

EXERCISE 3.4

1. $A = \begin{bmatrix} 1 & -2 & 5 \\ -2 & 3 & -1 \\ 5 & -1 & 0 \end{bmatrix}$ $B = \begin{bmatrix} -3 & 1 & -2 \\ 1 & 0 & -1 \\ -2 & -1 & 2 \end{bmatrix}$

Show that $(A+B)$ is symmetric

Sol. $A+B = \begin{bmatrix} 1 & -2 & 5 \\ -2 & 3 & -1 \\ 5 & -1 & 0 \end{bmatrix} + \begin{bmatrix} -3 & 1 & -2 \\ 1 & 0 & -1 \\ -2 & -1 & 2 \end{bmatrix}$

$= \begin{bmatrix} -2 & -1 & 3 \\ -1 & 3 & -2 \\ 3 & -2 & 2 \end{bmatrix}$

$(A+B)^t = \begin{bmatrix} -2 & -1 & 3 \\ -1 & 3 & -2 \\ 3 & -2 & 2 \end{bmatrix} = A+B$

Hence $(A+B)$ is symmetric matrix

2. If $A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 2 & -1 \\ -1 & 3 & 2 \end{bmatrix}$

Then show that

i) $A+A^t$ is symmetric

Sol. $A+A^t = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 2 & -1 \\ -1 & 3 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 3 & -1 \\ 2 & 2 & 3 \\ 0 & -1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 2 & 5 & -1 \\ 5 & 4 & 2 \\ -1 & 2 & 4 \end{bmatrix}$

$(A+A^t)^t = \begin{bmatrix} 2 & 5 & -1 \\ 5 & 4 & 2 \\ -1 & 2 & 4 \end{bmatrix} = A+A^t$

$\Rightarrow (A+A^t)$ is symmetric

ii) $A-A^t = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 2 & -1 \\ -1 & 3 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 3 & -1 \\ 2 & 2 & 3 \\ 0 & -1 & 2 \end{bmatrix}$
 $= \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -4 \\ -1 & 4 & 0 \end{bmatrix}$

$(A-A^t)^t = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 4 \\ 1 & -4 & 0 \end{bmatrix} = - \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -4 \\ -1 & 4 & 0 \end{bmatrix}$
 $= -(A-A^t)$

$\Rightarrow A-A^t$ is skew symmetric

3. i) If A is square matrix of order 3 show that

$A+A^t$ is symmetric

Sol. $(A+A^t)^t = A^t + (A^t)^t$

$= A^t + A = A+A^t$

$\Rightarrow A+A^t$ is symmetric

ii) $A-A^t$ is skew symmetric

$(A-A^t)^t = A^t - (A^t)^t = A^t - A$
 $= -(A-A^t)$

$\Rightarrow A-A^t$ is skew symmetric

4. If the matrices A and B are symmetric matrix $AB=BA$ show that AB is symmetric

Sol. $A^t = A$ $B^t = B$

To prove $(AB)^t = AB$

L.H.S $(AB)^t = B^t A^t = BA = AB = R.H.S$

Hence AB is symmetric

5. Show that AA^t and $A^t A$ are symmetric for any matrix of order 2×3

Sol. Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$

$AA^t = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} \begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \\ a_{13} & a_{23} \end{bmatrix}$

$= \begin{bmatrix} a_{11}^2 + a_{12}^2 + a_{13}^2 & a_{11}a_{21} + a_{12}a_{22} + a_{13}a_{23} \\ a_{21}a_{11} + a_{22}a_{12} + a_{23}a_{13} & a_{21}^2 + a_{22}^2 + a_{23}^2 \end{bmatrix}$

$(AA)^t = \begin{bmatrix} a_{11}^2 + a_{12}^2 + a_{13}^2 & a_{21}a_{11} + a_{22}a_{12} + a_{23}a_{13} \\ a_{11}a_{12} + a_{12}a_{22} + a_{13}a_{23} & a_{21}^2 + a_{22}^2 + a_{23}^2 \end{bmatrix}$

$= AA^t$

Hence AA^t is symmetric

$A^t A = \begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \\ a_{13} & a_{23} \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$

