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§ 3.2 FIELD: A non empty set F is called a field if

- (i) F is an abelian group under '+'
- (ii) $F - \{0\}$ an abelian group under multiplication
- (iii) Right distributive law holds in F

e.g. Set of real numbers R and set of complex numbers C are fields

EXERCISE 3.2

1. If $A = [a_{ij}]_{3 \times 4}$, show that (i) $I_3 A = A$, (ii) $A I_4 = A$.

Solution. $I_3 A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$

$$= \begin{bmatrix} a_{11} + 0 + 0 & a_{12} + 0 + 0 & a_{13} + 0 + 0 & a_{14} + 0 + 0 \\ 0 + a_{21} + 0 & 0 + a_{22} + 0 & 0 + a_{23} + 0 & 0 + a_{24} + 0 \\ 0 + 0 + a_{31} & 0 + 0 + a_{32} & 0 + 0 + a_{33} & 0 + 0 + a_{34} \end{bmatrix}$$

$$= \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix} = A$$

(ii) $A I_4 = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} a_{11}+0+0+0 & 0+a_{12}+0+0 & 0+0+a_{13}+0 & 0+0+0+a_{14} \\ a_{21}+0+0+0 & 0+a_{22}+0+0 & 0+0+a_{23}+0 & 0+0+0+a_{24} \\ a_{31}+0+0+0 & 0+a_{32}+0+0 & 0+0+a_{33}+0 & 0+0+0+a_{34} \end{bmatrix}$$

$$= \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix} = A$$

2. Find the inverse of the following matrices :

(i) $\begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix}$ (ii) $\begin{bmatrix} -2 & 3 \\ -4 & 5 \end{bmatrix}$ (iii) $\begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$ (iv) $\begin{bmatrix} 2 & 1 \\ 6 & 3 \end{bmatrix}$

Solution. Note that if $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $\Rightarrow |A| = ad - bc$;

$$\text{Adj } A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \text{ and } A^{-1} = \frac{1}{|A|} \text{adj. } A$$

(i) Let $A = \begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix} \Rightarrow |A| = 3 - (-2) = 5$; $\text{Adj } A = \begin{bmatrix} 1 & 1 \\ -2 & 3 \end{bmatrix}$

$$\therefore A^{-1} = \frac{1}{|A|} \text{adj. } A = \frac{1}{5} \begin{bmatrix} 1 & 1 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 1/5 & 1/5 \\ -2/5 & 3/5 \end{bmatrix}$$

(ii) Let $B = \begin{bmatrix} -2 & 3 \\ -4 & 5 \end{bmatrix} \Rightarrow |B| = -10 + 12 = 2$; $\text{Adj } B = \begin{bmatrix} 5 & -3 \\ 4 & -2 \end{bmatrix}$

$$\therefore B^{-1} = \frac{1}{|B|} \text{adj. } B = \frac{1}{2} \begin{bmatrix} 5 & -3 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 5/2 & -3/2 \\ 4/2 & -2/2 \end{bmatrix} = \begin{bmatrix} 5/2 & -3/2 \\ 2 & -1 \end{bmatrix}$$

(iii) Let $C = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix} \Rightarrow |C| = -2i^2 - i^2 = -3i^2 = 3$; $\text{Adj } C = \begin{bmatrix} -i & -i \\ -i & 2i \end{bmatrix}$

$$\therefore C^{-1} = \frac{1}{|C|} \text{adj. } C = \frac{1}{3} \begin{bmatrix} -i & -i \\ -i & 2i \end{bmatrix} = \begin{bmatrix} -i/3 & -i/3 \\ -i/3 & 2i/3 \end{bmatrix}$$

(iv) $D = \begin{bmatrix} 2 & 1 \\ 6 & 3 \end{bmatrix} \Rightarrow |D| = 6 - 6 = 0$, as $|D| = 0$ so D^{-1} does not exist.

