Cube Roots of Unity

Let x be cube root of unity

Then $x^3 = 1$

$\Rightarrow \quad x^3 - 1 = 0$

$\Rightarrow \quad (x-1)(x^2+x+1) = 0$

$\Rightarrow \quad x - 1 = 0 \quad or \quad x^2+x+1 = 0$

$\Rightarrow \quad x = 1 \quad or \quad x^2+x+1 = 0$

Now solving; $x^2+x+1 = 0$

$\Rightarrow \quad x = \frac{-1\pm\sqrt{1-4(1)(1)}}{2}$

$\Rightarrow \quad x = \frac{-1\pm\sqrt{-3}}{2}$

$\Rightarrow \quad x = \frac{-1\pm\sqrt{3}\,i}{2}$

Thus cube roots of unity are 1 , $\frac{-1+\sqrt{3}\,i}{2}$ and $\frac{-1-\sqrt{3}\,i}{2}$

Note	$\omega = \frac{-1+\sqrt{3}\,i}{2}$	and $\omega^2 = \frac{-1-\sqrt{3}\,i}{2}$

Properties of Cube Roots of Unity

(i) Sum of all cube roots of unity is zero.
(ii) Product of all cube roots of unity is 1.
(iii) Each complex cube root of unity is square of the other.

Proofs: (i) Sum of all cube roots of unity is zero.

Since $1, \omega \; and \; \omega^2$ are cube roots of unity

Now; $1 + \omega + \omega^2 = 1 + \left(\frac{-1+\sqrt{3}\,i}{2}\right) + \left(\frac{-1-\sqrt{3}\,i}{2}\right)$

$= \frac{2-1+\sqrt{3}\,i-1-\sqrt{3}\,i}{2}$

$= \frac{0}{2}$

$= 0$

(ii) Product of all cube roots of unity is 1.

Since $1, \omega \; and \; \omega^2$ are cube roots of unity

$1.\omega.\omega^2 = 1.\left(\frac{-1+\sqrt{3}\,i}{2}\right).\left(\frac{-1-\sqrt{3}\,i}{2}\right)$

$1.\omega.\omega^2 = \left(\frac{-1+\sqrt{3}\,i}{2}\right).\left(\frac{-1-\sqrt{3}\,i}{2}\right)$

$1.\omega.\omega^2 = \left(\frac{1-3i^2}{4}\right)$

$1.\omega.\omega^2 = \left(\frac{1+3}{4}\right) \qquad \because i^2 = -1$

$1.\omega.\omega^2 = \frac{4}{4}$

$1.\omega.\omega^2 = 1$ as required **Note:** $\boldsymbol{\omega^3 = 1}$

Four Fourth Roots of Unity

Let x be fourth root of unity

$\Rightarrow \quad x^4 = 1$

$\Rightarrow \quad x^4 - 1 = 0$

$\Rightarrow \quad (x^2-1)(x^2+1) = 0$

$\Rightarrow \quad x^2 - 1 = 0 \quad or \quad x^2+1 = 0$

$\Rightarrow \quad x^2 = 1 \quad or \quad x^2 = -1$

$\Rightarrow \quad x = \pm 1 \quad or \quad x = \pm i$

i.e; fourth roots of unity are 1, -1, $-i$ and i

Properties of Fourth roots of unity:

(i) Sum of all fourth roots of unity is 0
(ii) Every real fourth root of unity is additive inverse of the other
(iii) Every complex fourth root of unity is conjugate of the other
(iv) Product of all fourth root of unity is -1

Proofs: (i) Sum of all fourth roots of unity is 0

Since 1, -1, $-i$ and i are fourth roots of unity

$\Rightarrow \quad Sum = -1 + 1 - i + i$

$\Rightarrow \quad Sum = 0$

(ii) Real fourth roots of unity are additive inverses of each other

Since 1, -1 are real fourth roots of unity

$\Rightarrow \quad -1 + 1 = 1 + (-1) = 0$

$\Rightarrow \quad -1 + 1 = 1 + (-1) = 0$

$\Rightarrow$ Real fourth roots of unity are additive inverses of each other.

(iv) Complex fourth roots of unity are Conjugate of each other

i and $-i$ are conjugate of each other.

(v) Product of all fourth roots of unity is -1.

