Exercise 4.5

Use remainder theorem to find the remainder when the first polynomial is divided by the second polynomial.

1. $\boldsymbol{x^2 + 3x + 7, x + 1}$

Solution: Let $p(x) = x^2 + 3x + 7$

Since divisor is $x - a = x + 1$

$\Rightarrow \quad a = -1$

Now by remainder theorem

$p(-1) = (-1)^2 + 3(-1) + 7$

$p(-1) = 1 - 3 + 7$

$p(-1) = 5$ is the remainder.

2. $\boldsymbol{x^3 - x^2 + 5x + 4, x - 2}$

Solution: Let $p(x) = x^3 - x^2 + 5x + 4$

Since divisor is $x - a = x - 2$
$\Rightarrow \quad a = 2$
Now by remainder theorem
$p(2) = 2^3 - 2^2 + 5(2) + 4$
$p(2) = 8 - 4 + 10 + 4$
$p(2) = 18$ is the remainder.

3. $\boldsymbol{3x^4 + 4x^3 + x - 5, \ x + 1}$

Solution: Let $p(x) = 3x^4 + 4x^3 + x - 5$

Since divisor is $x - a = x + 1$
$\Rightarrow \quad a = -1$
Now by remainder theorem
$p(-1) = 3(-1)^4 + 4(-1)^3 + (-1) - 5$
$p(-1) = 3 - 4 - 1 - 5$
$p(-1) = -7$ is the remainder

4. $\boldsymbol{x^3 - 2x^2 + 3x + 3, x - 3}$

Solution: Let $p(x) = x^3 - 2x^2 + 3x + 3$

Since divisor is $x - a = x - 3$
$\Rightarrow \quad a = 3$
Now by remainder theorem
$p(3) = 3^3 - 2(3)^2 + 3(3) + 3$
$p(3) = 27 - 18 + 9 + 3$
$p(3) = 21$ is the remainder

Use factor theorem to determine if the first polynomial is a factor of second polynomial.

5. $\boldsymbol{x - 1, x^2 + 4x - 5}$

Solution: Let $p(x) = x^2 + 4x - 5$

Since divisor is $x - a = x - 1$
$\Rightarrow \quad a = 1$

Now ; $p(1) = 1^2 + 4(1) - 5$

$p(1) = 1 + 4 - 5$
$p(1) = 0$
$\Rightarrow R = 1$

Hence by factor theorem $x - 1$ is the factor of $x^2 + 4x - 5$.

6. $\boldsymbol{x - 2, x^3 + x^2 - 7x + 2}$

Solution: Let $p(x) = x^3 + x^2 - x + 2$

Since divisor is $x - a = x - 2$
$\Rightarrow \quad a = 2$
Now ; $p(2) = 2^3 + 2^2 - 7(2) + 2$
$p(2) = 14 - 14$
$p(2) = 0$
$\Rightarrow R = 0$

Hence by factor theorem $x - 2$ is the factor of $x^3 + x^2 - 7x + 2$.

7. $\boldsymbol{w + 2, 2w^3 + w^2 - 4w + 4}$

Solution: Let $p(w) = 2w^3 + w^2 - 4w + 4$
Since divisor is $x - a = w + 2$
$\Rightarrow \quad a = -2$
$p(-2) = 2(-2)^3 + (-2)^2 - 4(-2) + 4$
$p(-2) = -16 + 4 + 8 + 4$
$p(-2) = 0$
$\Rightarrow R = 0$
Hence by factor theorem $w + 2$ is the factor of $2w^3 + w^2 - 4w + 4$.

8. $\boldsymbol{x - a, x^n - a^n, n \text{ is a positive integer}}$.

Solution: let $p(x) = x^n - a^n$
Now $p(a) = a^n - a^n$
$p(a) = a^n - a^n$
$p(a) = 0$
$\Rightarrow R = 0$

Hence by factor theorem $x-a$ is the factor of x^n-a^n.

9. $x+a, x^n+a^n$, n is an odd integer.

Solution: let $p(x)=x^n+a^n$
Now; $p(-a)=(-a)^n-a^n$
$p(-a)=(-a)^n+a^n$
$p(-a)=-a^n+a^n$
$\Rightarrow R=0$
Hence by factor theorem $x+a$ is the factor of x^n+a^n.

10. **When $x^4+2x^3+kx^2+3$ is divided by $x-2$, the remainder is 1. Find the value of k.**

Solution:

Let $P(x)=x^4+2x^3+kx^2+3$
Since the divisor is $x-a=x-2$
$\Rightarrow a=2$
$P(2)=1$
Therefore by remainder theorem
$P(2)=2^4+2(2)^3+k(2)^2+3$
$1=2^4+2(2)^3+k(2)^2+3$
$1=16+16+4k+3$
$4k=1-35$
$4k=-34$
$k=-\frac{17}{2}$

11. **When the polynomial x^3+2x^2+kx+4 is divided by $x-2$, the remainder is 14. Find the value of k.**

Solution:

Let $P(x)=x^3+2x^2+kx+4$

Since the divisor is $x-a=x-2$

$\Rightarrow a=2$
$P(2)=14$
Therefore by remainder theorem
$P(2)=x^3+2x^2+kx+4$

$14=2^3+2(2)^2+k(2)+4$
$14=8+8+2k+4$
$2k=14-20$
$2k=-6$
$k=-3$

Use synthetic division to show that x is the solution of the polynomial and use the result to factorize the polynomial completely.

