x + 10 \\ &= x(x+5) + 2(x+5) \\ &= (x+5)(x+2) \end{aligned}$$

Solution

$$\frac{3x^2 - 4x - 5}{(x-2)(x^2 + 7x + 10)} = \frac{3x^2 - 4x - 5}{(x-2)(x+5)(x+2)}$$

Now resolving into partial fraction.

$$\frac{3x^2 - 4x - 5}{(x-2)(x+5)(x+2)} = \frac{A}{x-2} + \frac{B}{x+5} + \frac{C}{x+2}$$

$$\left[\begin{array}{l} \text{Do yourself. You will get} \\ A = -\frac{1}{28}, B = \frac{30}{7}, C = -\frac{5}{4} \end{array} \right]$$

Question # 5

$$\frac{1}{(x-1)(2x-1)(3x-1)}$$

Solution

$$\frac{1}{(x-1)(2x-1)(3x-1)} = \frac{A}{x-1} + \frac{B}{2x-1} + \frac{C}{3x-1}$$

Multiplying both side by $(x-1)(2x-1)(3x-1)$.

$$1 = A(2x-1)(3x-1) + B(x-1)(3x-1) + C(2x-1)(3x-1) \dots\dots\dots(i)$$

Put $x-1=0 \Rightarrow x=1$ in equation (i)

$$1 = A(2(1)-1)(3(1)-1) + B(0) + C(0) \Rightarrow 1 = A(1)(2) + 0 + 0$$

$$\Rightarrow 1 = 2A \Rightarrow \boxed{A = \frac{1}{2}}$$

Put $2x-1=0 \Rightarrow 2x=1 \Rightarrow x=\frac{1}{2}$ in equation (i)

$$1 = A(0) + B\left(\frac{1}{2}-1\right)\left(3\left(\frac{1}{2}\right)-1\right) + C(0) \Rightarrow 1 = 0 + B\left(-\frac{1}{2}\right)\left(\frac{1}{2}\right) + 0$$

$$\Rightarrow 1 = -\frac{1}{4}B \Rightarrow \boxed{B = -4}$$

Put $3x-1=0 \Rightarrow 3x=1 \Rightarrow x=\frac{1}{3}$ in equation (i)

$$1 = A(0) + B(0) + C\left(\frac{1}{3}-1\right)\left(2\left(\frac{1}{3}\right)-1\right) \Rightarrow 1 = 0 + 0 + C\left(-\frac{2}{3}\right)\left(-\frac{1}{3}\right)$$

$$\Rightarrow 1 = \frac{2}{9}C \Rightarrow \boxed{C = \frac{9}{2}}$$

Hence
$$\frac{1}{(x-1)(2x-1)(3x-1)} = \frac{1/2}{x-1} + \frac{-4}{2x-1} + \frac{9/2}{3x-1}$$

$$= \frac{1}{2(x-1)} - \frac{4}{2x-1} + \frac{9}{2(3x-1)} \quad \text{Answer}$$

Question # 6

$$\frac{x}{(x-a)(x-b)(x-c)}$$

Solution

$$\frac{x}{(x-a)(x-b)(x-c)} = \frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c}$$

Multiplying both sides by $(x-a)(x-b)(x-c)$.

$$x = A(x-b)(x-c) + B(x-a)(x-c) + C(x-a)(x-b) \dots\dots\dots(i)$$

Put $x-a=0 \Rightarrow x=a$ in equation (i)

$$a = A(a-b)(a-c) + B(0) + C(0)$$

$$\Rightarrow a = A(a-b)(a-c) + 0 + 0 \quad \Rightarrow \boxed{A = \frac{a}{(a-b)(a-c)}}$$

Now put $x-b=0 \Rightarrow x=b$ in equation (i)

$$b = A(0) + B(b-a)(b-c) + C(0)$$

$$\Rightarrow a = 0 + B(b-a)(b-c) + 0 \quad \Rightarrow \boxed{B = \frac{b}{(b-a)(b-c)}} \quad \text{Now put}$$

$x-c=0 \Rightarrow x=c$ in equation (i)

$$c = A(0) + B(0) + C(c-a)(c-b)$$

$$\Rightarrow c = 0 + 0 + C(c-a)(c-b) \quad \Rightarrow \boxed{C = \frac{c}{(c-a)(c-b)}}$$

