Exercise 4.10

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1. **The product of one less than a certain positive number and two less than three times the number is 14. Find the number.**

Solution: Let x be the required positive number.

According to given condition

$$(x-1)(3x-2)=14$$

$$3x^2-5x+2=14$$

$$3x^2-5x-12=0$$

$$3x^2-9x+4x-12=0$$

$$3x(x-3)+4(x-3)=0$$

$$(x-3)(3x+4)=0$$

$$x-3=0 \ \ or \ \ 3x+4=0$$

$$x=3 \ \ or \ \ x=-\frac{4}{3}$$

Since $x=-\frac{4}{3}$ is not positive, so required number is $x=3$.

2. **The sum of a positive number and its square is 380. Find the number.**

Solution: Let x be the required positive number.

According to given condition

$x+x^2=380$

$x^2+x-380=0$

$a=1, b=1, c=-380$

Using quadratic formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-1\pm\sqrt{1-4(1)(-380)}}{2}$$

$$x=\frac{-1\pm\sqrt{1+1520}}{2}$$

$$x=\frac{-1\pm\sqrt{1521}}{2}$$

$$x=\frac{-1\pm 39}{2}$$

$$x=\frac{-1+39}{2} \text{ and } x=\frac{-1-39}{2}$$

$$x=19 \ \ or \ \ x=-20$$

Since $x=-20$ is not positive, so required number is $x=19$.

3. **Divide 40 into two parts such that the sum of their squares is greater than two times their product by 100.**

Solution: Let two parts of 40 be x and $40-x$.

According to given condition

$$x^2+(40-x)^2=2(x)(40-x)+100$$

$$x^2-80x+1600+x^2=80x-2x^2+100$$

$$4x^2-160x+1500=0$$

$$x^2-40x+375=0$$

$$a=1, b=-40, c=375$$

Using quadratic formula

$$x=\frac{-(-40)\pm\sqrt{(-40)^2-4(1)(375)}}{2(1)}$$

$$x=\frac{40\pm\sqrt{1600-1500}}{2}$$

$$x=\frac{40\pm\sqrt{100}}{2}$$

$$x=\frac{40\pm 10}{2}$$

$$x=\frac{40+10}{2} \text{ and } x=\frac{40-10}{2}$$

$$x=25 \ \ or \ \ x=15$$

If $x=25$, then $40-x=40-25=15$

If $x=15$ then $40-x=40-15=25$

Hence required parts of 40 are 25 and 15.

4. The sum of a positive number and its reciprocal is $\frac{26}{5}$. Find the number.

Solution:

Let x be the required positive number such that ,

$$x+\frac{1}{x}=\frac{26}{5}$$

$$5x^2+5=26x$$

$$5x^2-26x+5=0$$

$$a=5, b=-26, c=5$$

Using quadratic formula

$$x=\frac{-(-26)\pm\sqrt{(-26)^2-4(5)(5)}}{2(1)}$$

$$x=\frac{26\pm\sqrt{676-100}}{10}$$

$$x=\frac{26\pm\sqrt{576}}{10}$$

$$x=\frac{26\pm 24}{10}$$

$$x=\frac{26+24}{10} \text{ and } x=\frac{26-24}{10}$$

$$x=5 \ \ or \ \ x=\frac{2}{10}=\frac{1}{5}$$

Numbers is 5 or $\frac{1}{5}$.

5. A number exceeds its square root by 56. Find the number.

Solution: Let x be the required number.

According to given condition

$$x-\sqrt{x}=56$$

$$x-56=\sqrt{x}$$

Squaring on both sides

$$(x-56)^2=x$$

$$x^2-112x+3136=x$$

$$x^2-113x+3136=0$$

$$x^2-64x-49x+3136=0$$

$$x(x-64)-49(x-64)=0$$

$$(x-64)(x-49)=0$$

$$x-64=0 \ \ or \ \ x-49=0$$

$$x=64 \ \ or \ \ x=49$$

$x=49$ does not satisfies the given condition, hence required number is $x=64$.

6. Find the two consecutive numbers whose product is 132.

Solution: Let x and $x+1$ be required consecutive numbers.

$$x(x+1)=132$$

$$x^2+x-132=0$$

$$a=1, b=1, c=-132$$

Using quadratic formula

$$x=\frac{-1\pm\sqrt{1^2-4(1)(-132)}}{2(1)}$$

$$x=\frac{-1\pm\sqrt{1+528}}{2}$$

$$x=\frac{-1\pm\sqrt{529}}{2}$$

$$x=\frac{-1\pm 23}{2}$$

$$x=\frac{-1+23}{2} \text{ and } x=\frac{-1-23}{2}$$

$$x=11 \ \ or \ \ x=-12$$

If $x=11$, $x+1=12$, Numbers are 11 and 12.

