

Quadratic Equation

An equation containing one or more terms in which the variable is raised to maximum positive power two. In general;

$ax^2 + bx + c = 0$ where $a \neq 0$ is called Quadratic Equation in variable x .

3 Methods.

To solve Quadratic Equation there are three different methods named as;

1. Factorization method.
2. Completing Square method.
3. Quadratic Formula method.

Example 1

Solve by Factorization $x^2 - 7x + 10 = 0$

$$\begin{aligned} x^2 - 2x - 5x + 10 &= 0 \\ x(x-2) - 5(x-2) &= 0 \\ (x-2)(x-5) &= 0 \\ x-2 &= 0, \quad x-5 = 0 \\ \Rightarrow x &= 2, \quad \Rightarrow x = 5 \end{aligned}$$

$\begin{array}{c} 10x^2 \\ \wedge \\ -2x \quad -5x \end{array}$

$\{2, 5\}$

Example 2

Solve $x^2 + 4x - 437 = 0$ by Completing Sq.

$$x^2 + 4x = 437$$

Adding $(\frac{4}{2})^2 = (2)^2$ on both sides.

$$\begin{aligned} x^2 + 4x + (2)^2 &= 437 + (2)^2 \\ (x+2)^2 &= 437 + 4 \\ (x+2)^2 &= 441 \\ x+2 &= \pm 21 \quad \because \sqrt{441} = 21 \\ x+2 &= 21, \quad x+2 = -21 \\ x &= 21-2, \quad x = -21-2 \\ x &= 19, \quad x = -23 \\ &\{19, -23\} \end{aligned}$$

Example 3

Solve $6x^2 + x - 15 = 0$ by Q. Formula

Comparing $6x^2 + x - 15 = 0$
with $ax^2 + bx + c = 0$

we have $a = 6, b = 1, c = -15$

By using Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-1 \pm \sqrt{(1)^2 - 4(6)(-15)}}{2(6)}$$

$$x = \frac{-1 \pm \sqrt{1+360}}{12} \Rightarrow x = \frac{-1 \pm \sqrt{361}}{12}$$

$$x = \frac{-1 \pm 19}{12} \quad \because \sqrt{361} = 19$$

$$x = \frac{-1+19}{12}, \quad x = \frac{-1-19}{12}$$

$$x = \frac{18}{12}, \quad x = \frac{-20}{12}$$

$$x = \frac{3}{2}, \quad x = \frac{-5}{3} \quad \left\{ \frac{3}{2}, \frac{-5}{3} \right\}$$

Example 4

Solve $8x^2 - 14x - 15 = 0$ by Quadratic For

Comparing $8x^2 - 14x - 15 = 0$

with $ax^2 + bx + c = 0$

We have $a = 8, b = -14, c = -15$

By using $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-(-14) \pm \sqrt{(-14)^2 - 4(8)(-15)}}{2(8)}$$

$$x = \frac{14 \pm \sqrt{196+480}}{16} \Rightarrow x = \frac{14 \pm \sqrt{676}}{16}$$

$$x = \frac{14 \pm 26}{16} \quad \because \sqrt{676} = 26$$

$$x = \frac{14+26}{16}, \quad x = \frac{14-26}{16}$$

$$x = \frac{40}{16}, \quad x = \frac{-12}{16}$$

$$x = \frac{5}{2}, \quad x = \frac{-3}{4} \quad \left\{ \frac{5}{2}, \frac{-3}{4} \right\}$$

Q Derive the Quadratic Formula.

Quadratic Equation in standard form
is $ax^2 + bx + c = 0$

Dividing by a

$$\frac{ax^2}{a} + \frac{bx}{a} + \frac{c}{a} = \frac{0}{a}$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

Now $x^2 + \frac{b}{a}x = -\frac{c}{a}$

Adding $(\frac{1}{2} \cdot \frac{b}{a})^2$ on both sides

$$x^2 + \frac{b}{a}x + (\frac{1}{2} \frac{b}{a})^2 = (\frac{1}{2} \frac{b}{a})^2 - \frac{c}{a}$$

$$x^2 + x \cdot \frac{b}{a} + (\frac{b}{2a})^2 = \frac{b^2}{4a^2} - \frac{c}{a}$$

$$(x)^2 + 2(x)(\frac{b}{2a}) + (\frac{b}{2a})^2 = \frac{b^2 - 4ac}{4a^2}$$

As $y^2 + 2yz + z^2 = (y+z)^2$

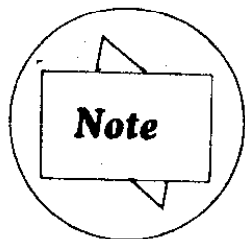
So $(x + \frac{b}{2a})^2 = \frac{b^2 - 4ac}{4a^2}$

$$x + \frac{b}{2a} = \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

This is called Quadratic Formula.