So

$$\frac{x}{(x-a)(x-b)(x-c)} = \frac{a/(a-b)(a-c)}{x-a} + \frac{b/(b-a)(b-c)}{x-b} + \frac{c/(c-a)(c-b)}{x-c}$$

$$= \frac{a}{(a-b)(a-c)(x-a)} + \frac{b}{(b-a)(b-c)(x-b)} + \frac{c}{(c-a)(c-b)(x-c)}$$

Answer

Question # 7

$$\frac{6x^3 + 5x^2 - 7}{2x^2 - x - 1}$$

Solution

$$\frac{6x^3 + 5x^2 - 7}{2x^2 - x - 1}$$

$$\begin{aligned}
 &= 3x + 4 + \frac{7x-3}{2x^2-x-1} \\
 &= 3x + 4 + \frac{7x-3}{2x^2-2x+x-1} \\
 &= 3x + 4 + \frac{7x-3}{2x(x-1)+1(x-1)} \\
 &= 3x + 4 + \frac{7x-3}{(x-1)(2x+1)}
 \end{aligned}$$

$$\begin{array}{r}
 2x^2 - x - 1 \overline{) 6x^3 + 5x^2 - 7} \\
 \underline{6x^3 - 3x^2 - 3x} \\
 8x^2 + 3x - 7 \\
 \underline{8x^2 - 4x - 4} \\
 7x - 3
 \end{array}$$

Now Consider

$$\frac{7x-3}{(x-1)(2x+1)} = \frac{A}{x-1} + \frac{B}{2x+1}$$

$$\left[\begin{array}{l} \text{Find value of A \& B yourself} \\ \text{You will get } A = \frac{4}{3} \text{ and } B = \frac{13}{3} \end{array} \right]$$

so

$$\frac{7x-3}{(x-1)(2x+1)} = \frac{\frac{4}{3}}{x-1} + \frac{\frac{13}{3}}{2x+1} = \frac{4}{3(x-1)} + \frac{13}{3(2x+1)}$$

Hence

$$\frac{6x^3 + 5x^2 - 7}{2x^2 - x - 1} = 3x + 4 + \frac{4}{3(x-1)} + \frac{13}{3(2x+1)} \quad \text{Answer}$$

Question # 8

$$\frac{2x^3 + x^2 - 5x + 3}{2x^3 + x^2 - 3x}$$

$$\begin{array}{r}
 2x^3 + x^2 - 3x \overline{) 2x^3 + x^2 - 5x + 3} \\
 \underline{2x^3 + x^2 - 3x} \\
 -2x + 3
 \end{array}$$

Solution

$$\frac{2x^3 + x^2 - 5x + 3}{2x^3 + x^2 - 3x}$$

$$= 1 + \frac{-2x + 3}{2x^3 + x^2 - 3x}$$

$$= 1 + \frac{-2x + 3}{x(2x^2 + x - 3)} = 1 + \frac{-2x + 3}{x(2x^2 + 3x - 2x - 3)}$$

$$= 1 + \frac{-2x + 3}{x(x(2x + 3) - 1(2x + 3))} = 1 + \frac{-2x + 3}{x(2x + 3)(x - 1)}$$

Now consider

$$\frac{3-2x}{x(2x+3)(x-1)} = \frac{A}{x} + \frac{B}{2x+3} + \frac{C}{x-1}$$

$$\Rightarrow 3-2x = A(2x+3)(x-1) + Bx(x-1) + Cx(2x+3) \dots\dots\dots (i)$$

Put $x=0$ in equation (i)

$$3-2(0) = A(2(0)+3)((0)-1) + B(0)+C(0) \Rightarrow 3-0 = A(0+3)(-1) + 0 + 0$$

$$\Rightarrow 3 = -3A \Rightarrow \boxed{A = -1}$$

Now put $2x+3=0 \Rightarrow 2x=-3 \Rightarrow x=-\frac{3}{2}$ in equation (i)

$$3-2\left(-\frac{3}{2}\right)=A(0)+B\left(-\frac{3}{2}\right)\left(-\frac{3}{2}-1\right)+C(0) \Rightarrow 3+3=0+B\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)+0$$

$$\Rightarrow 6=\frac{15}{4}B \Rightarrow B=(6)\left(\frac{4}{15}\right) \Rightarrow \boxed{B=\frac{8}{5}}$$