3. Solve the following system of linear equations :

(i) $2x_1 - 3x_2 = 5$ (ii) $4x_1 + 3x_2 = 5$ (iii) $3x - 5y = 1$

$$5x_1 + x_2 = 4 \quad 3x_1 - x_2 = 7 \quad -2x + y = -3$$

Solution. (i) Writing in matrix form, we get

$$\begin{bmatrix} 2 & -3 \\ 5 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$$

$$\Rightarrow AX = B \Rightarrow X = A^{-1}B \dots (1)$$

Here $A = \begin{bmatrix} 2 & -3 \\ 5 & 1 \end{bmatrix} \Rightarrow |A| = \begin{vmatrix} 2 & -3 \\ 5 & 1 \end{vmatrix} = 2 + 15 = 17 \neq 0$

$$\text{So } A^{-1} \text{ exists } \therefore A^{-1} = \frac{1}{|A|} \text{adj. } A = \frac{1}{17} \begin{bmatrix} 1 & 3 \\ -5 & 2 \end{bmatrix}$$

$$\text{From (1) } X = A^{-1}B = \frac{1}{17} \begin{bmatrix} 1 & 3 \\ -5 & 2 \end{bmatrix} \begin{bmatrix} 5 \\ 4 \end{bmatrix} = \frac{1}{17} \begin{bmatrix} 5 + 12 \\ -25 + 8 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \frac{1}{17} \begin{bmatrix} 17 \\ -17 \end{bmatrix} = \begin{bmatrix} 17/17 \\ -17/17 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\therefore \boxed{x_1 = 1, x_2 = -1}$$

(ii) $4x_1 + 3x_2 = 5$, $3x_1 - x_2 = 7$

Writing in matrix form, we get

$$\begin{bmatrix} 4 & 3 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \end{bmatrix}$$

$$\Rightarrow AX = B \Rightarrow X = A^{-1}B \dots (1)$$

Here $A = \begin{bmatrix} 4 & 3 \\ 3 & -1 \end{bmatrix} \Rightarrow |A| = \begin{vmatrix} 4 & 3 \\ 3 & -1 \end{vmatrix} = -4 - 9 = -13$

$$A^{-1} = \frac{1}{|A|} \text{adj. } A = \frac{1}{-13} \begin{bmatrix} -1 & -3 \\ -3 & 4 \end{bmatrix}$$

$$\text{From (1) } X = A^{-1}B = \frac{1}{-13} \begin{bmatrix} -1 & -3 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} 5 \\ 7 \end{bmatrix} = \frac{1}{-13} \begin{bmatrix} -5 - 21 \\ -15 + 28 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \frac{1}{-13} \begin{bmatrix} -26 \\ 13 \end{bmatrix} = \begin{bmatrix} -26/-13 \\ 13/-13 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

$$\therefore \boxed{x_1 = 2, x_2 = -1}$$

(iii) $3x - 5y = 1$, $-2x + y = -3$

Writing in matrix form, we get

$$\begin{bmatrix} 3 & -5 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

$$\Rightarrow AX = B \Rightarrow X = A^{-1}B \dots (1)$$

$$\text{where } A = \begin{bmatrix} 3 & -5 \\ -2 & 1 \end{bmatrix} \Rightarrow |A| = \begin{vmatrix} 3 & -5 \\ -2 & 1 \end{vmatrix} = 3 - 10 = -7$$

$$\therefore A^{-1} = \frac{1}{|A|} \text{adj. } A = \frac{1}{-7} \begin{bmatrix} 1 & 5 \\ 2 & 3 \end{bmatrix}$$

$$\text{From (1)} \quad X = A^{-1}B = \frac{1}{-7} \begin{bmatrix} 1 & 5 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ -3 \end{bmatrix} = \frac{1}{-7} \begin{bmatrix} 1 - 15 \\ 2 - 9 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \frac{1}{-7} \begin{bmatrix} -14 \\ -7 \end{bmatrix} = \begin{bmatrix} -14/-7 \\ -7/-7 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\therefore \boxed{x = 2, y = 1}$$

$$4. \text{ If } A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 2 & 5 \\ -1 & 0 & 4 \end{bmatrix}, B = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 3 & 4 \\ -1 & 2 & 1 \end{bmatrix} \text{ and } C = \begin{bmatrix} 1 & 3 & -2 \\ -1 & 2 & 0 \\ 3 & 4 & -1 \end{bmatrix}$$

