Type v

Radical Equations

Radical equations are equations involving the radical symbol "$\sqrt{\ }$". In this section we will discuss four types of Radical Equations. First we obtain radical free equation then solve it. The solutions of radical-free equation contain the solutions of given radical equation. Solutions of radical free equations, which do not satisfy the given radical equation, are called extraneous roots, because of involvement of extraneous roots we have to carry out the process of checking. Only roots satisfying the given equation are to be considered as the root of equation and be written in solution set.

Exercise 4.3

Q# 1: $\mathbf{3x^2 + 2x - \sqrt{3x^2 + 2x - 1} = 3}$

Solution:

$3x^2 + 2x - \sqrt{3x^2 + 2x - 1} = 3 \qquad \dashrightarrow (1)$

$Let\ \sqrt{3x^2 + 2x - 1} = y$

Putting values in (1)

$3x^2 + 2x - 1 = y^2$

$\Rightarrow \quad 3x^2 + 2x = y^2 + 1$

$\Rightarrow \quad y^2 + 1 - y = 3$

$\Rightarrow \quad y^2 - y - 2 = 0$

$\Rightarrow \quad y^2 - 2y + y - 2 = 0$

$\Rightarrow \quad y(y-2) + 1(y-2) = 0$

$\Rightarrow \quad (y-2)(y+1) = 0$

$\Rightarrow \quad y - 2 = 0 \quad or \quad y + 1 = 0$

$\Rightarrow \quad y = 2 \quad or \quad y = -1$

When $y = 2 \quad \Rightarrow \sqrt{3x^2 + 2x - 1} = 2$

$\Rightarrow 3x^2 + 2x - 1 = 4$

$\Rightarrow 3x^2 + 2x - 5 = 0$

$\Rightarrow 3x^2 + 5x - 3x - 5 = 0$

$\Rightarrow x(3x+5) - 1(3x+5) = 0$

$\Rightarrow (3x+5)(x-1) = 0$

$\Rightarrow 3x + 5 = 0 \ \ or \ \ x - 1 = 0$

$\Rightarrow x = -\frac{5}{3} \ \ or \ \ x = 1$

When $y = -1 \Rightarrow \sqrt{3x^2 + 2x - 1} = -1$

$\Rightarrow 3x^2 + 2x - 1 = 1$

$\Rightarrow 3x^2 + 2x - 2 = 0$

$\Rightarrow \quad x = \frac{-2 \pm \sqrt{4 - 4(3)(-2)}}{6}$

$\Rightarrow \quad x = \frac{-2 \pm \sqrt{4+24}}{6}$

$\Rightarrow \quad x = \frac{-2 \pm \sqrt{28}}{6}$

$\Rightarrow \quad x = \frac{-2 \pm 2\sqrt{7}}{6}$

$\Rightarrow \quad x = \frac{-1 \pm \sqrt{7}}{3}$

CHECKING: $\boldsymbol{let}\ x = 1$

$\Rightarrow \quad 3 + 2 - \sqrt{3 + 2 - 1} = 3$

$\Rightarrow \quad 5 - 2 = 3$

$\Rightarrow \quad 3 = 3$ True

$\Rightarrow \quad x = 1\ is\ root\ of\ equation\ (1)$

$\boldsymbol{let}\ x = -\ \frac{5}{3}$

$\Rightarrow 3\left(\frac{25}{9}\right) + 2\left(-\frac{5}{3}\right) - \sqrt{3\left(\frac{25}{9}\right) + 2\left(-\frac{5}{3}\right) - 1} = 3$

$\Rightarrow \frac{25}{3} - \frac{10}{3} - \sqrt{\frac{25}{3} - \frac{10}{3} - 1} = 3$

$\Rightarrow \frac{25-10}{3} - \sqrt{\frac{25-10-3}{3}} = 3$

$\Rightarrow 5 - \sqrt{4} = 3$

$\Rightarrow 3 = 3$ True

$\Rightarrow \quad x = -\frac{5}{3}$ *is root of equation* (1)

let $x = \frac{-1+\sqrt{7}}{3}$

$\Rightarrow 3\left(\frac{-1+\sqrt{7}}{3}\right)^2 + 2\left(\frac{-1+\sqrt{7}}{3}\right) - \sqrt{3\left(\frac{-1+\sqrt{7}}{3}\right)^2 + 2\left(\frac{-1+\sqrt{7}}{3}\right) - 1} = 3$

$\Rightarrow 3\,\frac{1+7-2\sqrt{7}}{9} + \frac{2(-1+\sqrt{7})}{3} - \sqrt{3\,\frac{1+7-2\sqrt{7}}{9} + \frac{2(-1+\sqrt{7})}{3} - 1} = 3$

$\Rightarrow \frac{8-2\sqrt{7}-2+2\sqrt{7}}{3} - \sqrt{\frac{8-2\sqrt{7}-2+2\sqrt{7}-3}{3}} = 3$

