

{ Question # 1

(i) Let x be a cube root of 8 then

$$\begin{aligned} x &= (8)^{\frac{1}{3}} && \Rightarrow x^3 = 8 \\ \Rightarrow x^3 - 8 &= 0 && \Rightarrow (x)^3 - (2)^3 = 0 \\ \Rightarrow (x-2)(x^2 + 2x + 4) &= 0 \\ \Rightarrow x-2 = 0 & \text{ or } x^2 + 2x + 4 = 0 \\ \Rightarrow x = 2 & \text{ or } x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(4)}}{2(1)} \\ &= \frac{-2 \pm \sqrt{4 - 16}}{2} = \frac{-2 \pm \sqrt{-12}}{2} \\ &= \frac{-2 \pm 2\sqrt{-3}}{2} = 2 \left(\frac{-1 \pm \sqrt{-3}}{2} \right) \\ \Rightarrow x &= 2 \left(\frac{-1 + \sqrt{-3}}{2} \right) && \text{ or } x = 2 \left(\frac{-1 - \sqrt{-3}}{2} \right) \\ \Rightarrow x &= 2\omega && \text{ or } x = 2\omega^2 \end{aligned}$$

Review:

$$\omega = \frac{-1 + \sqrt{-3}}{2}$$

$$\omega^2 = \frac{-1 - \sqrt{-3}}{2}$$

Hence cube root of 8 are $2, 2\omega$ and $2\omega^2$.

(ii) Hint

Considering x as a cube root of -8 and Solving as above you will get the following values of x

$$\begin{aligned} x &= -2, \quad x = \frac{2 + 2\sqrt{-3}}{2}, \quad x = \frac{2 - 2\sqrt{-3}}{2} \\ \Rightarrow x &= -2 \left(\frac{-1 - \sqrt{-3}}{2} \right), \quad x = -2 \left(\frac{-1 + \sqrt{-3}}{2} \right) \\ \Rightarrow x &= -2\omega^2, \quad x = -2\omega \end{aligned}$$

Hence cube root of -8 are $-2, -2\omega$ and $-2\omega^2$.

(iii) Do yourself as (iv) below.

(iv) Let x be a cube root of -27 then

$$\begin{aligned} x &= (-27)^{\frac{1}{3}} && \Rightarrow x^3 = -27 \\ \Rightarrow x^3 + 27 &= 0 && \Rightarrow (x)^3 + (3)^3 = 0 \\ \Rightarrow (x+3)(x^2 - 3x + 9) &= 0 \\ \Rightarrow x+3 = 0 & \text{ or } x^2 - 3x + 9 = 0 \\ \Rightarrow x = -3 & \text{ or } x = \frac{3 \pm \sqrt{(-3)^2 - 4(1)(9)}}{2(1)} \\ &= \frac{3 \pm \sqrt{9 - 36}}{2} = \frac{3 \pm \sqrt{-27}}{2} = \frac{3 \pm 3\sqrt{-3}}{2} \\ \Rightarrow x &= \frac{3 + 3\sqrt{-3}}{2} && \text{ or } x = \frac{3 - 3\sqrt{-3}}{2} \end{aligned}$$

$$\Rightarrow x = -3\left(\frac{-1 - \sqrt{-3}}{2}\right) \quad \text{or} \quad x = -3\left(\frac{-1 + \sqrt{-3}}{2}\right)$$

$$\Rightarrow x = -3\omega^2 \quad \text{or} \quad x = -3\omega$$

Hence cube root of -27 are $-3, -3\omega$ and $-3\omega^2$.