If $x=-12$, $x+1=-12+1=-11$, Numbers are -12 and -11.

7. The difference between the cubes of two consecutive even numbers is 296. Find the numbers.

Solution:

Let x and $x+2$ be two consecutive even numbers. According to given condition.

$$(x+2)^3-x^3=296$$

$$x^3+8+3x^2(2)+3x(4)-x^3=296$$

$$6x^2 + 12x - 288 = 0$$

$$x^2 + 2x - 48 = 0$$

$$x^2 + 8x - 6x - 48 = 0$$

$$x(x+8) - 6(x+8) = 0$$

$$(x+8)(x-6) = 0$$

$$x + 8 = 0 \ \ or \ \ x - 6 = 0$$
$$x = -8 \ \ or \ \ x = 6$$

As x can't be negative, so required number is $x = 6$.

8. A farmer bought some sheep for Rs. 9000. If he had paid Rs. 100 less for each sheep, he would have got 3 sheep more for the same money. How many sheep did he buy, when the rate in each case is uniform?

Solution:

Let x be the number of sheep.

Price of x sheep =Rs. 9000

Price of 1 sheep = Rs. $\frac{9000}{x}$

Now if price of 1 sheep is $\frac{9000}{x} - 100$ then number of sheep is $x + 3$

According to given condition

$$\frac{9000}{x+3} = \frac{9000}{x} - 100$$

On multiplying by $x(x+3)$ on both sides, we get

$$9000x = 9000(x+3) - 100x(x+3)$$

$$9000x = 9000x + 27000 - 100x^2 - 300x$$

$$100x^2 + 300x - 27000 = 0$$

Dividing by 100

$$x^2 + 3x - 270 = 0$$

$$a = 1, b = 3 \text{ and } c = -270$$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(-270)}}{2(1)}$$

$$x = \frac{-3 \pm \sqrt{9 + 1080}}{2}$$

$$x = \frac{-3 \pm \sqrt{1089}}{2}$$

$$x = \frac{-3 \pm 33}{2}$$

$$x = \frac{-3 + 33}{2}, x = \frac{-3 - 33}{2}$$

$$x = 15, x = -18$$

Since x cannot be negative, so $x = 15$. i.e., he buy 15 sheep.

9. A man sold his stock of eggs for Rs. 240. If he had 2 dozen more, he would have got the same money by selling the whole for Rs. 0.50 per dozen cheaper. How many dozen eggs did he sell?

Solution: Let he sell x dozen eggs.

Since total price of x dozen eggs is Rs. 240, so price of 1 dozen eggs = $\frac{240}{x}$

If he has $x + 2$ dozen eggs then price of 1 dozen = $\frac{240}{x+2}$

Now according to given condition

$$\frac{240}{x} - 0.50 = \frac{240}{x+2}$$

$$\frac{240}{x} - \frac{50}{100} = \frac{240}{x+2}$$

$$\frac{240}{x} - \frac{1}{2} = \frac{240}{x+2}$$

Multiplying by $2x(x+2)$

$$480(x+2) - x(x+2) = 480x$$

$$480x + 960 - x^2 - 2x = 480x$$

$$960 - x^2 - 2x = 0$$

$$x^2 + 2x - 960 = 0$$

$$a = 1, b = 2 \text{ and } c = -960$$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(-960)}}{2(1)}$$

$$x = \frac{-2 \pm \sqrt{4+3840}}{2}$$

$$x = \frac{-2 \pm \sqrt{3844}}{2}$$

$$x = \frac{-2 \pm 62}{2}$$

$$x = \frac{-2+62}{2}, x = \frac{-2-62}{2}$$

$$x = 30, x = -32$$

x cannot be negative, so $x = 30$ dozen eggs he sell.

10. A cyclist travelled 48 km at a uniform speed. Had he travelled 2km /hr slower, he would have taken 2 hours more to perform the journey. How long did he take to cover 48 km?

Solution:

Let x be the speed and y be the time to cover 48 km.