$\Rightarrow 2 - \sqrt{1} = 3$

$\Rightarrow 1 = 3$ False

$\Rightarrow x = \frac{-1+\sqrt{7}}{3}$ *is an extraneous root of equation* (1).

let $x = \frac{-1-\sqrt{7}}{3}$

$\Rightarrow 3\left(\frac{-1-\sqrt{7}}{3}\right)^2 + 2\left(\frac{-1-\sqrt{7}}{3}\right) - \sqrt{3\left(\frac{-1-\sqrt{7}}{3}\right)^2 + 2\left(\frac{-1-\sqrt{7}}{3}\right) - 1} = 3$

$\Rightarrow 3\,\frac{1+7+2\sqrt{7}}{9} + \frac{2(-1-\sqrt{7})}{3} - \sqrt{3\,\frac{1+7+2\sqrt{7}}{9} + \frac{2(-1-\sqrt{7})}{3} - 1} = 3$

$\Rightarrow \frac{8+2\sqrt{7}-2-2\sqrt{7}}{3} - \sqrt{\frac{8+2\sqrt{7}-2-2\sqrt{7}-3}{3}} = 3$

$\Rightarrow 2 - \sqrt{1} = 3$

$\Rightarrow 1 = 3$ False

$\Rightarrow \quad x = \frac{-1-\sqrt{7}}{3}$ *is an extraneous root of equation* (1).

Hence solution set = $\left\{-\frac{5}{3}, 1\right\}$

Q# 2: $\boldsymbol{x^2 - \frac{x}{2} - 7 = x - 3\sqrt{2x^2 - 3x + 2}}$

Solution:

$x^2 - \frac{x}{2} - 7 = x - 6\sqrt{2x^2 - 3x + 2} \quad \dashrightarrow (1)$

$2x^2 - x - 14 = 2x - 6\sqrt{2x^2 - 3x + 2}$

$2x^2 - x - 2x - 14 + 6\sqrt{2x^2 - 3x + 2} = 0$

$2x^2 - 3x - 14 + 6\sqrt{2x^2 - 3x + 2} = 0 \quad \dashrightarrow (2)$

$Let\ \sqrt{2x^2-3x+2}=y$

Putting values in (2)

$$2x^2-3x+2=y^2$$

$\Rightarrow\quad 2x^2-3x=y^2-2$

$\Rightarrow\quad y^2-2-14+6y=0$

$\Rightarrow\quad y^2+6y-16=0$

$\Rightarrow\quad y=\frac{-6\pm\sqrt{36-4(1)(-16)}}{2}$

$\Rightarrow\quad y=\frac{-6\pm\sqrt{36+64}}{2}$

$\Rightarrow\quad y=\frac{-6\pm\sqrt{100}}{2}$

$\Rightarrow\quad y=\frac{-6\pm10}{2}$

$\Rightarrow\quad y=2\qquad or\qquad y=-8$

When $y=2\quad\Rightarrow\sqrt{2x^2-3x+2}=2$

$\Rightarrow\ 2x^2-3x+2=4$

$\Rightarrow\ 2x^2-3x+2-4=0$

$\Rightarrow\ 2x^2-3x-2=0$

$\Rightarrow\quad x=\frac{3\pm\sqrt{9-4(2)(-2)}}{4}$

$\Rightarrow\quad x=\frac{3\pm\sqrt{9+16}}{4}$

$\Rightarrow\quad x=\frac{3\pm\sqrt{25}}{4}$

$\Rightarrow\quad x=\frac{3\pm5}{4}$

$\Rightarrow\quad x=\frac{3+5}{4}\qquad or\quad x=\frac{3-5}{4}$

$\Rightarrow\quad x=2\qquad or\quad x=-\frac{1}{2}$

When $y=-8\ \Rightarrow\sqrt{2x^2-3x+2}=-8$

$\Rightarrow\ 2x^2-3x+2=64$

$\Rightarrow\ 2x^2-3x+2-64=0$

$\Rightarrow\ 2x^2-3x-62-=0$

$\Rightarrow\quad x=\frac{3\pm\sqrt{9-4(2)(-62)}}{4}$

$\Rightarrow\quad x=\frac{3\pm\sqrt{9+496}}{4}$

$\Rightarrow\quad x=\frac{3\pm\sqrt{505}}{4}$

CHECKING: $\boldsymbol{let}\ x=2$

$\Rightarrow\quad 4-\frac{2}{2}-7=2-3\sqrt{8-6+2}$

$\Rightarrow\quad 4-\ 1-7=2-3\sqrt{4}$

$\Rightarrow\quad -4=-4$

$\Rightarrow\quad x=2\ is\ root\ of\ equation\ (1)$

$\boldsymbol{let}\ x=-\frac{1}{2}$

$\Rightarrow\quad \frac{1}{4}+\frac{1}{4}-7=-\frac{1}{2}-3\sqrt{\frac{1}{2}+\frac{3}{2}+2}$