(v) Let x be a cube root of 64 then

$$x = (64)^{\frac{1}{3}} \quad \Rightarrow \quad x^3 = 64$$

$$\Rightarrow x^3 - 64 = 0 \quad \Rightarrow \quad (x)^3 - (4)^3 = 0$$

$$\Rightarrow (x - 4)(x^2 + 4x + 16) = 0$$

$$\Rightarrow x - 4 = 0 \quad \text{or} \quad x^2 + 4x + 16 = 0$$

$$\Rightarrow x = 4 \quad \text{or} \quad x = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(16)}}{2(1)}$$

$$= \frac{-4 \pm \sqrt{16 - 64}}{2} = \frac{-4 \pm \sqrt{-48}}{2}$$

$$= \frac{-4 \pm 4\sqrt{-3}}{2} \quad \because 48 = 16 \times 3$$

$$\Rightarrow x = \frac{-4 + 4\sqrt{-3}}{2} \quad \text{or} \quad x = \frac{-4 - 4\sqrt{-3}}{2}$$

$$\Rightarrow x = 4\left(\frac{-1 + \sqrt{-3}}{2}\right) \quad \text{or} \quad x = 4\left(\frac{-1 - \sqrt{-3}}{2}\right)$$

$$\Rightarrow x = 4\omega \quad \text{or} \quad x = 4\omega^2$$

Hence cube root of 64 are $4, 4\omega$ and $4\omega^2$.

{ Question # 2

(i) $(1 + \omega - \omega^2)^8 = (1 + \omega + \omega^2 - 2\omega^2)^8$

$$= (0 - 2\omega^2)^8 \quad \because 1 + \omega + \omega^2 = 0$$

$$= (-2)^8(\omega^2)^8 = 256\omega^{16}$$

$$= 256\omega^{15} \cdot \omega = 256(\omega^3)^5 \cdot \omega$$

$$= 256(1)^5 \cdot \omega = 256\omega \quad \text{Answer} \quad \because \omega^3 = 1$$

(ii) $\omega^{28} + \omega^{29} + 1 = \omega^{27} \cdot \omega + \omega^{27} \cdot \omega^2 + 1$

$$= (\omega^3)^9 \cdot \omega + (\omega^3)^9 \cdot \omega^2 + 1$$

$$= (1)^9 \cdot \omega + (1)^9 \cdot \omega^2 + 1 \quad \because \omega^3 = 1$$

$$= \omega + \omega^2 + 1$$

$$= 0 \quad \text{Answer} \quad \because 1 + \omega + \omega^2 = 0$$

(iii) $(1 + \omega - \omega^2)(1 - \omega + \omega^2)$

$$= (1 + \omega + \omega^2 - 2\omega^2)(1 + \omega + \omega^2 - 2\omega)$$

$$\because 1 + \omega + \omega^2 = 0$$

$$= (0 - 2\omega^2)(0 - 2\omega) = (-2\omega^2)(-2\omega)$$

$$= 4\omega^3 = 4(1) = 4 \quad \text{Answer} \quad \because \omega^3 = 1$$

$$\begin{aligned}
 \text{(iv)} \quad & \left(\frac{-1 + \sqrt{-3}}{2} \right)^7 + \left(\frac{-1 - \sqrt{-3}}{2} \right)^7 \quad \left| \begin{array}{l} \because \omega = \frac{-1 + \sqrt{-3}}{2} \\ \omega^2 = \frac{-1 - \sqrt{-3}}{2} \end{array} \right. \\
 & = \omega^7 + (\omega^2)^7 \\
 & = \omega^7 + \omega^{14} \\
 & = \omega^6 \cdot \omega + \omega^{12} \cdot \omega^2 = (\omega^3)^3 \cdot \omega + (\omega^3)^4 \cdot \omega^2 \\
 & = (1)^3 \cdot \omega + (1)^4 \cdot \omega^2 \\
 & = \omega + \omega^2 = -1 \quad \text{Answer} \quad \because 1 + \omega + \omega^2 = 0
 \end{aligned}$$

