Exercise 4.6

Relation Between Roots and Coefficients of a Quadratic Equation:

General quadratic equation is

$$ax^2 + bx + c = 0 \longrightarrow (1)$$

a, b and c are coefficients and Roots of (1) are

$x = \frac{-b+\sqrt{b^2-4ac}}{2a}$ and $x = \frac{-b-\sqrt{b^2-4ac}}{2a}$.

If α, β are the roots of (1) then

$$\alpha = \frac{-b+\sqrt{b^2-4ac}}{2a} \text{ and } \beta = \frac{-b-\sqrt{b^2-4ac}}{2a}$$

Now

$$\alpha + \beta = \frac{-b+\sqrt{b^2-4ac}}{2a} + \frac{-b-\sqrt{b^2-4ac}}{2a}$$

$$\alpha + \beta = \frac{-b+\sqrt{b^2-4ac} - b - \sqrt{b^2-4ac}}{2a}$$

$$\alpha + \beta = \frac{-b-b}{2a}$$

$$\alpha + \beta = \frac{-2b}{2a}$$

$$\alpha + \beta = \frac{-b}{a}$$

$$\boldsymbol{\alpha + \beta = -\frac{\textbf{coefficient of } x}{\textbf{coefficient of } x^2}}$$

and

$$\alpha\beta = \frac{-b+\sqrt{b^2-4ac}}{2a} . \frac{-b-\sqrt{b^2-4ac}}{2a}$$

$$\alpha\beta = \frac{(-b)^2 - \left(\sqrt{b^2-4ac}\right)^2}{4a^2}$$

$$\alpha\beta = \frac{b^2 - (b^2-4ac)}{4a^2}$$

$$\alpha\beta = \frac{b^2 - b^2 + 4ac}{4a^2}$$

$$\alpha\beta = \frac{4ac}{4a^2}$$

$$\alpha\beta = \frac{c}{a}$$

$$\boldsymbol{\alpha\beta = -\frac{\textbf{coefficient of } x^0}{\textbf{coefficient of } x^2}}$$

Formation of Equation whose roots are given

Let α, β be roots of required equation then $x = \alpha$ and $x = \beta$

$\implies x - \alpha = 0$ and $x - \beta = 0$

$\implies (x-\alpha)(x-\beta) = 0$

$\implies x^2 - \alpha x - \beta x + \alpha\beta = 0$

$\implies x^2 - (\alpha+\beta)x + \alpha\beta = 0$

$\implies x^2 - \text{S}x + \text{P} = 0$

Where $\text{S} = \alpha + \beta$ and $\text{P} = \alpha\beta$.

1. If $\boldsymbol{\alpha, \beta}$ are the roots of $\boldsymbol{3x^2 - 2x + 4 = 0}$, find the values of

(i) $\boldsymbol{\frac{1}{\alpha^2} + \frac{1}{\beta^2}}$

Solution: Since α, β are roots of $3x^2 - 2x + 4 = 0$.

$$\implies \alpha + \beta = \frac{2}{3} \text{ and } \alpha\beta = \frac{4}{3}$$

Now, $\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\beta^2 + \alpha^2}{\alpha^2\beta^2}$

$$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2 + \beta^2}{\alpha^2\beta^2}$$

$$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2 + \beta^2 + 2\alpha\beta - 2\alpha\beta}{\alpha^2\beta^2}$$

$$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}$$

$$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\left(\frac{2}{3}\right)^2 - 2\left(\frac{4}{3}\right)}{\left(\frac{4}{3}\right)^2}$$

$$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\frac{4}{9} - \frac{8}{3}}{\frac{16}{9}}$$

$$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\frac{4-24}{9}}{\frac{16}{9}}$$

$$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\frac{-20}{9}}{\frac{16}{9}}$$

$$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{-20}{16}$$

$$\boldsymbol{\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{-5}{4}}$$

(ii) $\boldsymbol{\frac{\alpha}{\beta} + \frac{\beta}{\alpha}}$

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta}$$

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2 + 2\alpha\beta - 2\alpha\beta}{\alpha\beta}$$

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$$

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\left(\frac{2}{3}\right)^2 - 2\left(\frac{4}{3}\right)}{\frac{4}{3}}$$

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\frac{4}{9} - \frac{8}{3}}{\frac{4}{3}}$$

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\frac{4-24}{9}}{\frac{4}{3}}$$

$$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{\frac{-20}{9}}{\frac{4}{3}}$$
$$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{-20}{9}.\frac{3}{4}$$
$$\boldsymbol{\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{-5}{3}}$$