In Quadratic Equation $ax^2 + bx + c = 0$

1. a, b and c are real numbers.
2. The answer of Quadratic Equation are called its Roots.
3. Another name of Quadratic Equation is Second Degree Polynomial.

EXERCISE. 4.1

Solve by FACTORIZATION.

Q.1 $3x^2 + 4x + 1 = 0$

$$3x^2 + 3x + x + 1 = 0$$

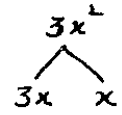
$$3x(x+1) + 1(x+1) = 0$$

$$(x+1)(3x+1) = 0$$

$$x+1 = 0, 3x+1 = 0$$

$$x = -1, 3x = -1$$

$$x = -\frac{1}{3}$$



$$\{-1, -\frac{1}{3}\}$$

Q.2

$$x^2 + 7x + 12 = 0$$

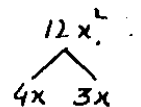
$$x^2 + 4x + 3x + 12 = 0$$

$$x(x+4) + 3(x+4) = 0$$

$$(x+3)(x+4) = 0$$

$$x+3 = 0, x+4 = 0$$

$$x = -3, x = -4$$



$$\{-3, -4\}$$

Q.3

$$9x^2 - 12x - 5 = 0$$

$$9x^2 + 3x - 15x - 5 = 0$$

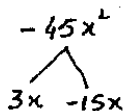
$$3x(3x+1) - 5(3x+1) = 0$$

$$(3x+1)(3x-5) = 0$$

$$3x+1 = 0, 3x-5 = 0$$

$$3x = -1, 3x = 5$$

$$x = -\frac{1}{3}, x = \frac{5}{3}$$



$$\{-\frac{1}{3}, \frac{5}{3}\}$$

Q.4

$$x^2 - x = 2$$

$$x^2 - x - 2 = 0$$

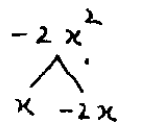
$$x^2 + x - 2x - 2 = 0$$

$$x(x+1) - 2(x+1) = 0$$

$$(x-2)(x+1) = 0$$

$$x-2 = 0, x+1 = 0$$

$$\{2, -1\}$$



Q.5

$$x(x+7) = (2x-1)(x+4)$$

$$x^2 + 7x = 2x^2 + 8x - x - 4$$

$$x^2 + 7x = 2x^2 + 7x - 4$$

$$2x^2 - x^2 + 7x - 7x - 4 = 0$$

$$x^2 - 4 = 0$$

$$x^2 - (2)^2 = 0$$

$$(x+2)(x-2) = 0$$

$$x+2 = 0, x-2 = 0$$

$$x = -2, x = 2$$

$$\{2, -2\}$$

$$\text{Q.6} \quad \frac{x}{x+1} + \frac{x+1}{x} = \frac{5}{2}$$

Multiplying by $2x(x+1)$

$$2x(x+1) \cdot \frac{x}{x+1} + 2x(x+1) \cdot \frac{x+1}{x} = 2x(x+1) \frac{5}{2}$$