Since $1, -1, -i$ and i are fourth roots of unity

$\Rightarrow \quad Product = -1 \times 1 \times -i \times i$

$\Rightarrow \quad Product = -1$

EXERCISE 4.4

Q# 1: Find cube roots of $8, -8, 27, -27, 64$

Solution: Let x be cube root of 8

Then $x^3 = 8$

$\Rightarrow \quad x^3 - 8 = 0$

$\Rightarrow \quad x^3 - 2^3 = 0$

$\Rightarrow \quad (x-2)(x^2+2x+4) = 0$

$\Rightarrow \quad x - 2 = 0 \quad or \quad x^2 + 2x + 4 = 0$

$\Rightarrow \quad x = 2 \quad or \quad x^2 + 2x + 4 = 0$

Now solving; $x^2 + 2x + 4 = 0$

$\Rightarrow \quad x = \frac{-2\pm\sqrt{4-4(1)(4)}}{2}$

$\Rightarrow \quad x = \frac{-2\pm\sqrt{12}}{2}$

$\Rightarrow \quad x = \frac{-2\pm2\sqrt{3}\,i}{2}$

$\Rightarrow \quad x = 2\left(\frac{-1\pm\sqrt{3}\,i}{2}\right)$

$\Rightarrow \quad 2, 2\omega \; and \; 2\omega^2$ are cube roots of 8.

Let x be cube root of -8

Then $x^3 = -8$

$\Rightarrow \quad x^3 + 8 = 0$

$\Rightarrow \quad x^3 + 2^3 = 0$

$\Rightarrow \quad (x+2)(x^2-2x+4) = 0$

$\Rightarrow \quad x + 2 = 0 \quad or \quad x^2 - 2x + 4 = 0$

$\Rightarrow \quad x = -2 \quad or \quad x^2 - 2x + 4 = 0$

Now solving; $x^2 - 2x + 4 = 0$

$\Rightarrow \quad x = \frac{2\pm\sqrt{4-4(1)(4)}}{2}$

$\Rightarrow \quad x = \frac{2\pm\sqrt{12}}{2}$

$\Rightarrow \quad x = \frac{2\pm2\sqrt{3}\,i}{2}$

$\Rightarrow \quad x = -2\left(\frac{-1\pm\sqrt{3}\,i}{2}\right)$

$\Rightarrow \quad -2, -2\omega \; and \; -2\omega^2$ are cube roots of -8.

Let x be cube root of 27

Then $x^3 = 27$

$\Rightarrow \quad x^3 - 27 = 0$

$\Rightarrow \quad x^3 - 3^3 = 0$

$\Rightarrow \quad (x-3)(x^2+3x+9) = 0$

$\Rightarrow \quad x - 3 = 0 \quad or \quad x^2 - 3x + 9 = 0$

$\Rightarrow \quad x = 3 \quad or \quad x^2 + 3x + 9 = 0$

Now solving; $x^2 + 3x + 9 = 0$

$\Rightarrow \quad x = \frac{-3\pm\sqrt{9-4(1)(9)}}{2}$

$\Rightarrow \quad x = \frac{-3\pm\sqrt{-27}}{2}$

$\Rightarrow \quad x = \frac{-3\pm3\sqrt{3}\,i}{2}$

$\Rightarrow \quad x = 3\left(\frac{-1\pm\sqrt{3}\,i}{2}\right)$

$\Rightarrow \quad 3, 3\omega \; and \; 3\omega^2$ are cube roots of 27.

Let x be cube root of -27

Then $x^3 = -27$

$\Rightarrow \quad x^3 + 27 = 0$

$\Rightarrow \quad x^3 + 3^3 = 0$

$\Rightarrow \quad (x+3)(x^2-3x+9) = 0$

$\Rightarrow \quad x + 3 = 0 \quad or \quad x^2 - 3x + 9 = 0$

$\Rightarrow \quad x = -3 \quad or \quad x^2 - 3x + 9 = 0$

Now solving; $x^2 - 3x + 9 = 0$

$\Rightarrow \quad x = \frac{3 \pm \sqrt{9-4(1)(9)}}{2}$

$\Rightarrow \quad x = \frac{3 \pm \sqrt{-27}}{2}$

$\Rightarrow \quad x = \frac{3 \pm 3\sqrt{3}\, i}{2}$

$\Rightarrow \quad x = -3\left(\frac{-1 \pm \sqrt{3}\, i}{2}\right)$

$\Rightarrow \quad -3, -3\omega \; and \; -3\omega^2$ are cube roots of -27.