(iii) $\boldsymbol{\alpha^4+\beta^4}$

$$\alpha^4+\beta^4=(\alpha^2)^2+(\beta^2)^2$$
$$\alpha^4+\beta^4=(\alpha^2)^2+(\beta^2)^2+2\alpha^2\beta^2-2\alpha^2\beta^2$$
$$\alpha^4+\beta^4=(\alpha^2+\beta^2)^2-2\alpha^2\beta^2$$
$$\alpha^4+\beta^4=(\alpha^2+\beta^2+2\alpha\beta-2\alpha\beta)^2-2\alpha^2\beta^2$$
$$\alpha^4+\beta^4=((\alpha+\beta)^2-2\alpha\beta)^2-2(\alpha\beta)^2$$
$$\alpha^4+\beta^4=\left(\left(\frac{2}{3}\right)^2-2\left(\frac{4}{3}\right)\right)^2-2\left(\frac{4}{3}\right)^2$$
$$\alpha^4+\beta^4=\left(\frac{4}{9}-\frac{8}{3}\right)^2-2\left(\frac{16}{9}\right)$$
$$\alpha^4+\beta^4=\left(\frac{4-24}{9}\right)^2-\frac{32}{9}$$
$$\alpha^4+\beta^4=\left(\frac{-20}{9}\right)^2-\frac{32}{9}$$
$$\alpha^4+\beta^4=\frac{400}{81}-\frac{32}{9}$$
$$\alpha^4+\beta^4=\frac{400-288}{81}$$
$$\alpha^4+\beta^4=\frac{112}{81}$$

(iv) $\boldsymbol{\alpha^3+\beta^3}$

Solution:

$$\alpha^3+\beta^3=(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)$$
$$\alpha^3+\beta^3=(\alpha+\beta)(\alpha^2+\beta^2-\alpha\beta)$$
$$\alpha^3+\beta^3=(\alpha+\beta)(\alpha^2+\beta^2+2\alpha\beta-2\alpha\beta-\alpha\beta)$$
$$\alpha^3+\beta^3=(\alpha+\beta)((\alpha+\beta)^2-3\alpha\beta)$$
$$\alpha^3+\beta^3=\left(\frac{2}{3}\right)\left(\left(\frac{2}{3}\right)^2-3\left(\frac{4}{3}\right)\right)$$
$$\alpha^3+\beta^3=\left(\frac{2}{3}\right)\left(\frac{4}{9}-4\right)$$
$$\alpha^3+\beta^3=\left(\frac{2}{3}\right)\left(\frac{4-36}{9}\right)$$
$$\alpha^3+\beta^3=\left(\frac{2}{3}\right)\left(\frac{-32}{9}\right)$$
$$\alpha^3+\beta^3=\left(\frac{2}{3}\right)\left(\frac{-32}{9}\right)$$
$$\alpha^3+\beta^3=\frac{-64}{27}$$

(v) $\boldsymbol{\frac{1}{\alpha^3}+\frac{1}{\beta^3}}$

Solution:

$$\frac{1}{\alpha^3}+\frac{1}{\beta^3}=\frac{\beta^3+\alpha^3}{\alpha^3\beta^3}$$
$$\frac{1}{\alpha^3}+\frac{1}{\beta^3}=\frac{\alpha^3+\beta^3}{\alpha^3\beta^3}$$
$$\frac{1}{\alpha^3}+\frac{1}{\beta^3}=\frac{\alpha^3+\beta^3}{(\alpha\beta)^3}$$
$$\frac{1}{\alpha^3}+\frac{1}{\beta^3}=\frac{(\alpha+\beta)(\alpha^2+\beta^2-\alpha\beta)}{(\alpha\beta)^3}$$
$$\frac{1}{\alpha^3}+\frac{1}{\beta^3}=\frac{(\alpha+\beta)(\alpha^2+\beta^2+2\alpha\beta-2\alpha\beta-\alpha\beta)}{(\alpha\beta)^3}$$
$$\frac{1}{\alpha^3}+\frac{1}{\beta^3}=\frac{(\alpha+\beta)((\alpha+\beta)^2-3\alpha\beta)}{(\alpha\beta)^3}$$
$$\frac{1}{\alpha^3}+\frac{1}{\beta^3}=\frac{\left(\frac{2}{3}\right)\left(\left(\frac{2}{3}\right)^2-3\left(\frac{4}{3}\right)\right)}{\left(\frac{4}{3}\right)^3}$$
$$\frac{1}{\alpha^3}+\frac{1}{\beta^3}=\frac{\left(\frac{2}{3}\right)\left(\frac{4}{9}-4\right)}{\frac{64}{27}}$$
$$\frac{1}{\alpha^3}+\frac{1}{\beta^3}=\frac{\left(\frac{2}{3}\right)\left(\frac{4-36}{9}\right)}{\frac{64}{27}}$$
$$\frac{1}{\alpha^3}+\frac{1}{\beta^3}=\frac{\left(\frac{2}{3}\right)\left(\frac{-32}{9}\right)}{\frac{64}{27}}$$
$$\frac{1}{\alpha^3}+\frac{1}{\beta^3}=-\frac{\frac{64}{27}}{\frac{64}{27}}$$
$$\boldsymbol{\frac{1}{\alpha^3}+\frac{1}{\beta^3}=-1}$$