$$2x^2 + 2(x+1)(x+1) = 5x(x+1)$$

$$2x^2 + 2(x^2 + 2x + 1) = 5x^2 + 5x$$

$$2x^2 + 2x^2 + 4x + 2 = 5x^2 + 5x$$

$$4x^2 + 4x + 2 = 5x^2 + 5x$$

$$5x^2 - 4x^2 + 5x - 4x - 2 = 0$$

$$x^2 + x - 2 = 0$$

$$x^2 - x + 2x - 2 = 0$$

$$x(x-1) + 2(x-1) = 0$$

$$(x-1)(x+2) = 0$$

$$x-1 = 0, \quad x+2 = 0$$

$$\Rightarrow x = 1, \quad x = -2 \quad \{1, -2\}$$

$$\text{Q.7} \quad \frac{1}{x+1} + \frac{2}{x+2} = \frac{7}{x+5}$$

Multiplying by $(x+1)(x+2)(x+5)$

$$(x+1)(x+2)(x+5) \cdot \frac{1}{x+1} + (x+1)(x+2)(x+5) \cdot \frac{2}{x+2}$$

$$= (x+1)(x+2)(x+5) \cdot \frac{7}{x+5}$$

$$(x+2)(x+5) + 2(x+1)(x+5) = 7(x+1)(x+2)$$

$$x^2 + 5x + 2x + 10 + 2(x^2 + 5x + x + 5) = 7(x^2 + 2x + x + 2)$$

$$x^2 + 7x + 10 + 2x^2 + 12x + 10 = 7x^2 + 21x + 14$$

$$3x^2 + 19x + 20 = 7x^2 + 21x + 14$$

$$7x^2 - 3x^2 + 21x - 19x + 14 - 20 = 0$$

$$4x^2 + 2x - 6 = 0$$

$$2x^2 + x - 3 = 0$$

$$2x^2 - 2x + 3x - 3 = 0$$

$$2x(x-1) + 3(x-1) = 0$$

$$(x-1)(2x+3) = 0$$

$$x-1 = 0, \quad 2x+3 = 0$$

$$x = 1, \quad 2x = -3$$

$$x = -\frac{3}{2} \quad \left\{1, -\frac{3}{2}\right\}$$

$$\text{Q.8} \quad \frac{a}{ax-1} + \frac{b}{bx-1} = a+b$$

$$\frac{a}{ax-1} - b + \frac{b}{bx-1} - a = 0$$

$$\frac{a-b(ax-1)}{ax-1} + \frac{b-a(bx-1)}{bx-1} = 0$$

$$\frac{a-abx+b}{ax-1} + \frac{b-abx+a}{bx-1} = 0$$

$$\frac{a+b-abx}{ax-1} + \frac{a+b-abx}{bx-1} = 0$$

$$(a+b-abx) \left\{ \frac{1}{ax-1} + \frac{1}{bx-1} \right\} = 0$$

$$(a+b-abx) \left\{ \frac{bx-1+ax-1}{(ax-1)(bx-1)} \right\} = 0$$

$$(a+b-abx)(ax+bx-2) = 0 \quad (ax-1)(bx-1)$$

$$(a+b-abx)(ax+bx-2) = 0$$

Either $a+b-abx = 0$ or $ax+bx-2 = 0$

$$\Rightarrow abx = a+b, \quad (a+b)x = 2$$

$$\Rightarrow x = \frac{a+b}{ab}, \quad x = \frac{2}{a+b}$$

$$\left\{ \frac{a+b}{ab}, \frac{2}{a+b} \right\}$$

★ Solve By Completing Square.