Speed × Time = Distance

$$xy = 48 \longrightarrow (1)$$

Now if speed is $(x-2)$ km/hr then time to cover distance of 48 km is $(y+2)$ hours.

i.e., $(x-2)(y+2) = 48$

$$xy + 2x - 2y - 4 = 48$$

$$48 + 2x - 2y - 4 = 48$$

$$2x - 2y - 4 = 0$$

$$x - y - 2 = 0$$

$x = y + 2 \longrightarrow (2)$ put in (1)

$$(y+2)y = 48$$

$$y^2 + 2y - 48 = 0$$

$$y^2 + 8y - 6y - 48 = 0$$

$$y(y+8) - 6(y+8) = 0$$

$$(y+8)(y-6) = 0$$

$$y + 8 = 0 \;\; or \;\; y - 6 = 0$$

$$y = -8 \;\; or \;\; y = 6$$

y cannot be negative. So $y = 6$ hours is the time to cover 48 km.

11. The area of a rectangular field is 297 square meters. Had it been 3 meter longer and 1 meter shorter, the area would have been 3 square meter more. Find its length and breadth.

Solution: Let x be the length and y be the breadth then

$$xy = 297 \longrightarrow (1)$$

Now if length is $(x+3)$ and breadth is $(y-1)$ then area would be 300 square meters, which can be written as

$$(x+3)(y-1) = 300$$

$$xy + 3y - x - 3 = 300$$

$297 + 3y - x - 3 = 300$ from (1)

$$294 + 3y - x = 300$$

$$3y - x = 6$$

$x = 3y - 6$ put in (1)

$$(3y-6)y = 297$$

$$3y^2 - 6y = 297$$

$$3y^2 - 6y - 297 = 0$$

$$3y^2 - 6y - 297 = 0$$

$$y = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(3)(-297)}}{2(3)}$$

$$y = \frac{6 \pm \sqrt{36+3564}}{6}$$

$$y = \frac{6 \pm \sqrt{3600}}{6}$$

$$y = \frac{6 \pm 60}{6}$$

$$y = \frac{6+60}{6}, y = \frac{6-60}{6}$$

$$y = 11, \quad y = -9$$

As y cannot be negative, so $y = 11$. Put in (1)

$xy = 297$

$11x = 297$

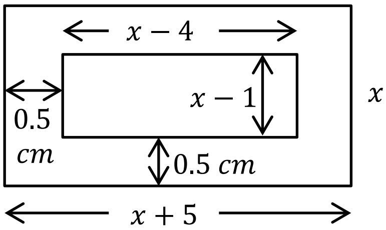
$x = 27$

Hence length is 27meter and breadth is 11 meter.

12. The length of a rectangular piece of paper exceeds its breadth by 5cm. If a strip of 0.5 cm wide be cut all around the piece of paper, the area of the remaining part would be 500 square cms. Find its original dimensions.

Solution: Let x is the breadth of rectangular piece of paper and $x + 5$ is the length.

On cutting a strip of 0.5cm wide all around the piece of paper, length and breadth of the piece



of paper are $x + 4$ and $x - 1$.

Now, $(x + 4)(x - 1) = 500$

$x^2 + 3x - 4 = 500$

$x^2 + 3x - 504 = 0$

$x^2 + 24x - 21x - 504 = 0$

$x(x + 24) - 21(x + 24) = 0$

$(x + 24)(x - 21) = 0$

$x + 24 = 0 \ \ or \ \ x - 21 = 0$

$x = -24 \ \ or \ \ x = 21$

$x = 21$, because x can't be negative.

$x + 5 = 26$

Hence length and breadth of rectangle are 26 and 21 cm.

13. A number consists of two digits whose product is 18. If the digits are interchanged, the new number becomes 27 less than the original number. Find the number.

Solution:

Let x be the unit place digit and y be tens place digit.

Number = $x + 10y$

According to given condition.

$xy = 18 \longrightarrow (1)$

And $x + 10y - 27 = y + 10x$

$x - 10x + 10y - y = 27$

$-9x + 9y = 27$

$y - x = 3$

$y = x + 3 \longrightarrow (2)$

Putting value from (2) in (1)

$x(x + 3) = 18$

$x^2 + 3x - 18 = 0$

$x^2 + 6x - 3x - 18 = 0$

$x(x + 6) - 3(x + 6) = 0$

$(x + 6)(x - 3) = 0$

$x + 6 = 0 \ \ or \ \ x - 3 = 0$

$x = -6 \ \ or \ \ x = 3$

$x = 3$, because x can't be negative.

Using value in (2), $y = 3 + 3 = 6$

Hence Number = $x + 10y = 3 + 10(6) = 63$

14. A number consists of two digits whose product is 14. If the digits are interchanged, the resulting number will exceed the original number by 45. Find the number.

Solution:

Let x be the unit place digit and y be tens place digit.