$$\begin{aligned}
 \text{(v)} \quad & (-1 + \sqrt{-3})^5 + (-1 - \sqrt{-3})^5 \\
 & = \left(2 \cdot \frac{-1 + \sqrt{-3}}{2} \right)^5 + \left(2 \cdot \frac{-1 - \sqrt{-3}}{2} \right)^5 \quad \left| \begin{array}{l} \because \omega = \frac{-1 + \sqrt{-3}}{2} \\ \omega^2 = \frac{-1 - \sqrt{-3}}{2} \end{array} \right. \\
 & = (2 \cdot \omega)^5 + (2 \cdot \omega^2)^5 \\
 & = 32\omega^5 + 32\omega^{10} = 32\omega^3 \cdot \omega^2 + 32\omega^9 \cdot \omega^1 \\
 & = 32(1) \cdot \omega^2 + 32(1) \cdot \omega \quad \because \omega^9 = (\omega^3)^3 = (1)^3 = 1 \\
 & = 32(\omega + \omega^2) \\
 & = 32(-1) = -32 \quad \because 1 + \omega + \omega^2 = 0
 \end{aligned}$$

{ Question # 3

$$\begin{aligned}
 \text{(i)} \quad \text{R.H.S} &= (x - y)(x - \omega y)(x - \omega^2 y) \\
 &= (x - y)[x(x - \omega^2 y) - \omega y(x - \omega^2 y)] \\
 &= (x - y)[x^2 - \omega^2 xy - \omega xy + \omega^3 y^2] \\
 &= (x - y)[x^2 - (\omega^2 + \omega)xy + (1)y^2] \quad \because \omega^3 = 1 \\
 &= (x - y)[x^2 - (-1)xy + y^2] \\
 &= (x - y)[x^2 + xy + y^2] \quad \left| \begin{array}{l} \because 1 + \omega + \omega^2 = 0 \\ \therefore \omega + \omega^2 = -1 \end{array} \right. \\
 &= x^3 - y^3 = \text{L.H.S}
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad \text{R.H.S} &= (x + y + z)(x + \omega y + \omega^2 z)(x + \omega^2 y + \omega z) \\
 &= (x + y + z)[x^2 + \omega^2 xy + \omega xz + \omega xy + \omega^3 y^2 + \omega^2 yz \\
 & \quad + \omega^2 xz + \omega^4 yz + \omega^3 z^2] \\
 &= (x + y + z)[x^2 + (\omega^2 + \omega)xy + (\omega + \omega^2)xz + (\omega^2 + \omega^4)yz \\
 & \quad + \omega^3 y^2 + \omega^3 z^2] \\
 &= (x + y + z)[x^2 + (-1)xy + (-1)xz + (\omega^2 + \omega)yz + (1)y^2 + (1)z^2] \\
 & \quad \because \omega^4 = \omega \ \& \ \omega + \omega^2 = -1 \\
 &= (x + y + z)[x^2 + y^2 + z^2 - xy + (-1)yz - xz] \\
 &= (x + y + z)[x^2 + y^2 + z^2 - xy - yz - xz] \\
 &= x^3 + y^3 + z^3 - 3xyz = \text{L.H.S}
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii)} \quad \text{L.H.S} &= (1 + \omega)(1 + \omega^2)(1 + \omega^4)(1 + \omega^8) \dots \dots \dots 2n \text{ factors} \\
 &= [(1 + \omega)(1 + \omega^2)][(1 + \omega^4)(1 + \omega^8)] \dots \dots \dots n \text{ factors} \\
 &= [(1 + \omega)(1 + \omega^2)][(1 + \omega^3 \cdot \omega)(1 + \omega^6 \cdot \omega^2)] \dots \dots \dots n \text{ factors} \\
 &= [(1 + \omega)(1 + \omega^2)][(1 + \omega^3 \cdot \omega)(1 + (\omega^3)^2 \cdot \omega^2)] \dots \dots \dots n \text{ factors}
 \end{aligned}$$

$$\begin{aligned}
 &= [(1 + \omega)(1 + \omega^2)][(1 + 1 \cdot \omega)(1 + (1)^2 \cdot \omega^2)] \dots \dots \dots n \text{ factors} \\
 &= [(1 + \omega)(1 + \omega^2)][(1 + \omega)(1 + \omega^2)] \dots \dots \dots n \text{ factors} \\
 &= [(1 + \omega)(1 + \omega^2)]^n = [1 + \omega + \omega^2 + \omega^3]^n \\
 &= [0 + 1]^n \qquad \qquad \qquad \because 1 + \omega + \omega^2 = 0 \ , \ \omega^3 = 1 \\
 &= [1]^n = 1 = \text{R.H.S}
 \end{aligned}$$