$$\text{Q.9} \quad x^2 - 2x - 899 = 0$$

$$x^2 - 2x = 899$$

Adding $\left(\frac{2}{2}\right)^2 = (1)^2$ on both sides

$$x^2 - 2x + (-1)^2 = 899 + (-1)^2$$

$$(x-1)^2 = 899 + 1$$

$$(x-1)^2 = 900$$

$$\Rightarrow x-1 = \pm 30$$

$$x-1 = 30, \quad x-1 = -30$$

$$x = 30+1, \quad x = -30+1$$

$$x = 31, \quad x = -29 \quad \{31, -29\}$$

$$\text{Q.10} \quad x^2 + 4x - 1085 = 0$$

$$x^2 + 4x = 1085$$

Adding $\left(\frac{4}{2}\right)^2 = (2)^2$ on both sides

$$x^2 + 4x + (2)^2 = 1085 + (2)^2$$

$$(x+2)^2 = 1085+4$$

$$(x+2)^2 = 1089$$

$$\Rightarrow x+2 = \pm 33.$$

$$x+2 = 33, \quad x+2 = -33$$

$$x = 33-2, \quad x = -33-2$$

$$x = 31, \quad x = -35$$

$$\{31, -35\}$$

Q.11

$$x^2 + 6x - 567 = 0$$

$$x^2 + 6x = 567$$

Adding $(\frac{6}{2})^2 = (3)^2$ on both sides

$$x^2 + 6x + (3)^2 = 567 + (3)^2$$

$$(x+3)^2 = 567+9$$

$$(x+3)^2 = 576$$

$$x+3 = \pm 24$$

$$x+3 = 24, \quad x+3 = -24$$

$$x = 24-3, \quad x = -24-3$$

$$x = 21, \quad x = -27$$

$$\{21, -27\}$$

Q.12

$$x^2 - 3x - 648 = 0$$

$$x^2 - 3x = 648$$

Adding $(\frac{3}{2})^2$ on both sides

$$x^2 - 3x + (\frac{3}{2})^2 = 648 + (\frac{3}{2})^2$$

$$(x - \frac{3}{2})^2 = 648 + \frac{9}{4}$$

$$(x - \frac{3}{2})^2 = \frac{2592+9}{4}$$

$$(x - \frac{3}{2})^2 = \frac{2601}{4}$$

$$\Rightarrow x - \frac{3}{2} = \pm \frac{51}{2}$$

$$x - \frac{3}{2} = \frac{51}{2}, \quad x - \frac{3}{2} = -\frac{51}{2}$$

$$x = \frac{51}{2} + \frac{3}{2}, \quad x = -\frac{51}{2} + \frac{3}{2}$$

$$x = \frac{51+3}{2}, \quad x = \frac{-51+3}{2}$$

$$x = \frac{54}{2}, \quad x = -\frac{48}{2}$$

$$x = 27, \quad x = -24 \quad \{27, -24\}$$

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Q.13 $x^2 - x - 1806 = 0$

$$x^2 - x = 1806$$

Adding $(\frac{1}{2})^2$ on both sides

$$x^2 - x + (\frac{1}{2})^2 = 1806 + (\frac{1}{2})^2$$

$$(x - \frac{1}{2})^2 = 1806 + \frac{1}{4}$$

$$(x - \frac{1}{2})^2 = \frac{7224+1}{4}$$

$$(x - \frac{1}{2})^2 = \frac{7225}{4}$$

$$\Rightarrow x - \frac{1}{2} = \pm \frac{85}{2}$$

$$x - \frac{1}{2} = \frac{85}{2}, \quad x - \frac{1}{2} = -\frac{85}{2}$$

$$x = \frac{85}{2} + \frac{1}{2}, \quad x = -\frac{85}{2} + \frac{1}{2}$$

$$x = \frac{85+1}{2}, \quad x = \frac{-85+1}{2}$$

$$x = \frac{86}{2}, \quad x = -\frac{84}{2}$$

$$x = 43, \quad x = -42 \quad \{43, -42\}$$

Q.14

$$2x^2 + 12x - 110 = 0$$

Dividing by 2. $x^2 + 6x - 55 = 0$

$$x^2 + 6x = 55$$

Adding $(\frac{6}{2})^2 = (3)^2$ on both sides

$$x^2 + 6x + (3)^2 = 55 + (3)^2$$

$$(x+3)^2 = 55+9$$

$$(x+3)^2 = 64$$

$$x+3 = \pm 8$$

$$x+3 = 8, \quad x+3 = -8$$

$$x = 8-3, \quad x = -8-3$$

$$x = 5, \quad x = -11 \quad \{5, -11\}$$

* Find roots by using Q. Formula.

Q.15

$$5x^2 - 13x + 6 = 0$$

Comparing $ax^2 + bx + c = 0$

we have $a = 5, b = -13, c = 6$

Using $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-(-13) \pm \sqrt{(-13)^2 - 4(5)(6)}}{2(5)}$$

$$x = \frac{13 \pm \sqrt{169 - 120}}{10}$$

$$x = \frac{13 \pm \sqrt{49}}{10} \Rightarrow x = \frac{13 \pm 7}{10}$$

$$x = \frac{13+7}{10}, \quad x = \frac{13-7}{10}$$

$$x = 2\frac{0}{10}, \quad x = \frac{6}{10}$$

$$x = 2, \quad x = \frac{3}{5} \quad \left\{ 2, \frac{3}{5} \right\}$$

Q.16 $4x^2 + 7x - 1 = 0$

Comparing $ax^2 + bx + c = 0$

we get $a=4, b=7, c=-1$

Using $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-7 \pm \sqrt{(7)^2 - 4(4)(-1)}}{2(4)}$$

$$x = \frac{-7 \pm \sqrt{49 + 16}}{8}$$

$$x = \frac{-7 \pm \sqrt{65}}{8}, \quad \left\{ \frac{-7 \pm \sqrt{65}}{8} \right\}$$

Q.17 $15x^2 + 2ax - a^2 = 0$

Comparing $ax^2 + bx + c = 0$

$a=15, b=2a, c=-a^2$

Using $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-2a \pm \sqrt{(2a)^2 - 4(15)(-a^2)}}{2(15)}$$

$$x = \frac{-2a \pm \sqrt{4a^2 + 60a^2}}{30}$$

$$x = \frac{-2a \pm \sqrt{64a^2}}{30} \Rightarrow x = \frac{-2a \pm 8a}{30}$$

$$x = \frac{-2a + 8a}{30}, \quad x = \frac{-2a - 8a}{30}$$

$$x = \frac{6a}{30}, \quad x = \frac{-10a}{30}$$

$$x = \frac{a}{5}, \quad x = -\frac{a}{3} \quad \left\{ \frac{a}{5}, -\frac{a}{3} \right\}$$