(vi) $\boldsymbol{\alpha^2-\beta^2}$

Solution

Now, $(\alpha^2-\beta^2)^2=\alpha^4+\beta^4-2\alpha^2\beta^2$

$$(\alpha^2-\beta^2)^2=(\alpha^2)^2+(\beta^2)^2+2\alpha^2\beta^2-2\alpha^2\beta^2-2\alpha^2\beta^2$$
$$(\alpha^2-\beta^2)^2=(\alpha^2+\beta^2)^2-4\alpha^2\beta^2$$
$$(\alpha^2-\beta^2)^2=(\alpha^2+\beta^2+2\alpha\beta-2\alpha\beta)^2-4\alpha^2\beta^2$$
$$(\alpha^2-\beta^2)^2=((\alpha+\beta)^2-2\alpha\beta)^2-4(\alpha\beta)^2$$
$$(\alpha^2-\beta^2)^2=\left(\left(\frac{2}{3}\right)^2-2\left(\frac{4}{3}\right)\right)^2-4\left(\frac{4}{3}\right)^2$$
$$(\alpha^2-\beta^2)^2=\left(\frac{4}{9}-\frac{8}{3}\right)^2-4\left(\frac{16}{9}\right)$$
$$(\alpha^2-\beta^2)^2=\left(\frac{4-24}{9}\right)^2-\frac{64}{9}$$

$(\alpha^2-\beta^2)^2=\left(\frac{-20}{9}\right)^2-\frac{64}{9}$
$(\alpha^2-\beta^2)^2=\frac{400}{81}-\frac{64}{9}$
$(\alpha^2-\beta^2)^2=\frac{400-576}{81}$
$(\alpha^2-\beta^2)^2=\frac{-176}{81}$
$\alpha^2-\beta^2=\pm\frac{\sqrt{-176}}{9}$
$\alpha^2-\beta^2=\pm\frac{4\sqrt{11}\,i}{9}$

2. If α,β are roots of $x^2-px-p-c=0$, prove that $(1+\alpha)(1+\beta)=1-c$.

Solution:

$x^2-px-p-c=0 \quad \rightarrow (1)$

Since α,β are the roots of (1)

$\Longrightarrow \alpha+\beta=\frac{-(-p)}{1}=p$
$\Longrightarrow \alpha\beta=\frac{-p-c}{1}=-p-c$

Now , $(1+\alpha)(1+\beta)=1+\alpha+\beta+\alpha\beta$

$(1+\alpha)(1+\beta)=1+p-p-c$
$(1+\alpha)(1+\beta)=1-c$
As required.

3. Find the condition that one root of $x^2+px+q=0$ is
(i) Double the other
(ii) square of the other
(iii) additive inverse of the other
(iv) multiplicative inverse of the other

solution:

$x^2+px+q=0 \quad \rightarrow (1)$

(i) Let $\alpha, 2\alpha$ be roots of (1)
$\Longrightarrow \alpha+2\alpha=-p$
$\Longrightarrow 3\alpha=-p$
$\Longrightarrow \alpha=\frac{-p}{3}$
$\Longrightarrow \alpha.2\alpha=q$
$2\alpha^2=q$
$2\left(\frac{-p}{3}\right)^2=q$
$2\frac{p^2}{9}=q$
$2p^2=9q$ is required condition.