{ Question # 4 (i)

Let $x^2 + x + 1 = 0 \dots \dots \dots (i)$

Since ω is root of (i) therefore

$$\omega^2 + \omega + 1 = 0 \dots \dots \dots (ii)$$

To prove ω^2 is root of (i)

Consider $(\omega^2)^2 + \omega^2 + 1 = \omega^4 + 2\omega^2 + 1 - \omega^2$

$$\begin{aligned}
 &= (\omega^2 + 1)^2 - \omega^2 = (\omega^2 + 1 + \omega)(\omega^2 + 1 - \omega) \\
 &= (0)(\omega^2 + 1 - \omega) \qquad \qquad \qquad \text{from (i)}
 \end{aligned}$$

$$\Rightarrow (\omega^2)^2 + \omega^2 + 1 = 0 \dots \dots \dots (iii)$$

$\Rightarrow \omega^2$ is the root of the equation (i).

Now subtracting (ii) from (iii)

$$\begin{array}{r}
 (\omega^2)^2 + \omega^2 + 1 = 0 \\
 \underline{\omega^2 + \omega + 1 = 0} \\
 \omega^4 - \omega = 0 \\
 \Rightarrow \omega(\omega^3 - 1) = 0 \\
 \Rightarrow \omega^3 - 1 = 0 \quad \text{as } \omega \neq 0 \\
 \Rightarrow \boxed{\omega^3 = 1}
 \end{array}$$

{ Question # 5

Let x be a cube root of -1 then

$$\begin{aligned}
 x &= (-1)^{\frac{1}{3}} \quad \Rightarrow \quad x^3 = -1 \\
 \Rightarrow x^3 + 1 &= 0 \quad \Rightarrow \quad (x)^3 + (1)^3 = 0 \\
 \Rightarrow (x + 1)(x^2 - x + 1) &= 0 \\
 \Rightarrow x + 1 = 0 \quad \text{or} \quad x^2 - x + 1 &= 0 \\
 \Rightarrow x = -1 \quad \text{or} \quad x &= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(1)}}{2(1)} \\
 &= \frac{1 \pm \sqrt{1 - 4}}{2} = \frac{1 \pm \sqrt{-3}}{2} \\
 \Rightarrow x &= \frac{1 + \sqrt{-3}}{2} \quad \text{or} \quad x = \frac{1 - \sqrt{-3}}{2} \\
 \Rightarrow x &= \frac{1 + \sqrt{3}i}{2} \quad \text{or} \quad x = \frac{1 - \sqrt{3}i}{2}
 \end{aligned}$$

Hence complex cube root of -1 are $\frac{1 + \sqrt{3}i}{2}$ and $\frac{1 - \sqrt{3}i}{2}$.

{ Question # 6

Since 2ω and $2\omega^2$ are roots of required equation, therefore

$$\begin{aligned} (x - 2\omega)(x - 2\omega^2) &= 0 \\ \Rightarrow x^2 - 2\omega x - 2\omega^2 x + 4\omega^3 &= 0 \\ \Rightarrow x^2 - 2x(\omega + \omega^2) + 4(1) &= 0 & \because \omega^3 = 1 \\ \Rightarrow x^2 - 2x(-1) + 4 &= 0 & \because 1 + \omega + \omega^2 = 0 \\ \Rightarrow x^2 + 2x + 4 &= 0 \end{aligned}$$

is the required equation.