Q.18 $16x^2 + 8x + 1 = 0$

Comparing $ax^2 + bx + c = 0$.

we get $a=16, b=8, c=1$

Using $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

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$$x = \frac{-8 \pm \sqrt{(8)^2 - 4(16)(1)}}{2(16)}$$

$$x = \frac{-8 \pm \sqrt{64 - 64}}{32} \Rightarrow x = \frac{-8 \pm \sqrt{0}}{32}$$

$$x = \frac{-8}{32} \Rightarrow x = -\frac{1}{4} \quad \left\{ -\frac{1}{4} \right\}$$

Q.19

$$(x-a)(x-b) + (x-b)(x-c) + (x-c)(x-a) = 0$$

Simplyfying

$$x^2 - bx - ax + ab + x^2 - cx - bx + bc + x^2 - ax - cx + ac = 0$$

$$3x^2 - 2ax - 2bx - 2cx + ab + bc + ac = 0$$

$$3x^2 - 2(a+b+c)x + ab + bc + ac = 0$$

Using $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-[-2(a+b+c)] \pm \sqrt{[-2(a+b+c)]^2 - 4(3)(ab+bc+ac)}}{2(3)}$$

$$x = \frac{2(a+b+c) \pm 2\sqrt{(a+b+c)^2 - 3(ab+bc+ac)}}{6}$$

$$x = \frac{(a+b+c) \pm \sqrt{a^2 + b^2 + c^2 + 2ab + 2bc + 2ca - 3ab - 3bc - 3ca}}{3}$$

$$x = \frac{(a+b+c) \pm \sqrt{a^2 + b^2 + c^2 - ab - bc - ca}}{3}$$

$$\left\{ \frac{(a+b+c) \pm \sqrt{a^2 + b^2 + c^2 - ab - bc - ca}}{3} \right\}$$

Q.20 $(a+b)x^2 + (a+2b+c)x + b+c = 0$

Using $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-(a+2b+c) \pm \sqrt{(a+2b+c)^2 - 4(a+b)(b+c)}}{2(a+b)}$$

$$x = \frac{-(a+2b+c) \pm \sqrt{(a+2b+c)^2 - 4(ab+ac+b^2+bc)}}{2(a+b)}$$

$$x = \frac{-(a+2b+c) \pm \sqrt{a^2 + 4b^2 + c^2 + 4ab + 4bc + 2ac - 4ab - 4ac - 4b^2 - 4bc}}{2(a+b)}$$

$$x = \frac{-(a+2b+c) \pm \sqrt{a^2 + c^2 - 2ac}}{2(a+b)}$$

$$x = \frac{-(a+2b+c) \pm \sqrt{(a-c)^2}}{2(a+b)}$$

$$x = \frac{-(a+2b+c) \pm (a-c)}{2(a+b)}$$

$$x = \frac{-(a+2b+c)+a-c}{2(a+b)}, \quad x = \frac{-(a+2b+c)-a+c}{2(a+b)}$$

$$x = \frac{-a-2b-c+a-c}{2(a+b)}, \quad x = \frac{-a-2b-c-a+c}{2(a+b)}$$

$$x = \frac{-2b-2c}{2(a+b)}, \quad x = \frac{-2b-2a}{2(a+b)}$$

$$x = \frac{-2(b+c)}{2(a+b)}, \quad x = \frac{-2(a+b)}{2(a+b)}$$

$$x = \frac{-(b+c)}{a+b}, \quad x = -1$$

$$\left\{ -\frac{(b+c)}{a+b}, -1 \right\}$$

1)

put $x^{1/4} = y$ then

$$y^2 - y - 6 = 0$$

factorizing,

$$y^2 + 2y - 3y - 6 = 0$$

$$y(y+2) - 3(y+2) = 0$$

$$(y+2)(y-3) = 0$$

$$y+2=0, \quad y-3=0$$

$$y = -2, \quad y = 3$$

if $y = -2$, if $y = 3$ Then $x^{1/4} = -2$ Then $x^{1/4} = 3$

$$(x^{1/4})^4 = (-2)^4, \quad (x^{1/4})^4 = (3)^4$$

$$x = 16, \quad x = 81$$

$$\{16, 81\}$$

Time No 2

$$\begin{array}{c} -6y^2 \\ \swarrow \quad \searrow \\ 2y \quad -3y \end{array}$$

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