(ii) Let α and α^2 be roots of equation (1)

$\Longrightarrow \alpha+\alpha^2=-p \longrightarrow (2)$
and $\alpha.\alpha^2=q$
$\Longrightarrow \alpha^3=q$
$\Longrightarrow \alpha=q^{\frac{1}{3}}$
Using value in (2)
$q^{\frac{1}{3}}+q^{\frac{2}{3}}=-p$
$\Longrightarrow q^{\frac{1}{3}}\left(1+q^{\frac{1}{3}}\right)=-p$
$\Longrightarrow q\left(1+q^{\frac{1}{3}}\right)^3=(-p)^3$
$\Longrightarrow q\left(1+q+3q^{\frac{1}{3}}\left(1+q^{\frac{1}{3}}\right)\right)=-p^3$
$\Longrightarrow q(1+q+3(-p))=-p^3$
$\Longrightarrow q+q^2-3pq+p^3=0$ is required condition.

(iii) Let α and $-\alpha$ be roots of equation (1)
$\Longrightarrow \alpha-\alpha=-p$
$\Longrightarrow 0=-p$
$\Longrightarrow p=0$ is required condition.

(iv) Let α and $\frac{1}{\alpha}$ be roots of equation (1)
$\Longrightarrow \alpha.\frac{1}{\alpha}=q$
$\Longrightarrow 1=q$
$\Longrightarrow q=1$ is required condition.

4. If the roots of the equation $x^2-px+q=0$ differ by unity, prove that $p^2=4q+1$.

Solution:

Let α and $\alpha-1$ be roots of the equation.
$\alpha+\alpha-1=-(-p)$
$2\alpha-1=p$
$2\alpha=p+1$
$2\alpha=p+1$
$\alpha=\frac{p+1}{2}$
And $\alpha(\alpha-1)=q$
$\frac{p+1}{2}\left(\frac{p+1}{2}-1\right)=q$
$\frac{p+1}{2}\left(\frac{p+1-2}{2}\right)=q$
$\frac{p+1}{2}\left(\frac{p-1}{2}\right)=q$
$\frac{p^2-1}{4}=q$
$p^2-1=4q$
$p^2=4q+1$ as required.

5. **Find the condition that $\frac{a}{x-a}+\frac{b}{x-b}=5$ may have roots equal in magnitude but opposite in signs.**

Solution:

$$\frac{a}{x-a}+\frac{b}{x-b}=5$$
$$\frac{a(x-b)+b(x-a)}{(x-a)(x-b)}=5$$
$$\frac{ax-ab+bx-ab}{(x-a)(x-b)}=5$$
$$ax-ab+bx-ab=5(x-a)(x-b)$$
$$ax-ab+bx-ab=5(x^2-ax-bx+ab)$$
$$ax-ab+bx-ab=5x^2-5ax-5bx+5ab$$
$$5x^2-5ax-5bx+5ab-ax+ab-bx+ab=0$$
$$5x^2-6ax-6bx+7ab=0$$
$$5x^2-6(a+b)x+7ab=0$$

Let $\alpha,-\alpha$ be roots of this equation

$$\alpha+(-\alpha)=\frac{-6(a+b)}{5}$$

$$0=\frac{-6(a+b)}{5}$$

$a+b=0$ is the required condition.

6. **If the roots of $px^2+qx+q=0$ are $,\beta$, prove that $\sqrt{\frac{\alpha}{\beta}}+\sqrt{\frac{\beta}{\alpha}}=\sqrt{\frac{\beta}{\alpha}}$.**

Solution: $px^2+qx+q=0 \longrightarrow (1)$

Since α,β are the roots of equation (1)

$$\Rightarrow \quad \alpha+\beta=-\frac{q}{p}$$

and $\alpha\beta=\frac{q}{p}$

$$\sqrt{\alpha\beta}=\sqrt{\frac{q}{p}}$$

$$\frac{\alpha+\beta}{\sqrt{\alpha\beta}}=-\frac{\frac{q}{p}}{\sqrt{\frac{q}{p}}}$$

$$\frac{\alpha}{\sqrt{\alpha\beta}}+\frac{\beta}{\sqrt{\alpha\beta}}=-\frac{\frac{q}{p}}{\sqrt{\frac{q}{p}}}$$

$$\frac{\sqrt{\alpha}\sqrt{\alpha}}{\sqrt{\alpha}\sqrt{\beta}}+\frac{\sqrt{\beta}\sqrt{\beta}}{\sqrt{\alpha}\sqrt{\beta}}=-\frac{\sqrt{\frac{q}{p}}\sqrt{\frac{q}{p}}}{\sqrt{\frac{q}{p}}}$$

$$\frac{\sqrt{\alpha}}{\sqrt{\beta}}+\frac{\sqrt{\beta}}{\sqrt{\alpha}}=-\sqrt{\frac{q}{p}}$$

$$\sqrt{\frac{\alpha}{\beta}}+\sqrt{\frac{\beta}{\alpha}}+\sqrt{\frac{q}{p}}=0$$

as required.