$\Rightarrow\quad \frac{1+1-28}{4}=-\frac{1}{2}-3\sqrt{\frac{1+3+4}{2}}$

$\Rightarrow\quad -\frac{26}{4}=-\frac{1}{2}-6$

$\Rightarrow\quad -\frac{13}{2}=-\frac{13}{2}$

$\Rightarrow\quad x=-\frac{1}{2}\ is\ root\ of\ equation\ (1)$

$\boldsymbol{let}\ x=\frac{3+\sqrt{505}}{4}$

$$\left(\frac{3+\sqrt{505}}{4}\right)^2-\frac{\frac{3+\sqrt{505}}{4}}{2}-7=\frac{3+\sqrt{505}}{4}-3\sqrt{2\left(\frac{3+\sqrt{505}}{4}\right)^2-3\left(\frac{3+\sqrt{505}}{4}\right)+2}$$

$\Rightarrow\quad \frac{9+6\sqrt{505}+505}{16}-\frac{3+\sqrt{505}}{8}-7=\frac{3+\sqrt{505}}{4}-3(8)$

$\Rightarrow\quad \frac{514+6\sqrt{505}-6-2\sqrt{505}-112}{16}=\frac{3+\sqrt{505}}{4}-24$

$\Rightarrow \frac{396+4\sqrt{505}}{16} = \frac{3+\sqrt{505}-96}{4}$

$\Rightarrow \frac{99+\sqrt{505}}{4} = \frac{-93+\sqrt{505}}{4}$ False

$\Rightarrow \quad x = \frac{3+\sqrt{505}}{4}$ *is an extraneous root*

Similarly on checking we come to know that $x = \frac{3-\sqrt{505}}{4}$ is also an extraneous root.

Hence solution set = $\left\{-\frac{1}{2}, 2\right\}$

Q# 3: $\sqrt{2x+8} + \sqrt{x+5} = 7$

Solution: $\sqrt{2x+8} + \sqrt{x+5} = 7 \quad \dashrightarrow (1)$

Squaring on both sides

$2x + 8 + x + 5 + 2\sqrt{2x+8}\,.\sqrt{x+5} = 49$

$3x + 13 + 2\sqrt{(2x+8)(x+5)} = 49$

$2\sqrt{(2x+8)(x+5)} = 49 - 3x - 13$

$2\sqrt{(2x+8)(x+5)} = 36 - 3x$

$2\sqrt{2x^2 + 18x + 40} = 3(12 - x)$

Again squaring on both sides

$\Rightarrow \quad 4(2x^2 + 18x + 40) = 9(12 - x)^2$

$\Rightarrow \quad 8x^2 + 72x + 160 = 9(144 + x^2 - 24x)$

$\Rightarrow \quad 8x^2 + 72x + 160 = 1296 + 9x^2 - 216x$

$\Rightarrow \quad 8x^2 + 72x + 160 - 1296 - 9x^2 + 216x = 0$

$\Rightarrow \quad -x^2 + 288x - 1136 = 0$

$\Rightarrow \quad x^2 - 288x + 1136 = 0$

$\Rightarrow \quad x^2 - 284x - 4x + 1136 = 0$

$\Rightarrow \quad x(x - 284) - 4(x - 284) = 0$

$\Rightarrow \quad (x - 284)(x - 4) = 0$

$\Rightarrow \quad x - 284 = 0 \quad or \quad x - 4 = 0$

$\Rightarrow \quad x = 284 \quad or \quad x = 4$

Checking: $\boldsymbol{let\ x = 4}$

$\Rightarrow \quad \sqrt{8+8} + \sqrt{4+5} = 7$

$\Rightarrow \quad \sqrt{16} + \sqrt{9} = 7$

$\Rightarrow \quad 4 + 3 = 7$

$\Rightarrow \quad 7 = 7$ True

$\Rightarrow \quad x = 4$ is root of equation (1)