Number = $x + 10y$

According to given condition.

$xy = 14 \longrightarrow (1)$

And $x + 10y + 45 = y + 10x$

$$x - 10x + 10y - y = -45$$

$$-9x + 9y = -45$$

$$-x + y = -5$$

$$x - y = 5$$

$$y = x - 5 \longrightarrow (2)$$

Putting value from (2) in (1)

$$x(x-5) = 14$$

$$x^2 - 5x - 14 = 0$$

$$x^2 - 7x + 2x - 14 = 0$$

$$x(x-7) + 2(x-7) = 0$$

$$(x-7)(x+2) = 0$$

$$x - 7 = 0 \ \ or \ \ x + 2 = 0$$

$$x = 7 \ \ or \ \ x = -2$$

$x = 7$, because x can't be negative.

Using value in (2), $y = 7 - 5 = 2$

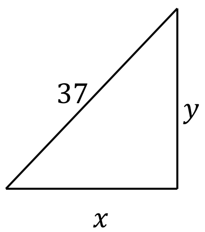
Hence Number $= x + 10y = 7 + 10(2) = 27$

15. The area of a right triangle is 210 square meters. If its hypotenuse is 37 meters long. Find the length of the base and the altitude.

Solution: Let x be base and y be altitude of the triangle.

$$x^2 + y^2 = 37^2$$

$$x^2 + y^2 = 1369 \ \longrightarrow (1)$$



As area of triangle is 210 square meters.

$$\frac{1}{2}xy = 210$$

$$xy = 420 \longrightarrow (2)$$

$$x^2 + y^2 - 2xy = 1369 - 2xy$$

$$(x-y)^2 = 1369 - 840$$

$$(x-y)^2 = 529$$

$$x - y = 23$$

$$y = x - 23 \longrightarrow (3)$$

Putting value from (3) in (2)

$$x(x - 23\ ) = 420$$

$$x^2 - 23x - 420 = 0$$

$$x^2 - 35x + 12x - 420 = 0$$

$$x(x-35) + 12(x-35) = 0$$

$$(x-35)(x+12) = 0$$

$$x - 35 = 0 \ \ or \ \ x + 12 = 0$$

$$x = 35 \ \ or \ \ x = -12$$

$x = 35$, because x can't be negative. Putting value in (3)

$$y = 35 - 23 = 12$$

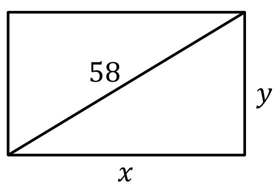
Hence length of base is 12 meter and altitude is 35 meter.

16. The area of a rectangle is 1680 square meters. If its diagonal is 58 meters long, find the length and the breadth of the rectangle.

Solution:

Let x and y be the length and breadth of the rectangle.

$$xy = 1680 \ \longrightarrow (1)$$



Also

$$x^2 + y^2 = (58)^2$$

$$x^2 + y^2 = (58)^2$$

$$x^2 + y^2 - 2xy = (58)^2 - 2xy$$

$$x^2 + y^2 - 2xy = (58)^2 - 2(1680)$$

$$x^2 + y^2 - 2xy = 3364 - 3360$$

$$(x-y)^2 = 4$$

$$x - y = 2$$

$y = x - 2$ put in (1)

$x(x-2) = 1680$

$x^2 - 2x = 1680$

$x^2 - 2x - 1680 = 0$

$x^2 - 42x + 40x - 1680 = 0$

$x(x-42) + 40(x-42) = 0$

$(x-42)(x+40) = 0$

$x - 42 = 0 \ or\ x + 40 = 0$

$x = 42\ or\ x = -40$

Since x can't be negative, so $x = 42$ and $y = 42 - 2 = 40$. Hence length and breadth are 42m and 40m.

17. To do a piece of work, A takes 10 days more than B. Together they finish the work in 12 days. How long would B take to finish it alone?

Solution:

Let B takes x days to complete the job. So A can do the job in $x + 10$ days.

One day work of A = $\frac{1}{x}$

One day work of B = $\frac{1}{x+10}$

Work done by both A and B in one day = $\frac{1}{x} + \frac{1}{x+10}$

Given that

A and B both can do the work in days = 12

One day work of both A and B = $\frac{1}{12}$

Thus, $\frac{1}{x} + \frac{1}{x+10} = \frac{1}{12}$

Multiplying by $12x(x+10)$

$12(x+10) + 12x = x(x+10)$

$12x + 120 + 12x = x^2 + 10x$

$x^2 + 10x - 24x - 120 = 0$

$x^2 - 14x - 120 = 0$

$x^2 - 20x + 6x - 120 = 0$

$x(x-20) + 6(x-20) = 0$

$(x-20)(x+6) = 0$

$x - 20 = 0\ or\ x + 6 = 0$

$x = 20\ or\ x = -6$

As x can't be negative, so $x = 20$ days.