Find (i) $A - B$ (ii) $B - A$ (iii) $(A - B) - C$ (iv) $A - (B - C)$.

$$\text{Solution. (i) } A - B = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 2 & 5 \\ -1 & 0 & 4 \end{bmatrix} - \begin{bmatrix} 2 & 1 & -1 \\ 1 & 3 & 4 \\ -1 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1-2 & -1-1 & 2+1 \\ 3-1 & 2-3 & 5-4 \\ -1+1 & 0-2 & 4-1 \end{bmatrix} = \begin{bmatrix} -1 & -2 & 3 \\ 2 & -1 & 1 \\ 0 & -2 & 3 \end{bmatrix}$$

$$\text{(ii) } B - A = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 3 & 4 \\ -1 & 2 & 1 \end{bmatrix} - \begin{bmatrix} 1 & -1 & 2 \\ 3 & 2 & 5 \\ -1 & 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2-1 & 1+1 & -1-2 \\ 1-3 & 3-2 & 4-5 \\ -1+1 & 2-0 & 1-4 \end{bmatrix} = \begin{bmatrix} 1 & 2 & -3 \\ -2 & 1 & -1 \\ 0 & 2 & -3 \end{bmatrix}$$

$$\text{(iii) } (A - B) - C = \begin{bmatrix} -1 & -2 & 3 \\ 2 & -1 & 1 \\ 0 & -2 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 3 & -2 \\ -1 & 2 & 0 \\ 3 & 4 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -1-1 & -2-3 & 3+2 \\ 2+1 & -1-2 & 1-0 \\ 0-3 & -2-4 & 3+1 \end{bmatrix} = \begin{bmatrix} -2 & -5 & 5 \\ 3 & -3 & 1 \\ -3 & -6 & 4 \end{bmatrix}$$

$$(iv) \text{ Now } B - C = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 3 & 4 \\ -1 & 2 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 3 & -2 \\ -1 & 2 & 0 \\ 3 & 4 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 2-1 & 1-3 & -1+2 \\ 1+1 & 3-2 & 4-0 \\ -1-3 & 2-4 & 1+1 \end{bmatrix} = \begin{bmatrix} 1 & -2 & 1 \\ 2 & 1 & 4 \\ -4 & -2 & 2 \end{bmatrix}$$

$$\therefore A - (B - C) = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 2 & 5 \\ -1 & 0 & 4 \end{bmatrix} - \begin{bmatrix} 1 & -2 & 1 \\ 2 & 1 & 4 \\ -4 & -2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1-1 & -1+2 & 2-1 \\ 3-2 & 2-1 & 5-4 \\ -1+4 & 0+2 & 4-2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 3 & 2 & 2 \end{bmatrix}$$

5. If $A = \begin{bmatrix} i & 2i \\ 1 & -i \end{bmatrix}$, $B = \begin{bmatrix} -i & 1 \\ 2i & i \end{bmatrix}$ and $C = \begin{bmatrix} 2i & -1 \\ -i & i \end{bmatrix}$, then show that

(i) $(AB)C = A(BC)$ (ii) $(A+B)C = AC + BC$.