$\boldsymbol{let\ x = 284}$

$\Rightarrow \quad \sqrt{568+8} + \sqrt{284+5} = 7$

$\Rightarrow \quad \sqrt{576} + \sqrt{289} = 7$

$\Rightarrow \quad 24 + 17 = 7$

$\Rightarrow \quad 41 = 7$ False

$\Rightarrow \quad x = 284$ is an extraneous root

Hence solution set = $\{4\}$

Q# 4: $\sqrt{3x+4} = 2 + \sqrt{2x-4}$

Solution: $\sqrt{3x+4} = 2 + \sqrt{2x-4} \quad \dashrightarrow (1)$

$\sqrt{3x+4} - \sqrt{2x-4} = 2$

Squaring on both sides

$3x + 4 + 2x - 4 - 2\sqrt{3x+4}\,.\sqrt{2x-4} = 4$

$5x - 2\sqrt{3x+4}\,.\sqrt{2x-4} = 4$

$-2\sqrt{3x+4}\,.\sqrt{2x-4} = 4 - 5x$

$-2\sqrt{(3x+4)(2x-4)} = 4 - 5x$

$-2\sqrt{6x^2 - 4x - 16} = 4 - 5x$

Again squaring on both sides

$\Rightarrow \quad 4(6x^2 - 4x - 16) = (4 - 5x)^2$

$\Rightarrow \quad 24x^2 - 16x - 64 = 16 + 25x^2 - 40x$

$\Rightarrow \quad 24x^2 - 16x - 64 - 16 - 25x^2 + 40x = 0$

$\Rightarrow \quad -x^2 + 24x - 80 = 0$

$\Rightarrow \quad x^2 - 24x + 80 = 0$

$\Rightarrow \quad x^2 - 20x - 4x + 80 = 0$

$\Rightarrow \quad x(x - 20) - 4(x - 20) = 0$

$\Rightarrow \quad (x - 20)(x - 4) = 0$

$\Rightarrow \quad x - 20 = 0 \quad or \quad x - 4 = 0$

$\Rightarrow \quad x = 20 \quad or \quad x = 4$

Checking: $let\ \ x = 20$

$\Rightarrow \quad \sqrt{60 + 4} = 2 + \sqrt{40 - 4}$

$\Rightarrow \quad \sqrt{64} = 2 + \sqrt{36}$

$\Rightarrow \quad 8 = 8$ True

$\Rightarrow \quad x = 20$ is root of equation (1)

$let\ \ x = 4$

$\Rightarrow \quad \sqrt{12 + 4} = 2 + \sqrt{8 - 4}$

$\Rightarrow \quad \sqrt{16} = 2 + \sqrt{4}$

$\Rightarrow \quad 4 = 4$ True

$\Rightarrow \quad x = 4$ is root of equation (1)

Hence solution set = $\{4, 20\}$

Q# 5: $\boldsymbol{\sqrt{x + 7} + \sqrt{x + 2} = \sqrt{6x + 13}}$

Solution:

$\sqrt{x + 7} + \sqrt{x + 2} = \sqrt{6x + 13} \qquad \cdots (1)$

Squaring on both sides

$x + 7 + x + 2 + 2\sqrt{x + 7}\,.\sqrt{x + 2} = 6x + 13$

$2x + 9 + 2\sqrt{(x + 7)(x + 2)} = 6x + 13$

$2\sqrt{(x + 7)(x + 2)} = 6x + 13 - 2x - 9$

$2\sqrt{(x + 7)(x + 2)} = 4x + 4$

$2\sqrt{x^2 + 9x + 14} = 4(x + 1)$

Again squaring on both sides

$\Rightarrow \quad 4(x^2 + 9x + 14) = 16(x + 1)^2$

$\Rightarrow \quad 4x^2 + 36x + 56 = 16(x^2 + 2x + 1)$

$\Rightarrow \quad 4x^2 + 36x + 56 = 16x^2 + 32x + 16$

$\Rightarrow \quad 4x^2 + 36x + 56 - 16x^2 - 32x - 16 = 0$

$\Rightarrow \quad -12x^2 + 4x + 40 = 0$

$\Rightarrow \quad -4(3x^2 - x - 10) = 0$

$\Rightarrow \quad 3x^2 - x - 10 = 0$

$\Rightarrow \quad 3x^2 - 6x + 5x - 10 = 0$

$\Rightarrow \quad 3x(x - 2) + 5(x - 2) = 0$

$\Rightarrow \quad (x - 2)(3x + 5) = 0$

$\Rightarrow \quad x - 2 = 0 \quad or \quad 3x + 5 = 0$

$\Rightarrow \quad x = 2 \quad or \quad x = -\frac{5}{3}$

Checking: $let\ \ x = 2$

$\Rightarrow \quad \sqrt{2 + 7} + \sqrt{2 + 2} = \sqrt{12 + 13}$

$\Rightarrow \quad \sqrt{9} + \sqrt{4} = \sqrt{12 + 13}$

$\Rightarrow \quad 3 + 2 = 5$

$\Rightarrow \quad 5 = 5$ True

$\Rightarrow \quad x = 2$ is root of equation (1)

$$let\ \ x = -\frac{5}{3}$$

$\Rightarrow \quad \sqrt{-\frac{5}{3} + 7} + \sqrt{-\frac{5}{3} + 2} = \sqrt{6\left(-\frac{5}{3}\right) + 13}$

$\Rightarrow \quad \sqrt{\frac{16}{3}} + \sqrt{\frac{1}{3}} = \sqrt{-10 + 13}$

$\Rightarrow \quad \frac{4}{\sqrt{3}} + \frac{1}{\sqrt{3}} = \sqrt{3}$

$\Rightarrow \quad \frac{5}{\sqrt{3}} = \frac{3}{\sqrt{3}}$ False

$\Rightarrow \quad x = -\frac{5}{3}$ is an extraneous root.