Let x be cube root of 64

Then $x^3 = 64$

$\Rightarrow \quad x^3 - 64 = 0$

$\Rightarrow \quad x^3 - 4^3 = 0$

$\Rightarrow \quad (x-4)(x^2 + 4x + 16) = 0$

$\Rightarrow \quad x - 4 = 0 \quad or \quad x^2 + 4x + 16 = 0$

$\Rightarrow \quad x = 4 \quad or \quad x^2 + 4x + 16 = 0$

Now solving; $x^2 + 4x + 16 = 0$

$\Rightarrow \quad x = \frac{-4 \pm \sqrt{16-4(1)(16)}}{2}$

$\Rightarrow \quad x = \frac{-4 \pm \sqrt{-48}}{2}$

$\Rightarrow \quad x = \frac{-4 \pm 4\sqrt{3}\, i}{2}$

$\Rightarrow \quad x = 4\left(\frac{-1 \pm \sqrt{3}\, i}{2}\right)$

$\Rightarrow \quad 4, 4\omega \; and \; 4\omega^2$ are cube roots of 64.

Q# 2: Evaluate

(i) $\left(\mathbf{1} + \boldsymbol{\omega} - \boldsymbol{\omega}^{\mathbf{2}}\right)^{\mathbf{8}}$

Solution: $\left(\mathbf{1} + \boldsymbol{\omega} - \boldsymbol{\omega}^{\mathbf{2}}\right)^{\mathbf{8}}$

$= (-\omega^2 - \omega^2)^8$

$= (-2\omega^2)^8$

$= (-2\omega^2)^8$

$= 2^8 \omega^{16}$

$= 256\omega \,.\, \omega^{15}$

$= 256\omega \,.\, (\omega^3)^5$

$= 256\omega \qquad \because \omega^3 = 1$

(ii) $\boldsymbol{\omega}^{\mathbf{28}} + \boldsymbol{\omega}^{\mathbf{29}} + \mathbf{1}$

Solution: $\boldsymbol{\omega}^{\mathbf{28}} + \boldsymbol{\omega}^{\mathbf{29}} + \mathbf{1}$

$= \omega . \omega^{27} + \omega^2 . \omega^{27} + 1$

$= \omega . (\omega^3)^9 + \omega^2 . (\omega^3)^9 + 1$

$= \omega . (1)^9 + \omega^2 . (1)^9 + 1$

$= \omega + \omega^2 + 1$

$= 1 + \omega + \omega^2$

$= 0$

(iii) $\left(\mathbf{1} + \boldsymbol{\omega} - \boldsymbol{\omega}^{\mathbf{2}}\right)\left(\mathbf{1} - \boldsymbol{\omega} + \boldsymbol{\omega}^{\mathbf{2}}\right)$

Solution: $\left(\mathbf{1} + \boldsymbol{\omega} - \boldsymbol{\omega}^{\mathbf{2}}\right)\left(\mathbf{1} - \boldsymbol{\omega} + \boldsymbol{\omega}^{\mathbf{2}}\right)$

$= (-\omega^2 - \omega^2)(1 + \omega^2 - \omega)$

$= (-\omega^2 - \omega^2)(-\omega - \omega)$

$= (-2\omega^2)(-2\omega)$

$= 4\omega^3 \qquad \because \omega^3 = 1$

$= 4$

(iv) $\left(\frac{\mathbf{-1+\sqrt{-3}}}{\mathbf{2}}\right)^{\mathbf{7}} + \left(\frac{\mathbf{-1-\sqrt{-3}}}{\mathbf{2}}\right)^{\mathbf{7}}$

Solution:

$$\left(\frac{\mathbf{-1+\sqrt{-3}}}{\mathbf{2}}\right)^{\mathbf{7}} + \left(\frac{\mathbf{-1-\sqrt{-3}}}{\mathbf{2}}\right)^{\mathbf{7}}$$

$= (\omega)^{\mathbf{7}} + (\omega^2)^{\mathbf{7}}$

$= \omega^{\mathbf{7}} + \omega^{\mathbf{14}}$

$= \omega . \omega^{\mathbf{6}} + \omega^2 . \omega^{12}$

$= \omega . \left(\omega^{\mathbf{3}}\right)^{\mathbf{2}} + \omega^2 . (\omega^3)^4$