$$\text{Solution. (i) } AB = \begin{bmatrix} i & 2i \\ 1 & -i \end{bmatrix} \cdot \begin{bmatrix} -i & 1 \\ 2i & i \end{bmatrix} = \begin{bmatrix} -i^2 + 4i^2 & i + 2i^2 \\ -i - 2i^2 & 1 - i^2 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & i - 2 \\ 2 - i & 2 \end{bmatrix} \quad \{ \text{by using } i^2 = -1 \}$$

$$\therefore (AB)C = \begin{bmatrix} -3 & i - 2 \\ 2 - i & 2 \end{bmatrix} \begin{bmatrix} 2i & -1 \\ -i & i \end{bmatrix} \\ = \begin{bmatrix} -6i + 2i - i^2 & 3 - 2i + i^2 \\ 4i - 2i^2 - 2i & -2 + i + 2i \end{bmatrix} = \begin{bmatrix} 1 - 4i & 2 - 2i \\ 2 + 2i & -2 + 3i \end{bmatrix} \dots (1)$$

$$\text{Again, } BC = \begin{bmatrix} -i & 1 \\ 2i & i \end{bmatrix} \cdot \begin{bmatrix} 2i & -1 \\ -i & i \end{bmatrix} = \begin{bmatrix} -2i^2 - i & i + i \\ 4i^2 - i^2 & -2i + i^2 \end{bmatrix}$$

$$= \begin{bmatrix} 2-i & 2i \\ -3 & -1-2i \end{bmatrix} \quad \{ \text{by using } i^2 = -1 \}$$

$$\therefore A(BC) = \begin{bmatrix} i & 2i \\ 1 & -i \end{bmatrix} \begin{bmatrix} 2-i & 2i \\ -3 & -1-2i \end{bmatrix}$$

$$= \begin{bmatrix} 2i - i^2 - 6i & 2i^2 - 2i - 4i^2 \\ 2-i+3i & 2i+i+2i^2 \end{bmatrix}$$

$$= \begin{bmatrix} 1-4i & 2-2i \\ 2+2i & -2+3i \end{bmatrix} \quad \{ \text{by using } i^2 = -1 \} \quad \dots(2)$$

From (1) and (2), we have $(AB)C = A(BC)$.

(ii) $(A+B)C = AC + BC$.

$$\text{Now } A+B = \begin{bmatrix} i & 2i \\ 1 & -i \end{bmatrix} + \begin{bmatrix} -i & 1 \\ 2i & i \end{bmatrix} = \begin{bmatrix} 0 & 1+2i \\ 1+2i & 0 \end{bmatrix}$$

$$(A+B)C = \begin{bmatrix} 0 & 1+2i \\ 1+2i & 0 \end{bmatrix} \begin{bmatrix} 2i & -1 \\ -i & i \end{bmatrix}$$

$$= \begin{bmatrix} 0-i-2i^2 & 0+i+2i^2 \\ 2i+4i^2+0 & -1-2i+0 \end{bmatrix} = \begin{bmatrix} -i-2i^2 & i+2i^2 \\ 2i+4i^2 & -1-2i \end{bmatrix}$$

$$= \begin{bmatrix} 2-i & i-2 \\ 2i-4 & -1-2i \end{bmatrix} \quad \dots (1) \quad \{ \text{by using } i^2 = -1 \}$$

$$\text{Also } AC = \begin{bmatrix} i & 2i \\ 1 & -i \end{bmatrix} \begin{bmatrix} 2i & -1 \\ -i & i \end{bmatrix} = \begin{bmatrix} 2i^2 - 2i^2 & -i + 2i^2 \\ 2i + i^2 & -1 - i^2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -2-i \\ -1+2i & 0 \end{bmatrix} \quad \{ \text{by using } i^2 = -1 \}$$

$$\text{and } BC = \begin{bmatrix} -i & 1 \\ 2i & i \end{bmatrix} \begin{bmatrix} 2i & -1 \\ -i & i \end{bmatrix} = \begin{bmatrix} -2i^2 - i & i+i \\ 4i^2 - i^2 & -2i+i^2 \end{bmatrix}$$

$$= \begin{bmatrix} 2-i & 2i \\ -3 & -1-2i \end{bmatrix} \quad \{ \text{by using } i^2 = -1 \}$$

$$\therefore AC + BC = \begin{bmatrix} 0 & -2-i \\ -1+2i & 0 \end{bmatrix} + \begin{bmatrix} 2-i & 2i \\ -3 & -1-2i \end{bmatrix}$$