Hence solution set = $\{2\}$

Q# 6: $\sqrt{x^2+x+1} - \sqrt{x^2+x-1} = 1$

Solution:

$\sqrt{x^2+x+1} - \sqrt{x^2+x-1} = 1 \quad \cdots\rightarrow(1)$

$let\ \sqrt{x^2+x+1} = a$

$\sqrt{x^2+x-1} = b$

$\Rightarrow \quad a - b = 1 \quad \cdots\rightarrow (2)$

$now\ \ a^2 - b^2 = 2 \quad \cdots\rightarrow (3)$

Dividing equation (3) by equation (2)

$\frac{a^2-b^2}{a-b} = \frac{2}{1}$

$\Rightarrow \quad \frac{(a+b)(a-b)}{a-b} = \frac{2}{1}$

$\Rightarrow \quad a + b = 2 \quad \cdots\rightarrow (4)$

Adding equation (2) and (4)

$\Rightarrow \quad 2a = 3$

$\Rightarrow \quad a = \frac{3}{2}$

$\Rightarrow \sqrt{x^2+x+1} = \frac{3}{2}$

$\Rightarrow \quad x^2 + x + 1 = \frac{9}{4}$

$\Rightarrow \quad 4x^2 + 4x + 4 = 9$

$\Rightarrow \quad 4x^2 + 4x - 5 = 0$

$\Rightarrow \quad x = \frac{-4 \pm \sqrt{16 - 4(4)(-5)}}{8}$

$\Rightarrow \quad x = \frac{-4 \pm \sqrt{16+80}}{8}$

$\Rightarrow \quad x = \frac{-4 \pm \sqrt{96}}{8}$

$\Rightarrow \quad x = \frac{-4 \pm 4\sqrt{6}}{8}$

$\Rightarrow \quad x = \frac{-1 \pm \sqrt{6}}{2}$

Checking: Do yourself.

Hence solution set = $\left\{\frac{-1\pm\sqrt{6}}{2}\right\}$

Q# 7: $\sqrt{x^2+2x-3} + \sqrt{x^2+7x-8} = \sqrt{5(x^2+3x-4)}$

Solution: $\sqrt{x^2+2x-3} + \sqrt{x^2+7x-8} = \sqrt{5(x^2+3x-4)} \quad \cdots\rightarrow(1)$

$\sqrt{x^2+3x-x-3} + \sqrt{x^2+8x-x-8} = \sqrt{5(x^2+4x-x-4)}$

$\sqrt{x(x+3)-1(x+3)} + \sqrt{x(x+8)-1(x+8)} = \sqrt{5(x(x+4)-1(x+4))}$

$\sqrt{(x+3)(x-1)} + \sqrt{(x+8)(x-1)} = \sqrt{5(x+4)(x-1)}$

$\sqrt{(x+3)(x-1)} + \sqrt{(x+8)(x-1)} - \sqrt{5(x+4)(x-1)} = 0$

$\sqrt{x-1}\ \left[\sqrt{x+3} + \sqrt{x+8} - \sqrt{5(x+4)}\ \right] = 0$

$\Rightarrow \quad \sqrt{x-1} = 0 \quad or \quad \sqrt{x+3} + \sqrt{x+8} - \sqrt{5(x+4)} = 0$

$\Rightarrow \quad x - 1 = 0 \quad or \quad \sqrt{x+3} + \sqrt{x+8} - \sqrt{5(x+4)} = 0$

$\Rightarrow \quad x = 1 \quad or \quad \sqrt{x+3} + \sqrt{x+8} = \sqrt{5(x+4)}$

Now solving $\sqrt{x+3}+\sqrt{x+8}=\sqrt{5(x+4)}$

Squaring on both sides

$x+3+x+8+2\sqrt{x+3}\,.\sqrt{x+8}=5(x+4)$

$2x+11+2\sqrt{(x+3)(x+8)}=5x+20$

$2\sqrt{(x+3)(x+8)}=5x+20-2x-11$

$2\sqrt{x^2+11x+24}=3x+9$

$2\sqrt{x^2+11x+24}=3(x+3)$

Again squaring on both sides

$\Rightarrow \quad 4(x^2+11x+24)=9(x+3)^2$

$\Rightarrow \quad 4x^2+44x+96=9(x^2+6x+9)$

$\Rightarrow \quad 4x^2+44x+96=9x^2+54x+81$

$\Rightarrow \quad 4x^2+44x+96-9x^2-54x-81=0$

$\Rightarrow \quad -5x^2-10x+15=0$

$\Rightarrow \quad -5(x^2+2x-3)=0$

$\Rightarrow \quad x^2+2x-3=0$

$\Rightarrow \quad x^2+3x-x-3=0$

$\Rightarrow \quad x(x+3)-1(x+3)=0$

$\Rightarrow \quad (x+3)(x-1)=0$

$\Rightarrow \quad x+3=0 \quad or \quad x-1=0$

$\Rightarrow \quad x=-3 \quad or \quad x=1$

Checking: $let\ \ x=-3$

$\Rightarrow \quad \sqrt{9-6-3}+\sqrt{9-21-8}=\sqrt{5(9-9-4)}$

$\Rightarrow \quad \sqrt{0}+\sqrt{-20}=\sqrt{5(-4)}$

$\Rightarrow \quad \sqrt{-20}=\sqrt{-20}$ True

$\Rightarrow \quad x=-3$ is root of equation (1)