Now put $x-1=0 \Rightarrow x=1$ in equation (i)

$$3-2(1)=A(0)+B(0)+C(1)(2(1)+3) \Rightarrow 1=0+0+5C \Rightarrow \boxed{C=\frac{1}{5}}$$

So
$$\frac{3-2x}{x(2x+3)(x-1)}=\frac{-1}{x}+\frac{\frac{8}{5}}{2x+3}+\frac{\frac{1}{5}}{x-1}=-\frac{1}{x}+\frac{8}{5(2x+3)}+\frac{1}{5(x-1)}$$

Hence
$$\frac{2x^3+x^2-5x+3}{2x^3+x^2-3x}=1-\frac{1}{x}+\frac{8}{5(2x+3)}+\frac{1}{5(x-1)} \quad \text{Answer}$$

Question # 9

$$\frac{(x-1)(x-3)(x-5)}{(x-2)(x-4)(x-6)}$$

Solution

$$\begin{aligned} \frac{(x-1)(x-3)(x-5)}{(x-2)(x-4)(x-6)} &= \frac{(x-1)(x^2-3x-5x+15)}{(x-2)(x^2-4x-6x+24)} \\ &= \frac{(x-1)(x^2-8x+15)}{(x-2)(x^2-10x+24)} = \frac{x^3-8x^2+15x-x^2+8x-15}{x^3-10x^2+24x-2x^2+20x-48} \\ &= \frac{x^3-9x^2+23x-15}{x^3-12x^2+44x-48} \\ &= 1 + \frac{3x^2-21x+33}{x^3-12x^2+44x-48} \qquad \begin{array}{r} 1 \\ \hline x^3-12x^2+44x-48 \end{array} \left. \begin{array}{l} x^3-9x^2+23x-15 \\ -x^3+12x^2-44x+48 \\ \hline \end{array} \right\} \begin{array}{l} x^3-9x^2+23x-15 \\ x^3-12x^2+44x-48 \\ \hline \end{array} \\ &= 1 + \frac{3x^2-21x+33}{(x-2)(x-4)(x-6)} \qquad \frac{3x^2-21x+33}{3x^2-21x+33} \end{aligned}$$

Now Suppose

$$\frac{3x^2-21x+33}{(x-2)(x-4)(x-6)} = \frac{A}{x-2} + \frac{B}{x-4} + \frac{C}{x-6}$$

$$\left[\begin{array}{l} \text{Find value of } A, B \text{ and } C \text{ yourself} \\ \text{You will get } A=\frac{3}{8}, B=\frac{3}{4}, C=\frac{15}{8} \end{array} \right]$$

So
$$\frac{3x^2-21x+33}{(x-2)(x-4)(x-6)} = \frac{\frac{3}{8}}{x-2} + \frac{\frac{3}{4}}{x-4} + \frac{\frac{15}{8}}{x-6}$$

$$= \frac{3}{8(x-2)} + \frac{3}{4(x-4)} + \frac{15}{8(x-6)}$$

Hence

$$\frac{(x-1)(x-3)(x-5)}{(x-2)(x-4)(x-6)} = 1 + \frac{3}{8(x-2)} + \frac{3}{4(x-4)} + \frac{15}{8(x-6)} \quad \text{Answer}$$

Question # 10

$$\frac{1}{(1-ax)(1-bx)(1-cx)}$$

Solution

$$\frac{1}{(1-ax)(1-bx)(1-cx)} = \frac{A}{1-ax} + \frac{B}{1-bx} + \frac{C}{1-cx}$$

Multiplying both sides by $(1-ax)(1-bx)(1-cx)$.

$$1 = A(1-bx)(1-cx) + B(1-ax)(1-cx) + C(1-ax)(1-bx) \dots\dots\dots(i)$$

Put $1-ax=0 \Rightarrow ax=1 \Rightarrow x=\frac{1}{a}$ in equation (i).