$= \omega . (\mathbf{1})^{\mathbf{2}} + \omega^2 . (1)^4$

$= \omega + \omega^2$

$= -1$

(vi) $\left(\mathbf{-1} + \sqrt{\mathbf{-3}}\right)^{\mathbf{5}} + \left(\mathbf{-1} - \sqrt{\mathbf{-3}}\right)^{\mathbf{5}}$

$= \left(-1 + \sqrt{-3}\right)^5 + \left(-1 - \sqrt{-3}\right)^5$

$= \left(2 . \frac{\mathbf{-1+\sqrt{-3}}}{\mathbf{2}}\right)^5 + \left(2 . \frac{\mathbf{-1-\sqrt{-3}}}{\mathbf{2}}\right)^5$

$= 32(\omega)^{\mathbf{5}} + 32(\omega^2)^{\mathbf{5}}$

$= 32\omega^5 + 32\omega^{10}$

$= 32\omega^5(1+\omega^5)$

$= 32\omega^2.\omega^3(1+\omega^2.\omega^3)$

$= 32\omega^2(1+\omega^2)$

$= 32\omega^2(-\omega)$

$= -32\omega^3$

$= -32$

Q#3 Show that

(i) $\boldsymbol{x^3 - y^3 = (x-y)(x-\omega y)(x-\omega^2 y)}$

Solution:

$$x^3 - y^3 = (x-y)(x-\omega y)(x-\omega^2 y)$$

$\boldsymbol{R.H.S} = (x-y)(x-\omega y)(x-\omega^2 y)$

$= (x-y)(x^2 - \omega xy - \omega^2 xy + \omega^3 y^2)$

$= (x-y)(x^2 - (\omega+\omega^2)xy + y^2)$

$= (x-y)(x^2 - (-1)xy + y^2)$

$= (x-y)(x^2 + xy + y^2)$

$= x^3 - y^3$

$= L.H.S$

(ii) $\boldsymbol{x^3 + y^3 + z^3 - 3xyz = (x+y+z)(x+\omega y+\omega^2 z)(x+\omega^2 y+\omega z)}$

Solution: $\boldsymbol{x^3 + y^3 + z^3 - 3xyz = (x+y+z)(x+\omega y+\omega^2 z)(x+\omega^2 y+\omega z)}$

$R.H.S = (x+y+z)(x+\omega y+\omega^2 z)(x+\omega^2 y+\omega z)$

$= (x+y+z)(x^2 + \omega^2 xy + \omega xz + \omega xy + \omega^3 y^2 + \omega^2 yz + \omega^2 zx + \omega^4 yz + \omega^3 z^2)$

$= (x+y+z)(x^2 + y^2 + z^2 + (\omega^2+\omega)xy + (\omega^2+\omega)zx + (\omega^2+\omega)yz)$

$= (x+y+z)(x^2 + y^2 + z^2 + (-1)xy + (-1)zx + (-1)yz)$

$= (x+y+z)(x^2 + y^2 + z^2 - xy - yz - zx)$

$= x^3 + y^3 + z^3 - 3xyz$

$= R.H.S$

(iii)

$\boldsymbol{(1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^8)\ \ldots .2n\ factors = 1}$

Solution:

$$(1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^8)\ \ldots .2n\ factors = 1$$

$L.H.S = (1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^8)\ \ldots .2n\ factors$

$= (1+\omega)(1+\omega^2)(1+\omega.\omega^3)(1+\omega.\omega^6)\ \ldots .2n\ factors$

$= (1+\omega)(1+\omega^2)(1+\omega)(1+\omega^2)\ \ldots .2n\ factors$

$= (-\omega^2)(-\omega)(-\omega^2)(-\omega)\ \ldots .2n\ factors$

$= \omega^3.\omega^3\ \ldots n\ factors$

$= 1.1.1\ \ldots .\ n\ factors$

$= 1$

$= R.H.S$

Q#4: If $\boldsymbol{\omega}$ is a root of $\boldsymbol{x^2+x+1=0}$, show that its other root is $\boldsymbol{\omega^2}$ and prove that $\boldsymbol{\omega^3=1}$.