$let\ \ x=1$

$\Rightarrow \quad \sqrt{1+2-3}+\sqrt{1+7-8}=\sqrt{5(9-9-4)}$

$\Rightarrow \quad \sqrt{0}+\sqrt{0}=\sqrt{5(0)}$

$\Rightarrow \quad 0=0$ True

$\Rightarrow \quad x=1$ is root of equation (1)

Hence solution set = $\{-3,1\}$

Q# 8: $\boldsymbol{\sqrt{2x^2-5x-3}+3\sqrt{2x+1}=\sqrt{2x^2+25x+12}}$

Solution: $\sqrt{2x^2-5x-3}+3\sqrt{2x+1}=\sqrt{2x^2+25x+12} \quad \rightarrow(1)$

$\sqrt{2x^2-6x+x-3}+3\sqrt{2x+1}=\sqrt{2x^2+24x+x+12}$

$\sqrt{2x(x-3)+1(x+3)}+3\sqrt{2x+1}=\sqrt{2x(x+12)+1(x+12)}$

$\sqrt{(x-3)(2x+1)}+3\sqrt{2x+1}=\sqrt{(x+12)(2x+1)}$

$\sqrt{(x-3)(2x+1)}+3\sqrt{2x+1}-\sqrt{(x+12)(2x+1)}=0$

$\sqrt{2x+1}\left[\sqrt{x-3}+3-\sqrt{x+12}\right]=0$

$\Rightarrow \quad \sqrt{2x+1}=0 \quad or \quad \sqrt{x-3}+3-\sqrt{x+12}=0$

$\Rightarrow \quad 2x+1=0 \quad or \quad \sqrt{x-3}-\sqrt{x+12}=-3$

$\Rightarrow \quad x=-\frac{1}{2} \quad or \quad \sqrt{x-3}-\sqrt{x+12}=-3$

Now solving $\sqrt{x-3}-\sqrt{x+12}=-3$

Squaring on both sides

$x - 3 + x + 12 + 2\sqrt{x-3}\,.\sqrt{x+12} = 9$

$2x + 9 + 2\sqrt{(x-3)(x+12)} = 9$

$2\sqrt{(x-3)(x+12)} = 2x$

$\sqrt{x^2 + 9x - 36} = x$

$x^2 + 9x - 36 = x^2$

$\Longrightarrow \quad 9x - 36 = 0$

$\Longrightarrow \quad 9x = 36$

$\Longrightarrow \quad x = 4$

Checking: $\quad let\ \ x = -\frac{1}{2}$

$\Longrightarrow \quad \sqrt{2\left(\frac{1}{4}\right) + \frac{5}{2} - 3} + 3\sqrt{2\left(-\frac{1}{2}\right) + 1} = \sqrt{2\left(\frac{1}{4}\right) - \frac{25}{2} + 12}$

$\Longrightarrow \quad \sqrt{\frac{1}{2} + \frac{5}{2} - 3} + 3\sqrt{-1+1} = \sqrt{\frac{1}{2} - \frac{25}{2} + 12}$

$\Longrightarrow \quad 0 = 0$ True

$\Longrightarrow \quad x = -\frac{1}{2}$ is root of equation (1)

$let\ \ x = 4$

$\Longrightarrow \sqrt{32 - 20 - 3} + 3\sqrt{9} = \sqrt{32 + 100 + 12}$

$\Longrightarrow \quad \sqrt{9} + 9 = \sqrt{144}$

$\Longrightarrow \quad 3 + 9 = 12$

$\Longrightarrow \quad 12 = 12$ True

$\Longrightarrow \quad x = 4$ is root of equation (1)

Hence solution set = $\left\{4\,, -\frac{1}{2}\right\}$

Q# 9: $\boldsymbol{\sqrt{3x^2 - 5x + 2} + \sqrt{6x^2 - 11x + 5} = \sqrt{5x^2 - 9x + 4}}$

Solution: $\sqrt{3x^2 - 5x + 2} + \sqrt{6x^2 - 11x + 5} = \sqrt{5x^2 - 9x + 4} \quad \dashrightarrow$ **(1)**

$\sqrt{3x^2 - 3x - 2x + 2} + \sqrt{6x^2 - 6x - 5x + 5} = \sqrt{5x^2 - 5x - 4x - 4)}$

$\sqrt{3x(x-1) - 2(x-1)} + \sqrt{6x(x-1) - 5(x-1)} = \sqrt{5x(x-1) - 4(x-1))}$

$\sqrt{(x-1)(3x-2)} + \sqrt{(x-1)(6x-5)} = \sqrt{(x-1)(5x-4)}$

$\sqrt{(x-1)(3x-2)} + \sqrt{(x-1)(6x-5)} - \sqrt{(x-1)(5x-4)} = 0$

$\sqrt{x-1}\left[\sqrt{3x-2} + \sqrt{6x-5} - \sqrt{5x-4}\right] = 0$

$\Longrightarrow \quad \sqrt{x-1} = 0 \quad or \quad \sqrt{3x-2} + \sqrt{6x-5} - \sqrt{5x-4} = 0$