$= \begin{bmatrix} a_{11}^2 + a_{21}^2 & a_{11}a_{12} + a_{21}a_{22} & a_{11}a_{13} + a_{21}a_{23} \\ a_{12}a_{11} + a_{22}a_{21} & a_{12}^2 + a_{22}^2 & a_{12}a_{13} + a_{22}a_{23} \\ a_{13}a_{11} + a_{23}a_{21} & a_{12}a_{13} + a_{22}a_{23} & a_{13}^2 + a_{23}^2 \end{bmatrix}$

$(A^t A)^t = \begin{bmatrix} a_{11}^2 + a_{21}^2 & a_{21}a_{11} + a_{22}a_{21} & a_{13}a_{11} + a_{23}a_{21} \\ a_{11}a_{12} + a_{21}a_{22} & a_{12}^2 + a_{22}^2 & a_{13}a_{12} + a_{23}a_{22} \\ a_{11}a_{13} + a_{21}a_{23} & a_{12}a_{13} + a_{22}a_{23} & a_{13}^2 + a_{23}^2 \end{bmatrix}$

$= A^t A$

Thus $A^t A$ is symmetric

6. If $A = \begin{pmatrix} i & 1+i \\ 1 & -i \end{pmatrix}$
 Show that $A + (\bar{A})^t$ is Hermitian

To prove $\overline{(A + (\bar{A})^t)}^t = A + (\bar{A})^t$

Sol. $\bar{A} = \begin{pmatrix} -i & 1-i \\ 1 & i \end{pmatrix}$ $(\bar{A})^t = \begin{pmatrix} -i & 1 \\ 1-i & i \end{pmatrix}$
 $A + (\bar{A})^t = \begin{pmatrix} i & 1+i \\ 1 & -i \end{pmatrix} + \begin{pmatrix} -i & 1 \\ 1-i & i \end{pmatrix} = \begin{pmatrix} 0 & 2+i \\ 2-i & 0 \end{pmatrix}$
 $\overline{(A + (\bar{A})^t)}^t = \begin{pmatrix} 0 & 2-i \\ 2+i & 0 \end{pmatrix}$
 $(\overline{(A + (\bar{A})^t)}^t)^t = \begin{pmatrix} 0 & 2+i \\ 2-i & 0 \end{pmatrix} = A + (\bar{A})^t$

Thus $A + (\bar{A})^t$ is Hermitian

ii) $A - (\bar{A})^t$ is skew Hermitian

To prove $\overline{(A - (\bar{A})^t)}^t = -(A - (\bar{A})^t)$
 $A - (\bar{A})^t = \begin{pmatrix} i & 1+i \\ 1 & -i \end{pmatrix} - \begin{pmatrix} -i & 1 \\ 1-i & i \end{pmatrix} = \begin{pmatrix} 2i & i \\ i & -2i \end{pmatrix}$
 $\overline{(A - (\bar{A})^t)}^t = \begin{pmatrix} -2i & -i \\ -i & 2i \end{pmatrix}$
 $(\overline{(A - (\bar{A})^t)}^t)^t = \begin{pmatrix} -2i & -i \\ -i & 2i \end{pmatrix} = -\begin{pmatrix} 2i & i \\ i & -2i \end{pmatrix} = -(A - (\bar{A})^t)$

Thus $A - (\bar{A})^t$ is skew Hermitian.

7. If A is symmetric or skew symmetric show that A^2 symmetric

Sol. If A is symmetric $A^t = A$

To prove $(A^2)^t = A^2$

$(A^2)^t = (A \cdot A)^t = A^t A^t = A A = A^2$

Thus A^2 is symmetric

If A is skew symmetric $A^t = -A$

$(A^2)^t = (A \cdot A)^t = A^t A^t = (-A)(-A) = A^2$

Thus A^2 is symmetric

8. If $A = \begin{pmatrix} 1 & -i \\ -i & i \end{pmatrix}$ $A(\bar{A})^t = ?$

Sol $\bar{A} = \begin{pmatrix} 1 & i \\ i & 1+i \end{pmatrix}$ $(\bar{A})^t = \begin{pmatrix} 1 & i \\ i & 1+i \end{pmatrix}$

$A(\bar{A})^t = \begin{pmatrix} 1 & -i \\ -i & i \end{pmatrix} \begin{pmatrix} 1 & i \\ i & 1+i \end{pmatrix}$