{ Question # 7

(i) Let x be a fourth root of 16 then

$$\begin{aligned} x &= (16)^{\frac{1}{4}} \Rightarrow x^4 = 16 \\ \Rightarrow x^4 - 16 &= 0 \Rightarrow (x^2)^2 - (4)^2 = 0 \\ \Rightarrow (x^2 + 4)(x^2 - 4) &= 0 \\ \Rightarrow x^2 + 4 = 0 \quad \text{or} \quad x^2 - 4 &= 0 \\ \Rightarrow x^2 = -4 \quad \text{or} \quad x^2 = 4 \\ \Rightarrow x = \pm\sqrt{-4} \quad \text{or} \quad x = \pm\sqrt{4} \\ \Rightarrow x = \pm 2i \quad \text{or} \quad x = \pm 2 \end{aligned}$$

Hence the four fourth root of 16 are $2, -2, 2i, -2i$.

(ii) Do yourself as above. **Hint:** $81 = (9)^2$

(iii) Let x be a fourth root of 625 then

$$\begin{aligned} x &= (625)^{\frac{1}{4}} \Rightarrow x^4 = 625 \\ \Rightarrow x^4 - 625 &= 0 \Rightarrow (x^2)^2 - (25)^2 = 0 \\ \Rightarrow (x^2 + 25)(x^2 - 25) &= 0 \\ \Rightarrow x^2 + 25 = 0 \quad \text{or} \quad x^2 - 25 &= 0 \\ \Rightarrow x^2 = -25 \quad \text{or} \quad x^2 = 25 \\ \Rightarrow x = \pm\sqrt{-25} \quad \text{or} \quad x = \pm\sqrt{25} \\ \Rightarrow x = \pm 5i \quad \text{or} \quad x = \pm 5 \end{aligned}$$

Hence the four fourth root of 625 are $5, -5, 5i, -5i$.

Question # 8

(i) $2x^4 - 32 = 0$
 $\Rightarrow 2(x^4 - 16) = 0 \Rightarrow x^4 - 16 = 0$

Now do you as in Question # 7 (i)

(ii) $3y^5 - 243y = 0$
 $\Rightarrow 3y(y^4 - 81) = 0$
 $\Rightarrow 3y = 0 \quad \text{or} \quad y^4 - 81 = 0$

$$\Rightarrow y = 0 \quad \text{or} \quad (y^2)^2 - (9)^2 = 0$$

$$\Rightarrow (y^2 + 9)(y^2 - 9) = 0$$

$$\Rightarrow y^2 + 9 = 0 \quad \text{or} \quad y^2 - 9 = 0$$

$$\Rightarrow y^2 = -9 \quad \text{or} \quad y^2 = 9$$

$$\Rightarrow y = \pm\sqrt{-9} \quad \text{or} \quad y = \pm\sqrt{9}$$

$$\Rightarrow y = \pm 3i \quad \text{or} \quad y = \pm 3$$

$$\text{Hence S.Set} = \{0, \pm 3, \pm 3i\}$$

$$\text{(iii)} \quad x^3 + x^2 + x + 1 = 0$$

$$\Rightarrow x^2(x+1) + 1(x+1) = 0$$

$$\Rightarrow (x+1)(x^2+1) = 0$$

$$\Rightarrow x+1 = 0 \quad \text{or} \quad x^2+1 = 0$$

$$\Rightarrow x = -1 \quad \text{or} \quad x^2 = -1 \Rightarrow x = \pm i$$

$$\text{Hence S.Set} = \{-1, \pm i\}$$

$$\text{(iv)} \quad 5x^5 - 5x = 0$$

$$\Rightarrow 5x(x^4 - 1) = 0$$

$$\Rightarrow 5x = 0 \quad \text{or} \quad x^4 - 1 = 0$$

$$\Rightarrow x = 0 \quad \text{or} \quad (x^2)^2 - (1)^2 = 0$$

$$\Rightarrow (x^2 + 1)(x^2 - 1) = 0$$

$$\Rightarrow x^2 + 1 = 0 \quad \text{or} \quad x^2 - 1 = 0$$

$$\Rightarrow x^2 = -1 \quad \text{or} \quad x^2 = 1$$

$$\Rightarrow x = \pm i \quad \text{or} \quad x = \pm 1$$

$$\text{Hence S.Set} = \{0, \pm 1, \pm i\}$$

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