$\Longrightarrow \quad x - 1 = 0 \quad or \quad \sqrt{3x-2} + \sqrt{6x-5} - \sqrt{5x-4} = 0$

$\Longrightarrow \quad x = 1 \qquad or \quad \sqrt{3x-2} + \sqrt{6x-5} = \sqrt{5x-4}$

Now solving $\sqrt{3x-2} + \sqrt{6x-5} = \sqrt{5x-4}$

Squaring on both sides

$3x - 2 + 6x - 5 + 2\sqrt{3x-2}\,.\sqrt{6x-5} = 5x - 4$

$9x - 7 + 2\sqrt{(3x-2)(6x-5)} = 5x - 4$

$2\sqrt{(3x-2)(6x-5)} = 5x - 4 - 9x + 7$

$2\sqrt{18x^2 - 27x + 10} = -4x + 3$

Again squaring on both sides

$\Longrightarrow \quad 4(18x^2 - 27x + 10) = (-4x + 3)^2$

$\Longrightarrow \quad 72x^2 - 108x + 40 = 16x^2 - 24x + 9$

$\Longrightarrow \quad 72x^2 - 108x + 40 - 16x^2 + 24x - 9 = 0$

$\Longrightarrow \quad 56x^2 - 84x + 31 = 0$

$\Longrightarrow \quad x = \frac{84 \pm \sqrt{7056 - 4(56)(31)}}{112}$

$\Rightarrow \quad x = \frac{84 \pm \sqrt{7056-6944}}{112}$

$\Rightarrow \quad x = \frac{84 \pm \sqrt{112}}{112}$

$\Rightarrow \quad x = \frac{84 \pm 4\sqrt{7}}{112}$

$\Rightarrow \quad x = \frac{21 \pm \sqrt{7}}{28}$

Checking: Do yourself

Hence solution set = $\{1\}$

Q# 10:

$(\boldsymbol{x+4})(\boldsymbol{x+1}) = \sqrt{\boldsymbol{x^2+2x-15}} + \boldsymbol{3x+31}$

Solution:

$(x+4)(x+1) = \sqrt{x^2+2x-15} + 3x + 31 \quad \dashrightarrow$ **(1)**

$x^2 + 5x + 4 = \sqrt{x^2+2x-15} + 3x + 31$

$x^2 + 5x + 4 - 3x - 31 - \sqrt{x^2+2x-15} = 0$

$x^2 + 2x - 27 - \sqrt{x^2+2x-15} = 0$

$Let \ \sqrt{x^2+2x-15} = y$

$\Rightarrow \quad x^2 + 2x - 15 = y^2$

$\Rightarrow \quad x^2 + 2x = y^2 + 15$

$\Rightarrow \quad y^2 + 15 - 27 - y = 0$

$\Rightarrow \quad y^2 - y - 12 = 0$

$\Rightarrow \quad y^2 + 3y - 4y - 12 = 0$

$\Rightarrow \quad y(y+3) - 4(y+3) = 0$

$\Rightarrow \quad (y+3)(y-4) = 0$

$\Rightarrow \quad y+3 = 0 \quad or \quad y-4 = 0$

$\Rightarrow \quad y = -3 \quad or \quad y = 4$

When $y = -3 \Rightarrow \sqrt{x^2+2x-15} = -3$

$\Rightarrow \quad x^2 + 2x - 15 = 9$

$\Rightarrow x^2 + 2x - 15 - 9 = 0$

$\Rightarrow x^2 + 2x - 24 = 0$

$\Rightarrow \quad x^2 + 6x - 4x - 24 = 0$

$\Rightarrow \quad x(x+6) - 4(x+6) = 0$

$\Rightarrow \quad (x+6)(x-4) = 0$

$\Rightarrow \quad x+6 = 0 \quad or \quad x-4 = 0$

$\Rightarrow \quad x = -6 \quad or \quad x = 4$

When $y = 4 \quad \Rightarrow \sqrt{x^2+2x-15} = 4$

$\Rightarrow x^2 + 2x - 15 = 16$

$\Rightarrow x^2 + 2x - 15 - 16 = 0$

$\Rightarrow x^2 + 2x - 31 = 0$

$\Rightarrow \quad x = \frac{-2 \pm \sqrt{4-4(1)(-31)}}{2}$

$\Rightarrow \quad x = \frac{-2 \pm \sqrt{128}}{2}$

$\Rightarrow \quad x = \frac{-2 \pm 8\sqrt{2}}{2}$

$\Rightarrow \quad x = -1 \pm 4\sqrt{2}$

Checking: Do yourself

Hence solution set = $\{-1 \pm 4\sqrt{2}\}$

Q# 11: $\sqrt{\boldsymbol{3x^2-2x+9}} + \sqrt{\boldsymbol{3x^2-2x-4}} = \boldsymbol{13}$