Solution: Since ω is a root of $x^2+x+1=0$

$\Longrightarrow\ \omega^2+\omega+1=0 \quad \rightarrow(1)$

Now to show that ω^2 is root of $x^2+x+1=0$

we have to show that $\omega^4+\omega^2+1=0$

$\omega^4+\omega^2+1 = (\omega^2)^2 + 1 + \omega^2$

$= (\omega^2)^2 + 1 + 2\omega^2 + \omega^2 - 2\omega^2$

$= (\omega^2+1)^2 - \omega^2$

$= (\omega^2+1+\omega)(\omega^2+1-\omega)$

$= (1+\omega+\omega^2)(\omega^2-\omega+1)$

$= (0)(\omega^2-\omega+1)$ from (1)

Hence $\omega^4+\omega^2+1=0$ ⇢(2)

Now subtracting equations (1) and (2)

$(\omega^4+\omega^2+1)-(\omega^2+\omega+1)=0$

$\omega^4-\omega=0$

$\omega\,(\omega^3-1)=0$

$\omega\neq 0 \quad \Longrightarrow \quad (\omega^3-1)=0$

$\Longrightarrow \quad \omega^3=1$ as required.

Q#5: Prove that complex cube roots of -1 are $\left(\frac{1+\sqrt{3}\,i}{2}\right)$ and $\left(\frac{1-\sqrt{3}\,i}{2}\right)$; and hence prove that $\left(\frac{1+\sqrt{-3}}{2}\right)^9+\left(\frac{1-\sqrt{-3}}{2}\right)^9=-2$

Solution: Let x be cube root of -1

$\Longrightarrow \quad x^3=-1$

$\Longrightarrow \quad x^3+1=0$

$\Longrightarrow \quad (x+1)(x^2-x+1)=0$

$\Longrightarrow \quad x+1=0 \quad or \quad x^2-x+1=0$

$\Longrightarrow \quad x=-1 \quad or \quad x^2-x+1=0$

Now solving; $x^2-x+1=0$

$\Longrightarrow \quad x=\frac{1\pm\sqrt{1-4(1)(1)}}{2}$

$\Longrightarrow \quad x=\frac{1\pm\sqrt{-3}}{2}$

$\Longrightarrow \quad x=\frac{1\pm\sqrt{3}\,i}{2}$

Thus cube roots of -1 are -1 , $\frac{1+\sqrt{3}\,i}{2}$ and $\frac{1-\sqrt{3}\,i}{2}$

Hence complex cube roots of -1 are $\frac{1+\sqrt{3}\,i}{2}$ and $\frac{1-\sqrt{3}\,i}{2}$

Now to prove $\left(\frac{1+\sqrt{-3}}{2}\right)^9+\left(\frac{1-\sqrt{-3}}{2}\right)^9=-2$

$$\text{L.H.S}=\left(\frac{1+\sqrt{-3}}{2}\right)^9+\left(\frac{1-\sqrt{-3}}{2}\right)^9$$

$=\left(-1.\frac{-1-\sqrt{-3}}{2}\right)^9+\left(-1.\frac{-1+\sqrt{-3}}{2}\right)^9$

$=(-\omega^2)^9+(-\omega)^9$

$=-\omega^{18}-\omega^9$

$=-(\omega^3)^6-(\omega^3)^3$

$=-(1)^6-(1)^3$

$=-1-1$

$=-2$

$=\text{R.H.S}$

Q#6: If ω is a cube root of unity, form an equation whose roots are 2ω and $2\omega^2$

Solution: An equation whose roots are 2ω and $2\omega^2$ is

$x^2-Sx+P=0$ ⇢(1)

Now $S=2\omega+2\omega^2$

$=2(\omega+\omega^2)$

$=2(-1)$

$=-2$

$P=2\omega.2\omega^2$

$P=4\omega^3$

$P=4$

Putting values in (1)

$\Longrightarrow \quad x^2+2x+4=0$ is required equation.

Q#7: Find four fourth roots of 16, 81, 625.

Solution: Let x be fourth root of 16.

$\Longrightarrow \quad x^4=16$

$\Longrightarrow \quad x^4-16=0$

$\Longrightarrow \quad (x^2)^2-(4)^2=0$

$\Longrightarrow \quad (x^2-4)(x^2+4)=0$

$\Longrightarrow \quad x^2-4=0 \quad or \quad x^2+4=0$

$\Longrightarrow \quad x^2=4 \quad or \quad x^2=-4$

$\Longrightarrow \quad x=\pm 2 \quad or \quad x=\pm 2i$

i.e; fourth roots of 16 are 2, -2, $-2i$ and $2i$.