$$= \begin{bmatrix} 2-i & -2+i \\ -4+2i & -1-2i \end{bmatrix} \quad \dots (2)$$

From (1) and (2), we have $(A+B)C = AC + BC$

6. If A and B are square matrices of the same order, then explain why, in general

$$(i) (A + B)^2 \neq A^2 + 2AB + B^2 \quad (ii) (A - B)^2 \neq A^2 - 2AB + B^2$$

$$(iii) (A + B)(A - B) \neq A^2 - B^2.$$

Solution.

$$(i) \text{ Now } (A + B)^2 = (A + B)(A + B) = AA + AB + BA + BB$$

$$= A^2 + AB + BA + B^2 \neq A^2 + AB + AB + B^2$$

$$(A + B)^2 \neq A^2 + 2AB + B^2 \quad \text{since, in general, } BA \neq AB$$

$$(ii) \text{ Now } (A - B)^2 = (A - B)(A - B) = AA - AB - BA + BB$$

$$= A^2 - AB - BA + B^2 \neq A^2 - AB - AB + B^2$$

$$(A - B)^2 \neq A^2 - 2AB + B^2 \quad \text{since, in general, } BA \neq AB$$

$$(iii) (A + B)(A - B) = AA - AB + BA - BB$$

$$\neq A^2 - AB + AB + B^2$$

$$(A + B)(A - B) \neq A^2 - B^2 \quad \text{since, in general, } BA \neq AB.$$

7. If $A = \begin{bmatrix} 2 & -1 & 3 & 0 \\ 1 & 0 & 4 & -2 \\ -3 & 5 & 2 & -1 \end{bmatrix}$, then find AA^t and A^tA .

Solution. Now $A^t = \begin{bmatrix} 2 & 1 & -3 \\ -1 & 0 & 5 \\ 3 & 4 & 2 \\ 0 & -2 & -1 \end{bmatrix}$

$$\therefore AA^t = \begin{bmatrix} 2 & -1 & 3 & 0 \\ 1 & 0 & 4 & -2 \\ -3 & 5 & 2 & -1 \end{bmatrix} \begin{bmatrix} 2 & 1 & -3 \\ -1 & 0 & 5 \\ 3 & 4 & 2 \\ 0 & -2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 4+1+9+0 & 2+0+12-0 & -6-5+6+0 \\ 2+0+12+0 & 1+0+16+4 & -3+0+8+2 \\ -6-5+6-0 & -3+0+8+2 & 9+25+4+1 \end{bmatrix} = \begin{bmatrix} 14 & 14 & -5 \\ 14 & 21 & 7 \\ -5 & 7 & 39 \end{bmatrix}$$

$$\text{Also } A^tA = \begin{bmatrix} 2 & 1 & -3 \\ -1 & 0 & 5 \\ 3 & 4 & 2 \\ 0 & -2 & -1 \end{bmatrix} \begin{bmatrix} 2 & -1 & 3 & 0 \\ 1 & 0 & 4 & -2 \\ -3 & 5 & 2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 4+1+9 & -2+0-15 & 6+4-6 & 0-2+3 \\ -2+0-15 & 1+0+25 & -3+0+10 & 0+0-5 \\ 6+4-6 & -3+0+10 & 9+16+4 & 0-8-2 \\ 0-2+3 & 0+0-5 & 0-8-2 & 0+4+1 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & -17 & 4 & 1 \\ -17 & 26 & 7 & -5 \\ 4 & 7 & 29 & -10 \\ 1 & -5 & -10 & 5 \end{bmatrix}$$

8. Solve the following matrix equations for X:

(i) $3X - 2A = B$ if $A = \begin{bmatrix} 2 & 3 & -2 \\ -1 & 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -3 & 1 \\ 5 & 4 & -1 \end{bmatrix}$