$A(\bar{A})^t = \begin{pmatrix} 1 & 1-i & -i \\ 1+i & 1-i^2 & -i-i^2 \\ i & i-i^2 & -i^2 \end{pmatrix}$
 $= \begin{pmatrix} 1 & 1-i & -i \\ 1+i & 2 & -i+1 \\ i & 1+i & 1 \end{pmatrix}$

9. Find the inverse by matrices, Row, Column

i) $A = \begin{pmatrix} 1 & 2 & -3 \\ 0 & -2 & 0 \\ -2 & -2 & 2 \end{pmatrix}$

Cofactors of A = $\begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{pmatrix}$

$A_{11} = (-1)^{1+1} \begin{vmatrix} -2 & 0 \\ 2 & 2 \end{vmatrix} = (-1)(-4-0) = -4$

$A_{12} = (-1)^{1+2} \begin{vmatrix} 0 & 0 \\ -2 & 2 \end{vmatrix} = (-1)(0+0) = 0$

$A_{13} = (-1)^{1+3} \begin{vmatrix} 0 & -2 \\ -2 & -2 \end{vmatrix} = (-1)(0-4) = -4$

$A_{21} = (-1)^{2+1} \begin{vmatrix} 2 & -3 \\ -2 & 2 \end{vmatrix} = (-1)(4-6) = 2$

$A_{22} = (-1)^{2+2} \begin{vmatrix} 1 & -3 \\ -2 & 2 \end{vmatrix} = (-1)(2-6) = -4$

$A_{23} = (-1)^{2+3} \begin{vmatrix} 1 & 2 \\ -2 & -2 \end{vmatrix} = (-1)(-2+4) = -2$

$A_{31} = (-1)^{3+1} \begin{vmatrix} 2 & -3 \\ -2 & 0 \end{vmatrix} = (-1)(0-6) = -6$

$A_{32} = (-1)^{3+2} \begin{vmatrix} 1 & -3 \\ 0 & 0 \end{vmatrix} = (-1)(0+0) = 0$

$A_{33} = (-1)^{3+3} \begin{vmatrix} 1 & 2 \\ 0 & -2 \end{vmatrix} = (-1)(-2-0) = -2$

Cofactors of A = $\begin{pmatrix} -4 & 0 & -4 \\ 2 & -4 & -2 \\ -6 & 0 & -2 \end{pmatrix}$

Adj A = (Cofactors of A)^t = $\begin{pmatrix} -4 & 2 & -6 \\ 0 & -4 & 0 \\ -4 & -2 & -2 \end{pmatrix}$

$|A| = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$

$= 1(-4) + 2(0) + (-3)(-4)$

$= -4 + 0 + 12 = 8$

$A^{-1} = \frac{1}{|A|} \text{Adj} A = \frac{1}{8} \begin{pmatrix} -4 & 2 & -6 \\ 0 & -4 & 0 \\ -4 & -2 & -2 \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} & \frac{1}{4} & -\frac{3}{4} \\ 0 & -\frac{1}{2} & 0 \\ -\frac{1}{2} & -\frac{1}{4} & -\frac{1}{4} \end{pmatrix}$

Find Inverse of A by Column operation

$$A = \begin{pmatrix} 1 & 2 & -3 \\ 0 & -2 & 0 \\ -2 & -2 & 2 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\sim C \begin{pmatrix} 1 & 0 & 0 \\ 0 & -2 & -4 \\ -2 & -2 & -4 \\ 1 & -2 & 3 \\ 0 & 0 & 1 \end{pmatrix} \begin{array}{l} \text{By} \\ C_2 - 2C_1 \rightarrow C_2' \\ C_3 + 2C_1 \rightarrow C_3' \end{array}$$

$$\sim C \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & -4 \\ -2 & -2 & -4 \\ 1 & -1 & 3 \\ 0 & -1/2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{array}{l} \text{By} \\ -\frac{1}{2}C_2 \rightarrow C_2' \end{array}$$

$$\sim C \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & -1 & -1 \\ -1 & -1 & -3/4 \\ 1 & -1/2 & 0 \\ 0 & 0 & -1/4 \end{pmatrix} \begin{array}{l} \text{By} \\ -\frac{1}{4}C_1 \rightarrow C_1' \end{array}$$