Let x be fourth root of 81.

$\Rightarrow \quad x^4 = 81$

$\Rightarrow \quad x^4 - 81 = 0$

$\Rightarrow \quad (x^2)^2 - (9)^2 = 0$

$\Rightarrow \quad (x^2 - 9)(x^2 + 9) = 0$

$\Rightarrow \quad x^2 - 9 = 0 \quad or \quad x^2 + 9 = 0$

$\Rightarrow \quad x^2 = 9 \quad or \quad x^2 = -9$

$\Rightarrow \quad x = \pm 3 \quad or \quad x = \pm 3i$

i.e; fourth roots of 81 are 3, -3, $-3i$ and $3i$.

Let x be fourth root of 625.

$\Rightarrow \quad x^4 = 625$

$\Rightarrow \quad x^4 - 625 = 0$

$\Rightarrow \quad (x^2)^2 - (25)^2 = 0$

$\Rightarrow \quad (x^2 - 25)(x^2 + 25) = 0$

$\Rightarrow \quad x^2 - 25 = 0 \quad or \quad x^2 + 25 = 0$

$\Rightarrow \quad x^2 = 25 \quad or \quad x^2 = -25$

$\Rightarrow \quad x = \pm 5 \quad or \quad x = \pm 5i$

i.e; fourth roots of 625 are 5, -5, $-5i$ and $5i$.

Q# 8: Solve the following equations.

(i) $\mathbf{4x^2 - 32 = 0}$

$\Rightarrow \quad 4(x^2 - 16) = 0$

$\Rightarrow \quad x^2 - 16 = 0$

$\Rightarrow \quad x^2 - 4^2 = 0$

$\Rightarrow \quad (x - 4)(x + 4) = 0$

$\Rightarrow \quad x - 4 = 0 \quad or \quad x + 4 = 0$

$\Rightarrow \quad x = 4 \quad or \quad x = -4$

Solution set is $\{-4, 4\}$

(ii) $\mathbf{3y^5 - 243y = 0}$

$\Rightarrow \quad 3y(y^4 - 81) = 0$

$\Rightarrow \quad 3y = 0 \ \ or \ \ y^4 - 81 = 0$

$\Rightarrow \quad y = 0 \ \ or \ \ y^4 = 81$

$\Rightarrow \quad y^4 - 81 = 0$

$\Rightarrow \quad (y^2)^2 - (9)^2 = 0$

$\Rightarrow \quad (y^2 - 9)(y^2 + 9) = 0$

$\Rightarrow \quad y^2 - 9 = 0 \quad or \quad y^2 + 9 = 0$

$\Rightarrow \quad y^2 = 9 \quad or \quad y^2 = -9$

$\Rightarrow \quad y = \pm 3 \quad or \quad y = \pm 3i$

Solution set is $\{0, \pm 3, \pm 3i\}$

(iii) $\mathbf{x^3 + x^2 + x + 1}$

Solution: $x^3 + x^2 + x + 1$

$\Rightarrow \quad x^2(x + 1) + 1(x + 1) = 0$

$\Rightarrow \quad (x + 1)(x^2 + 1) = 0$

$\Rightarrow \quad x + 1 = 0 \ \ or \ \ x^2 + 1 = 0$

$\Rightarrow \quad x = -1 \ \ or \ \ x^2 = -1$

$\Rightarrow \quad x = -1 \ \ or \ \ x = \pm i$

Solution set is $\{-1, \pm i\}$

(iv) $\mathbf{5x^5 - 5x = 0}$

$5x(x^4 - 1) = 0$

$5x = 0 \ \ or \ \ x^4 - 1 = 0$

$5x = 0 \ \ or \ \ x^4 - 1 = 0$

$x = 0 \ \ or \ \ x^4 - 1 = 0$

$\Rightarrow \quad x^4 - 1 = 0$

$\Rightarrow \quad (x^2 - 1)(x^2 + 1) = 0$

$\Rightarrow \quad x^2 - 1 = 0 \quad or \quad x^2 + 1 = 0$

$\Rightarrow \quad x^2 = 1 \quad or \quad x^2 = -1$

$\Rightarrow \quad x = \pm 1 \quad or \quad x = \pm i$

Solution set is $\{0, \pm 1, \pm i\}$

"If A is a success in life, then A equals x plus y plus z. Work is x; y is play; and z is keeping your mouth shut"

Albert Einstein