$$1 = A\left(1-b \cdot \frac{1}{a}\right)\left(1-c \cdot \frac{1}{a}\right) + B(0) + C(0) \quad \Rightarrow 1 = A\left(1-\frac{b}{a}\right)\left(1-\frac{c}{a}\right) + 0 + 0$$

$$\Rightarrow 1 = A\left(\frac{a-b}{a}\right)\left(\frac{a-c}{a}\right) \quad \Rightarrow 1 = A \frac{(a-b)(a-c)}{a^2} \quad \Rightarrow \boxed{A = \frac{a^2}{(a-b)(a-c)}}$$

$$\left[\begin{array}{l} \text{Find value of } B \text{ \& } C \text{ yourself as } A. \\ \text{You will get } B = \frac{b^2}{(b-a)(b-c)}, C = \frac{c^2}{(c-a)(c-b)} \end{array} \right]$$

$$\begin{aligned} \text{Hence } \frac{1}{(1-ax)(1-bx)(1-cx)} &= \frac{\frac{a^2}{(a-b)(a-c)}}{1-ax} + \frac{\frac{b^2}{(b-a)(b-c)}}{1-bx} + \frac{\frac{c^2}{(c-a)(c-b)}}{1-cx} \\ &= \frac{a^2}{(a-b)(a-c)(1-ax)} + \frac{b^2}{(b-a)(b-c)(1-bx)} + \frac{c^2}{(c-a)(c-b)(1-cx)} \end{aligned}$$

Answer**Question # 11**

$$\frac{x^2 + a^2}{(x^2 + b^2)(x^2 + c^2)(x^2 + d^2)}$$

Solution

$$\frac{x^2 + a^2}{(x^2 + b^2)(x^2 + c^2)(x^2 + d^2)}$$

Put $y = x^2$ in above.

$$\frac{y + a^2}{(y + b^2)(y + c^2)(y + d^2)}$$

Now consider

$$\frac{y + a^2}{(y + b^2)(y + c^2)(y + d^2)} = \frac{A}{y + b^2} + \frac{B}{y + c^2} + \frac{C}{y + d^2}$$

$$\Rightarrow y + a^2 = A(y + c^2)(y + d^2) + B(y + b^2)(y + d^2) + C(y + b^2)(y + c^2) \dots\dots\dots(i)$$

Put $y + b^2 = 0 \Rightarrow y = -b^2$ in equation (i)

$$-b^2 + a^2 = A(-b^2 + c^2)(-b^2 + d^2) + B(0) + C(0)$$

$$\Rightarrow a^2 - b^2 = A(c^2 - b^2)(d^2 - b^2) + 0 + 0 \Rightarrow \boxed{A = \frac{a^2 - b^2}{(c^2 - b^2)(d^2 - b^2)}}$$

Now put $y + c^2 = 0 \Rightarrow y = -c^2$ in equation (i)

$$-c^2 + a^2 = A(0) + B(-c^2 + b^2)(-b^2 + d^2) + C(0)$$

$$\Rightarrow a^2 - c^2 = 0 + B(b^2 - c^2)(d^2 - c^2) + 0 \Rightarrow \boxed{B = \frac{a^2 - c^2}{(b^2 - c^2)(d^2 - c^2)}}$$

Now put $y + d^2 = 0 \Rightarrow y = -d^2$ in equation (i)

$$-d^2 + a^2 = A(0) + B(0) + C(-d^2 + b^2)(-d^2 + c^2)$$

$$\Rightarrow a^2 - d^2 = 0 + 0 + C(b^2 - d^2)(c^2 - d^2) \Rightarrow \boxed{C = \frac{a^2 - d^2}{(b^2 - d^2)(c^2 - d^2)}}$$

Hence

$$\begin{aligned} \frac{y + a^2}{(y + b^2)(y + c^2)(y + d^2)} &= \frac{\frac{a^2 - b^2}{(c^2 - b^2)(d^2 - b^2)}}{y + b^2} + \frac{\frac{a^2 - c^2}{(b^2 - c^2)(d^2 - c^2)}}{y + c^2} + \frac{\frac{a^2 - d^2}{(b^2 - d^2)(c^2 - d^2)}}{y + d^2} \\ &= \frac{a^2 - b^2}{(c^2 - b^2)(d^2 - b^2)(y + b^2)} + \frac{a^2 - c^2}{(b^2 - c^2)(d^2 - c^2)(y + c^2)} + \frac{a^2 - d^2}{(b^2 - d^2)(c^2 - d^2)(y + d^2)} \end{aligned}$$

Since $y = x^2$

$$= \frac{a^2 - b^2}{(c^2 - b^2)(d^2 - b^2)(x^2 + b^2)} + \frac{a^2 - c^2}{(b^2 - c^2)(d^2 - c^2)(x^2 + c^2)} + \frac{a^2 - d^2}{(b^2 - d^2)(c^2 - d^2)(x^2 + d^2)}$$

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