$$\sim C \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1/2 & 1/4 & -3/4 \\ 0 & -1/2 & 0 \\ -1/2 & -1/4 & -1/4 \end{pmatrix}$$

$$\begin{array}{l} \text{By} \\ C_1 + 2C_3 \rightarrow C_1' \\ C_2 + C_3 \rightarrow C_2' \end{array}$$

$$\begin{pmatrix} -1/2 & 1/4 & -3/4 \\ 0 & -1/2 & 0 \\ -1/2 & -1/4 & -1/4 \end{pmatrix}$$

Hence the inverse of matrix A is

10. Find the rank of the matrices

(i) $\begin{pmatrix} 1 & -1 & 2 & 1 \\ 2 & -6 & 5 & -3 \\ 0 & 5 & 4 & 1 \\ 0 & 2 & -2 & -1 \end{pmatrix}$

Sol. $\sim R \begin{pmatrix} 1 & -1 & 2 & 1 \\ 0 & -4 & -2 & -1 \\ 0 & 5 & 4 & 1 \\ 0 & 2 & -2 & -1 \end{pmatrix}$ By $R_2 - 2R_1 \rightarrow R_2'$
 $R_3 - 3R_1 \rightarrow R_3'$

$\sim R \begin{pmatrix} 1 & -1 & 2 & 1 \\ 0 & -1 & -2 & -1 \\ 0 & 5 & 4 & 1 \\ 0 & 2 & -2 & -1 \end{pmatrix}$ By $-\frac{1}{4}R_2 \rightarrow R_2'$, $\frac{1}{2}R_3 \rightarrow R_3'$

$\sim R \begin{pmatrix} 1 & 0 & 7/4 & 5/4 \\ 0 & 1 & -1/4 & 1/4 \\ 0 & 0 & 0 & -4 \end{pmatrix}$ By $R_1 + R_2 \rightarrow R_1'$
 $R_3 - 4R_2 \rightarrow R_3'$

$\sim R \begin{pmatrix} 1 & 0 & 7/4 & 5/4 \\ 0 & 1 & -1/4 & 1/4 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ By $-\frac{1}{4}R_3 \rightarrow R_3'$

Hence the rank of given matrix is 3

(ii) $\begin{pmatrix} 1 & -4 & -7 \\ 2 & -5 & 1 \\ 1 & -2 & 3 \\ 3 & -7 & 4 \end{pmatrix}$

$$\sim R \begin{pmatrix} 1 & -4 & -7 \\ 0 & 3 & 15 \\ 0 & 2 & 10 \\ 0 & 5 & 25 \end{pmatrix} \begin{array}{l} R_2 - 2R_1 \rightarrow R_2' \\ R_3 - R_1 \rightarrow R_3' \\ R_4 - 3R_1 \rightarrow R_4' \end{array} \quad R \begin{pmatrix} 1 & -4 & -7 \\ 0 & 1 & 5 \\ 0 & 2 & 10 \\ 0 & 1 & 5 \end{pmatrix} \begin{array}{l} R_2 - R_3 \rightarrow R_2' \\ \frac{1}{5} R_3 \rightarrow R_3' \end{array}$$

$$\sim R \begin{pmatrix} 1 & 0 & 13 \\ 0 & 1 & 5 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{array}{l} \text{By } R_1 + 4R_2 \rightarrow R_1' \\ R_2 - 2R_2 \rightarrow R_2' \\ R_1 - R_2 \rightarrow R_4' \end{array}$$

Hence the rank of given matrix is 2

(iii) $\begin{pmatrix} 3 & -1 & 3 & 0 & -1 \\ 1 & 2 & -1 & -3 & -2 \\ 2 & 3 & 4 & -2 & 5 \\ 2 & 5 & -2 & -3 & 3 \end{pmatrix}$