$x = 6 \quad \Longrightarrow 2x = 12$

B would take 6 days and A would take 12 days to complete the job.

18. To complete a job, A and B take 4 days working together. A alone takes twice as long as B alone takes to finish the same job. How long would each one alone take to do the job?

Solution:

Let B takes x days to complete the job. So A can do the job in $2x$ days.

One day work of A = $\frac{1}{x}$

One day work of B = $\frac{1}{2x}$

Work done by both A and B in one day = $\frac{1}{x} + \frac{1}{2x}$

Given that

A and B both can do the work in days = 4

One day work of both A and B = $\frac{1}{4}$

Thus, $\frac{1}{x} + \frac{1}{2x} = \frac{1}{4}$

Multiplying by $4x$

$4 + 2 = x$

$x = 6$

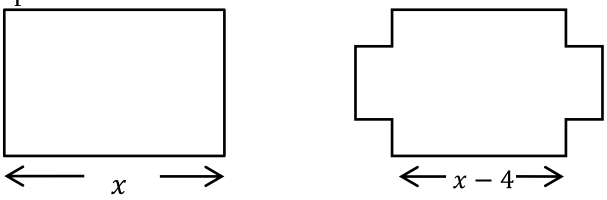
$2x = 12$

B would take 6 days and A would take 12 days to complete the job.

19. An open box is to be made from a square piece of tin by cutting a piece 2dm square from each corner and then folding the sides of the remaining piece. If the capacity of the box is

128 cubic dm, find the length of the sides of the piece.

Solution: Let x be the length and width of square piece of tin.



After cutting $2\ \text{dm}^2$ from each corner

Length and width of box becomes $x-4$ dm, height of box is 2 dm.

We know that volume of box $=$ length $\times$ width $\times$ height

$$128=\text{length}\times\text{width}\ \times\text{height}$$

$$128=2(x-4)(x-4)$$

$$128=2(x-4)^2$$

$$64=(x-4)^2$$

$$x-4=8$$

$$x=12$$

So length of piece of tin is 12 dm.

20. A man invests Rs. 100,000 in two companies. His total profit is Rs. 3080. If he receives Rs. 1980 from one company and at a rate 1% more from the other company. Find the amount of each investment.

Solution:

Let two companies are A and B.

Investment in company A $=$ Rs. x

Investment in company B $=$ Rs. $(100{,}000-x)$

Profit rate in company A $=y$ %

Profit rate in company B $=(y+1)$ %

As we know that

$$\text{profit}=\frac{\text{amount}\times\text{rate}\times\text{period}}{100}$$

$$1980=\frac{x\times y\times 1}{100}$$

$$198{,}000=xy$$

$$xy=198{,}000 \longrightarrow (1)$$

Also,

$$3080=\frac{(100{,}000-x)\times(y+1)\times 1}{100}$$

$$(100{,}000-x)\times(y+1)=308{,}000$$

$$100{,}000y+100{,}000-xy-x=308{,}000$$

$$100{,}000y-xy-x=208{,}000$$

$$100{,}000y-198{,}000-x=208{,}000$$

$$100{,}000y-x=208{,}000+198{,}000$$

$$100{,}000y-x=40{,}6000$$

$$x=100{,}000y-40{,}6000 \longrightarrow (2)$$

Putting value from (2) in (1)

$$(100{,}000y-40{,}6000)\ y=198{,}000$$

$$100{,}000y^2-40{,}6000y-198{,}000=0$$

Dividing by 20,000

$$50y^2-203y-99=0$$

$$y=\frac{-(-203)\pm\sqrt{(-203)^2-4(50)(-99)}}{2(50)}$$

$$y=\frac{203\pm\sqrt{41209+19800}}{100}$$

$$y=\frac{203\pm\sqrt{61009}}{100}$$

$$y=\frac{203\pm 247}{100}$$

$$y=\frac{203+247}{100}$$

because y can't be negative.

$y=4.5$ put in (1)

$$4.5x=198{,}000$$

$x = \frac{198,000}{4.5}$

$x = \text{Rs.} 44,000$

Investment in company A = Rs. 44,000

Investment in company B = Rs. $(100,000 - 44,000) = \text{Rs.} 56,000$