12. **$x^3-7x+6, x=2$**

Solution:

Let $P(x)=x^3-7x+6$
Using synthetic division

2	1	0	-7	6
		2	4	-6
	1	2	-3	0

As $R=0 \Rightarrow x=2$ is solution of $P(x)=0$
$\Rightarrow x-2$ is factor of $P(x)$
now depressed equation is
$x^2+2x-3=0$
$x^2+3x-x-3=0$
$x(x+3)-1(x+3)=0$
$(x+3)(x-1)=0$
Hence complete factorization of $P(x)$ is
$P(x)=(x-2)(x+3)(x-1)$

13. **$x^3-28x-48=0, x=-4$**

Solution: $x^3-28x-48=0 \longrightarrow (1)$

Using synthetic division

-2	1	0	-28	-48
		-2	4	48
	1	-2	-24	0

As $R=0 \Rightarrow x=-2$ is solution of (1)
$\Rightarrow x+2=0$ is factor of (1)
Now depressed equation is
$x^2-2x-24=0$
$x^2-6x+4x-24=0$
$x(x-6)-4(x-6)=0$
$(x-6)(x-4)=0$
Hence complete factorization of $P(x)$ is
$(x+2)(x-6)(x-4)=0$

14. **$2x^4+7x^3-4x^2-27x-18, x=2, x=-3$**

Solution:

Let $P(x)=2x^4+7x^3-4x^2-27x-18$
Using synthetic division

2	2	7	-4	-27	-18
		4	22	36	18
-3	2	11	18	9	0
		-6	-15	-9	
	2	5	3	0	

As remainder is zero, hence $x = 2$ and $x = -3$ are solutions of $2x^4 + 7x^3 - 4x^2 - 27x - 18$.

Now depressed equation is
$2x^2 + 5x + 3 = 0$

$$2x^2 + 2x + 3x + 3 = 0$$
$$2x(x + 1) + 3(x + 1) = 0$$
$$(x + 1)(2x + 3) = 0$$

Hence complete factorization of (1) is
$2x^4 + 7x^3 - 4x^2 - 27x - 18$
$= (x - 2)(x + 3(x + 1)(2x + 3)$

15. Use synthetic division to find the values of p and q if $x + 1$and $x - 2$ are the factors of the polynomial $x^3 + px^2 + qx + 6$.

Solution: Let $P(x) = x^3 + px^2 + qx + 6$
Using synthetic division

-1	1	p	q	6
		-1	$-p + 1$	$p - q - 1$
	1	$p - 1$	$-p + q + 1$	$p - q + 5$

Since $x + 1$ is factor of $P(x)$
$$p - q + 5 = 0$$
$$p - q = -5 \;\longrightarrow (1)$$

Again by synthetic division

2	1	p	q	6
		2	$2p + 4$	$4p + 2q + 8$
	1	$p + 2$	$2p + q + 4$	$4p + 2q + 14$

Since $x - 2$ is factor of $P(x)$
$$4p + 2q + 14 = 0$$
$$2p + q + 7 = 0$$
$$2p + q = -7 \longrightarrow (2)$$
Now adding equations (1) and (2)
$$3p = -12$$
$p = -4$ put in (1)

$$-4 - q = -5$$
$$-q = -1$$
$$q = 1$$

16. Find the values of a an b if -2 and 2 are the roots of polynomial eqution $x^3 - 4x^2 + ax + b = 0$

Solution: Let $P(x) = \boldsymbol{x^3 - 4x^2 + ax + b = 0}$

Using synthetic division

-2	1	-4	a	b
		-2	12	$-2a - 24$
	1	-6	$a + 12$	$-2a + b - 24$

Since $x = -2$ is root of $P(x) = 0$
$$-2a + b - 24 = 0$$
$$-2a + b = 24 \;\longrightarrow (1)$$

Again by synthetic division

2	1	-4	a	b
		2	-4	$2a - 8$
	1	-2	$a - 4$	$2a + b - 8$

Since $x = 2$ is factor of $P(x) = 0$
$$2a + b - 8 = 0$$
$$2a + b = 8 \;\longrightarrow (2)$$
Now adding equations (1) and (2)
$$2b = 32$$
$$b = 16$$
put in (1)

$$-2a + 16 = 24$$
$$2a = -8$$
$$a = -4$$