Sol. $\begin{pmatrix} 1 & -1 & 3 & 0 & -1 \\ 3 & 2 & -1 & -3 & -2 \\ 2 & 3 & 4 & -2 & 5 \\ 2 & 5 & -2 & -3 & 3 \end{pmatrix}$ By $R_1 \leftrightarrow R_2$

$$\sim R \begin{pmatrix} 1 & -1 & 3 & 0 & -1 \\ 0 & 2 & -1 & -3 & -2 \\ 0 & 6 & 9 & -3 & 5 \\ 0 & 6 & 8 & 3 & 7 \end{pmatrix} \begin{array}{l} \text{By } \\ R_2 - 3R_1 \rightarrow R_2' \\ R_3 - 2R_1 \rightarrow R_3' \\ R_4 - 2R_1 \rightarrow R_4' \end{array}$$

$$\sim R \begin{pmatrix} 1 & -1 & 3 & 0 & -1 \\ 0 & 2 & -1 & -3 & -2 \\ 0 & 0 & 6 & 9 & 5 \\ 0 & 0 & 6 & 9 & 5 \end{pmatrix} \text{By } R_2 \leftrightarrow R_4$$

$$\sim R \begin{pmatrix} 1 & -1 & 3 & 0 & -1 \\ 0 & 2 & -1 & -3 & -2 \\ 0 & 0 & 6 & 9 & 5 \\ 0 & 0 & 6 & 9 & 5 \end{pmatrix} \begin{array}{l} \text{By } \\ R_1 - 2R_2 \rightarrow R_1' \\ R_3 + R_2 \rightarrow R_3' \\ R_4 + 7R_2 \rightarrow R_4' \end{array}$$

$$\sim R \begin{pmatrix} 1 & 0 & 1 & -9 & -16 \\ 0 & 1 & 0 & 3 & 7 \\ 0 & 0 & 6 & 11 & 16 \\ 0 & 0 & 6 & 11 & 16 \end{pmatrix} \frac{1}{6} R_3 \rightarrow R_3'$$

$$\sim R \begin{pmatrix} 1 & 0 & 1 & -9 & -16 \\ 0 & 1 & 0 & 3 & 7 \\ 0 & 0 & 1 & 1 & 11 \\ 0 & 0 & 1 & 1 & 11 \end{pmatrix} R_3 \leftrightarrow R_4$$

$$\sim R \begin{pmatrix} 1 & 0 & 0 & -4 & -7 \\ 0 & 1 & 0 & 3 & 7 \\ 0 & 0 & 1 & 1 & 11 \\ 0 & 0 & 0 & -19 & -38 \end{pmatrix} \begin{array}{l} R_1 + R_3 \rightarrow R_1' \\ R_4 - 6R_3 \rightarrow R_4' \end{array}$$

$$\sim R \begin{pmatrix} 1 & 0 & 0 & -4 & -7 \\ 0 & 1 & 0 & 3 & 7 \\ 0 & 0 & 1 & 1 & 11 \\ 0 & 0 & 0 & -1 & -2 \end{pmatrix} -\frac{1}{19} R_4 \rightarrow R_4'$$

$$\sim R \begin{pmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 \end{pmatrix} \begin{array}{l} R_1 + 4R_4 \rightarrow R_1' \\ R_2 - 3R_4 \rightarrow R_2' \\ R_3 - 5R_4 \rightarrow R_3' \end{array}$$

Hence the rank of given matrix is 4.
 9 (ii) let $B = \begin{bmatrix} 1 & 2 & -\frac{1}{3} \\ 0 & -1 & \frac{3}{2} \\ 1 & 0 & 2 \end{bmatrix}$

Sol. Cofactors of $B = \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \end{bmatrix}$

$$B_{11} = (-1)^{1+1} \begin{vmatrix} -1 & \frac{3}{2} \\ 0 & 2 \end{vmatrix} = (-1)^2 (-2 - 0) = 1(-2) = -2$$

$$B_{12} = (-1)^{1+2} \begin{vmatrix} 0 & \frac{3}{2} \\ 1 & 2 \end{vmatrix} = (-1)^3 (0 - 3) = -1(-3) = 3$$

$$B_{13} = (-1)^{1+3} \begin{vmatrix} 0 & -1 \\ 1 & 0 \end{vmatrix} = (-1)^4 (0 + 1) = 1(1) = 1$$

$$B_{21} = (-1)^{2+1} \begin{vmatrix} 2 & -\frac{1}{3} \\ 0 & 2 \end{vmatrix} = (-1)^3 (4 + 0) = -1(4) = -4$$