7. **If α,β are roots of $ax^2+bx+c=0$, form the equations whose roots are.**

(i) $\alpha^2,\ \beta^2$

Solution: Since α,β are roots of $ax^2+bx+c=0$.

$$\alpha+\beta=-\frac{b}{a}$$
$$\alpha\beta=\frac{c}{a}$$

We have to form an equation, whose roots are $\alpha^2,\ \beta^2$

$$S=\alpha^2+\beta^2$$
$$S=\alpha^2+\beta^2+2\alpha\beta-2\alpha\beta$$
$$S=(\alpha+\beta)^2-2\alpha\beta$$
$$S=\left(-\frac{b}{a}\right)^2-2\left(\frac{c}{a}\right)$$
$$S=\frac{b^2}{a^2}-\frac{2c}{a}$$
$$S=\frac{b^2-2ac}{a^2}$$
$$P=\alpha^2\beta^2=(\alpha\beta)^2=\left(\frac{c}{a}\right)^2=\frac{c^2}{a^2}$$

Equation is $x^2-Sx+P=0$

$$x^2-\frac{b^2-2ac}{a^2}x+\frac{c^2}{a^2}=0$$
$$\boldsymbol{a^2x^2-(b^2-2ac)x+c^2=0}$$

(ii) $\frac{1}{\alpha},\frac{1}{\beta}$

$$S=\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\beta+\alpha}{\alpha\beta}=\frac{-\frac{b}{a}}{\frac{c}{a}}=-\frac{b}{c}$$
$$P=\frac{1}{\alpha}\frac{1}{\beta}=\frac{1}{\alpha\beta}=\frac{a}{c}$$

Equation is $x^2-Sx+P=0$

$$x^2-\left(-\frac{b}{c}\right)x+\frac{a}{c}=0$$
$$x^2+\frac{b}{c}x+\frac{a}{c}=0$$
$$cx^2+bx+a=0$$

(iii) $\frac{1}{\alpha^2},\frac{1}{\beta^2}$

Solution:

$$S=\frac{1}{\alpha^2}+\frac{1}{\beta^2}$$

$$= \frac{\beta^2 + \alpha^2}{\alpha^2\beta^2}$$
$$= \frac{\alpha^2 + \beta^2 + 2\alpha\beta - 2\alpha\beta}{(\alpha\beta)^2}$$
$$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{(\alpha\beta)^2} = \frac{\left(-\frac{b}{a}\right)^2 - 2\left(\frac{c}{a}\right)}{\left(\frac{c}{a}\right)^2}$$
$$= \frac{\frac{b^2}{a^2} - \frac{2c}{a}}{\frac{c^2}{a^2}} = \frac{\frac{b^2 - 2ac}{a^2}}{\frac{c^2}{a^2}} = \frac{b^2 - 2ac}{c^2}$$

$$P = \frac{1}{\alpha^2}\frac{1}{\beta^2} = \frac{1}{(\alpha\beta)^2} = \frac{a^2}{c^2}$$

Equation is $x^2 - Sx + P = 0$
$$x^2 - \left(\frac{b^2 - 2ac}{c^2}\right)x + \frac{a^2}{c^2} = 0$$
$$\boldsymbol{c^2x^2 - (b^2 - 2ac)x + a^2 = 0}$$

(iv) $\boldsymbol{\alpha^3, \beta^3}$

Solution:
$$S = \alpha^3 + \beta^3$$
$$= (\alpha+\beta)(\alpha^2 - \alpha\beta + \beta^2)$$
$$= (\alpha+\beta)(\alpha^2 + \beta^2 + 2\alpha\beta - 2\alpha\beta - \alpha\beta)$$
$$= (\alpha+\beta)((\alpha+\beta)^2 - 3\alpha\beta)$$
$$= \left(-\frac{b}{a}\right)\left(\left(-\frac{b}{a}\right)^2 - 3\frac{c}{a}\right)$$
$$= \left(-\frac{b}{a}\right)\left(\frac{b^2}{a^2} - \frac{3c}{a}\right)$$
$$= \left(-\frac{b}{a}\right)\left(\frac{b^2 - 3ac}{a^2}\right) = \frac{-b^3 + 3abc}{a^3}$$
$$P = \alpha^3\beta^3 = (\alpha\beta)^3 = \frac{c^3}{a^3}$$