Solution:

$\sqrt{3x^2-2x+9} + \sqrt{3x^2-2x-4} = 13 \quad \dashrightarrow$ **(1)**

$let \ \sqrt{3x^2-2x+9} = a$

$\sqrt{3x^2-2x-4} = b$

$\Rightarrow \quad a + b = 13 \quad \dashrightarrow (2)$

$now \ a^2 - b^2 = 13 \quad \dashrightarrow (3)$

Dividing equation (3) by equation (2)

$\frac{a^2-b^2}{a+b} = \frac{13}{13}$

$\Rightarrow \quad \frac{(a+b)(a-b)}{a+b} = 1$

$\Rightarrow \quad a - b = 1 \quad \dashrightarrow (4)$

Adding equation (2) and (4)

$\Longrightarrow \quad 2a = 14$

$\Longrightarrow \quad a = 7$

$a = 7 \Longrightarrow \sqrt{3x^2 - 2x + 9} = 7$

$\Longrightarrow \quad 3x^2 - 2x + 9 = 49$

$\Longrightarrow \quad 3x^2 - 2x + 9 - 49 = 0$

$\Longrightarrow \quad 3x^2 - 2x - 40 = 0$

$\Longrightarrow \quad x = \frac{2 \pm \sqrt{4 - 4(3)(-40)}}{6}$

$\Longrightarrow \quad x = \frac{2 \pm \sqrt{4 + 480}}{6}$

$\Longrightarrow \quad x = \frac{2 \pm \sqrt{484}}{6}$

$\Longrightarrow \quad x = \frac{2 \pm 22}{6}$

$\Longrightarrow \quad x = \frac{2+22}{6} \quad or \quad x = \frac{2-22}{6}$

$\Longrightarrow \quad x = 4 \quad or \quad x = -\frac{10}{3}$

Checking: Let $x = 4$

$\Longrightarrow \sqrt{48 - 8 + 9} + \sqrt{48 - 8 - 4} = 13$

$\Longrightarrow \sqrt{49} + \sqrt{36} = 13$

$\Longrightarrow 7 + 6 = 13$

$\Longrightarrow 13 = 13$ True

$\Longrightarrow x = 4$ is root of equation (1)

Let $x = -\frac{10}{3}$

$\Longrightarrow \sqrt{\frac{100}{3} + \frac{20}{3} + 9} + \sqrt{\frac{100}{3} + \frac{20}{3} - 4} = 13$

$\Longrightarrow \sqrt{\frac{100+20+27}{3}} + \sqrt{\frac{100+20-12}{3}} = 13$

$\Longrightarrow \sqrt{\frac{147}{3}} + \sqrt{\frac{108}{3}} = 13$

$\Longrightarrow \quad 7 + 6 = 13$

$\Longrightarrow 13 = 13$ True

$\Longrightarrow x = -\frac{10}{3}$ is root of equation (1)

Hence solution set = $\left\{4, -\frac{10}{3}\right\}$

Q# 12: $\sqrt{5x^2 + 7x + 2} - \sqrt{4x^2 + 7x + 18} = x - 4$

Solution:

$\sqrt{5x^2 + 7x + 2} - \sqrt{4x^2 + 7x + 18} = x - 4 \dashrightarrow$ **(1)**

$let \ \sqrt{5x^2 + 7x + 2} = a$

$\sqrt{4x^2 + 7x + 18} = b$

$\Longrightarrow \quad a - b = x - 4 \quad \dashrightarrow (2)$

$now \ a^2 - b^2 = x^2 - 16 \quad \dashrightarrow (3)$

Dividing equation (3) by equation (2)

$\frac{a^2 - b^2}{a - b} = \frac{x^2 - 16}{x - 4}$

$\Longrightarrow \quad \frac{(a+b)(a-b)}{a-b} = \frac{(x+4)(x-4)}{x-4}$

$\Longrightarrow \quad a + b = x + 4 \quad \dashrightarrow (4)$

Adding equation (2) and (4)

$\Longrightarrow \quad 2a = 2x$

$\Longrightarrow \quad a = x$

$\Longrightarrow \sqrt{5x^2 + 7x + 2} = x$

$\Longrightarrow \quad 5x^2 + 7x + 2 = x^2$

$\Longrightarrow \quad 4x^2 + 7x + 2 = 0$

$\Longrightarrow \quad x = \frac{-7 \pm \sqrt{49 - 4(4)(2)}}{8}$

$\Longrightarrow \quad x = \frac{-7 \pm \sqrt{49 - 32}}{8}$

$\Longrightarrow \quad x = \frac{-7 \pm \sqrt{17}}{8}$

Checking: Do yourself.

Hence solution set = $\left\{\frac{-7 \pm \sqrt{17}}{8}\right\}$