$$B_{22} = (-1)^{2+2} \begin{vmatrix} 1 & -\frac{1}{3} \\ 1 & 2 \end{vmatrix} = (-1)^4 (2 + 1) = 1(3) = 3$$

$$B_{23} = (-1)^{2+3} \begin{vmatrix} 1 & \frac{2}{0} \\ 1 & 2 \end{vmatrix} = (-1)^5 (0 - 2) = -1(-2) = 2$$

$$B_{31} = (-1)^{3+1} \begin{vmatrix} 2 & -\frac{1}{3} \\ -1 & \frac{3}{2} \end{vmatrix} = (-1)^4 (6 - 1) = 1(5) = 5$$

$$B_{32} = (-1)^{3+2} \begin{vmatrix} 1 & -\frac{1}{3} \\ 0 & \frac{3}{2} \end{vmatrix} = (-1)^5 (3 + 0) = -1(3) = -3$$

$$B_{33} = (-1)^{3+3} \begin{vmatrix} 1 & \frac{2}{0} \\ 0 & -1 \end{vmatrix} = (-1)^6 (-1 - 0) = 1(-1) = -1$$

$$\text{Cofactors of } B = \begin{bmatrix} -2 & 3 & 1 \\ -4 & 3 & 2 \\ 5 & -3 & -1 \end{bmatrix}$$

$$\text{Adj } B = (\text{Cofactors of } B)^t = \begin{bmatrix} -2 & -4 & 5 \\ 3 & 3 & -3 \\ 1 & 2 & -1 \end{bmatrix}$$

$$|B| = \begin{vmatrix} 1 & 2 & -\frac{1}{3} \\ 0 & -1 & \frac{3}{2} \\ 1 & 0 & 2 \end{vmatrix} = 1(-2 - 0) - 2(0 - 3) - 1(0 + 1)$$

$$= 1(-2) - 2(-3) - 1(1) = -2 + 6 - 1 = 3$$

$$B^{-1} = \frac{1}{|B|} \text{Adj } B = \frac{1}{3} \begin{bmatrix} -2 & -4 & 5 \\ 3 & 3 & -3 \\ 1 & 2 & -1 \end{bmatrix} = \begin{bmatrix} -2/3 & -4/3 & 5/3 \\ 1/3 & 2/3 & -1/3 \end{bmatrix}$$

Now B^{-1} By Row operations

$$B = \begin{bmatrix} 1 & 2 & -\frac{1}{3} & | & 1 & 0 & 0 \\ 0 & -1 & \frac{3}{2} & | & 0 & 1 & 0 \\ 1 & 0 & 2 & | & 0 & 0 & 1 \end{bmatrix}$$

$$\sim B \begin{bmatrix} 1 & 2 & -\frac{1}{3} & | & 1 & 0 & 0 \\ 0 & -1 & \frac{3}{2} & | & 0 & 1 & 0 \\ 0 & -2 & \frac{3}{3} & | & -1 & 0 & 1 \end{bmatrix} \quad R_3 - R_1 \rightarrow R_3$$

$$\sim R \begin{bmatrix} 1 & 2 & -\frac{1}{3} & | & 1 & 0 & 0 \\ 0 & -1 & \frac{3}{2} & | & 0 & 1 & 0 \\ 0 & -2 & 3 & | & -1 & 0 & 1 \end{bmatrix} \quad -R_2 \rightarrow R_2$$

$$\sim R \begin{bmatrix} 1 & 0 & 5 & | & 1 & 2 & 0 \\ 0 & -1 & -\frac{3}{2} & | & 0 & -1 & 0 \\ 0 & 0 & -\frac{3}{2} & | & -1 & 2 & 1 \end{bmatrix} \quad \begin{array}{l} R_1 - 2R_2 \rightarrow R_1 \\ R_3 + 2R_2 \rightarrow R_3 \end{array}$$

$$\sim R \begin{bmatrix} 1 & 0 & 5 & | & 1 & 2 & 0 \\ 0 & -1 & -\frac{3}{2} & | & 0 & -1 & 0 \\ 0 & 0 & -\frac{3}{2} & | & -1 & 2 & 1 \end{bmatrix} \quad -\frac{1}{3}R_3 \rightarrow R_3$$