Equation is $x^2 - \text{S}x + \text{P} = 0$
$$x^2 - \left(\frac{-b^3 + 3abc}{a^3}\right)x + \frac{c^3}{a^3} = 0$$
$$\boldsymbol{a^3x^2 + (b^3 - 3abc)x + c^3 = 0}$$

Note: Do remaining parts yourself.

8. If $\boldsymbol{\alpha, \beta}$ are roots of $\boldsymbol{5x^2 - x - 2 = 0}$, form an equation whose roots are $\boldsymbol{\frac{3}{\alpha}}$ and $\boldsymbol{\frac{3}{\beta}}$.

Solution $5x^2 - x - 2 = 0 \longrightarrow (1)$

Since α, β are roots of equation (1)
$$\Rightarrow \alpha + \beta = \frac{1}{5}$$
$$\alpha\beta = -\frac{2}{5}$$

Now we have to form an equation whose roots are $\frac{3}{\alpha}$ and $\frac{3}{\beta}$.
$$S = \frac{3}{\alpha} + \frac{3}{\beta} = \frac{3\beta + 3\alpha}{\alpha\beta} = \frac{3(\beta+\alpha)}{\alpha\beta} = \frac{3(\alpha+\beta)}{\alpha\beta} = \frac{3\left(\frac{1}{5}\right)}{-\frac{2}{5}}$$
$$= \frac{\frac{3}{5}}{-\frac{2}{5}} = -\frac{3}{2}$$
$$P = \frac{3}{\alpha}.\frac{3}{\beta} = \frac{9}{\alpha\beta} = \frac{9}{-\frac{2}{5}} = -\frac{45}{2}$$

Now equation is
$$x^2 - Sx + P = 0$$
$$x^2 - \left(-\frac{3}{2}\right)x - \frac{45}{2} = 0$$
$$2x^2 + 3x - 45 = 0$$

9. If $\boldsymbol{\alpha, \beta}$ are roots of $\boldsymbol{x^2 - 3x + 5 = 0}$, form an equation whose roots are $\boldsymbol{\frac{1-\alpha}{1+\alpha}}$ and $\boldsymbol{\frac{1-\beta}{1+\beta}}$.

Solution: $x^2 - 3x + 5 = 0 \longrightarrow (1)$

Since α, β are roots of equation (1)
$$\Rightarrow \alpha + \beta = 3$$
$$\alpha\beta = 5$$

Now we have to form an equation whose roots are $\frac{1-\alpha}{1+\alpha}$ and $\frac{1-\beta}{1+\beta}$.
$$S = \frac{1-\alpha}{1+\alpha} + \frac{1-\beta}{1+\beta}$$
$$S = \frac{(1-\alpha)(1+\beta) + (1+\alpha)(1-\beta)}{(1+\alpha)(1+\beta)}$$
$$S = \frac{1 - \alpha + \beta - \alpha\beta + 1 + \alpha - \beta - \alpha\beta}{1 + \alpha + \beta + \alpha\beta}$$
$$S = \frac{1 - \alpha\beta + 1 - \alpha\beta}{1 + \alpha + \beta + \alpha\beta}$$
$$S = \frac{2 - 2\alpha\beta}{1 + (\alpha+\beta) + \alpha\beta} = \frac{2 - 2(5)}{1 + (3) + 5} = -\frac{8}{9}$$
$$P = \left(\frac{1-\alpha}{1+\alpha}\right)\left(\frac{1-\beta}{1+\beta}\right)$$
$$P = \frac{1 - \alpha - \beta + \alpha\beta}{1 + \alpha + \beta + \alpha\beta}$$
$$P = \frac{1 - (\alpha+\beta) + \alpha\beta}{1 + (\alpha+\beta) + \alpha\beta}$$
$$P = \frac{1 - 3 + 5}{1 + 3 + 5} = \frac{3}{9} = \frac{1}{3}$$

Now equation is
$$x^2 - Sx + P = 0$$
$$x^2 - \left(-\frac{8}{9}\right)x + \frac{1}{3} = 0$$
$$9x^2 + 8x + 3 = 0$$