Solution. (i) $3X = 2A + B = 2 \begin{bmatrix} 2 & 3 & -2 \\ -1 & 1 & 5 \end{bmatrix} + \begin{bmatrix} 2 & -3 & 1 \\ 5 & 4 & -1 \end{bmatrix}$

$$3X = \begin{bmatrix} 4 & 6 & -4 \\ -2 & 2 & 10 \end{bmatrix} + \begin{bmatrix} 2 & -3 & 1 \\ 5 & 4 & -1 \end{bmatrix} = \begin{bmatrix} 6 & 3 & -3 \\ 3 & 6 & 9 \end{bmatrix}$$

$$\therefore X = \frac{1}{3} \begin{bmatrix} 6 & 3 & -3 \\ 3 & 6 & 9 \end{bmatrix} = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 2 & 3 \end{bmatrix}$$

(ii) (ii) $2X - 3A = B$ if $A = \begin{bmatrix} 1 & -1 & 2 \\ -2 & 4 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix}$

$$2X = 3A + B = 3 \begin{bmatrix} 1 & -1 & 2 \\ -2 & 4 & 5 \end{bmatrix} + \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix}$$

$$2X = \begin{bmatrix} 3 & -3 & 6 \\ -6 & 12 & 15 \end{bmatrix} + \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 6 & -4 & 6 \\ -2 & 14 & 16 \end{bmatrix}$$

$$\therefore X = \frac{1}{2} \begin{bmatrix} 6 & -4 & 6 \\ -2 & 14 & 16 \end{bmatrix} = \begin{bmatrix} 3 & -2 & 3 \\ -1 & 7 & 8 \end{bmatrix}$$

9. Solve the following matrix equations for A:

(i) $\begin{bmatrix} 4 & 3 \\ 2 & 2 \end{bmatrix} A - \begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} -1 & -4 \\ 3 & 6 \end{bmatrix}$

Solution. (i) $\begin{bmatrix} 4 & 3 \\ 2 & 2 \end{bmatrix} A = \begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix} + \begin{bmatrix} -1 & -4 \\ 3 & 6 \end{bmatrix} = \begin{bmatrix} 2-1 & 3-4 \\ -1+3 & -2+6 \end{bmatrix}$

i.e. $\begin{bmatrix} 4 & 3 \\ 2 & 2 \end{bmatrix} A = \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix}$

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $\begin{bmatrix} 4 & 3 \\ 2 & 2 \end{bmatrix} A = \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix}$

gives $\begin{bmatrix} 4 & 3 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix}$

or $\begin{bmatrix} 4a + 3c & 4b + 3d \\ 2a + 2c & 2b + 2d \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix}$

$\Rightarrow 4a + 3c = 1 \dots (1), \quad 4b + 3d = -1 \dots (2)$

$2a + 2c = 2 \dots (3), \quad 2b + 2d = 4 \dots (4)$

Eqns. (1) and (3) give $4a + 3c = 1 \dots (1)$ and $a + c = 1 \dots (3)$

To find a and c , put $c = 1 - a$ in (3), then $4a + 3(1 - a) = 1 \Rightarrow a = -2$

Then $c = 1 - a = 1 - (-2) = 1 + 2 = 3$

Eqns. (2) and (4) give

$4b + 3d = -1 \dots (2)$ and $b + d = 2 \dots (4)$

To find b and d , put $d = 2 - b$ from (4) in (2), then

$4b + 3(2 - b) = -1 \Rightarrow 4b - 3b = -1 - 6 \Rightarrow b = -7$

Then $d = 2 - b = 2 - (-7) = 2 + 7 = 9$

Hence $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} -2 & -7 \\ 3 & 9 \end{bmatrix}$

(ii) $A \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} - \begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ -1 & 5 \end{bmatrix}$

$A \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ -1 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 6 \end{bmatrix}$

$\therefore A \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 6 \end{bmatrix}$

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $A \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 6 \end{bmatrix}$ gives

$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 6 \end{bmatrix}$

$$\begin{bmatrix} 3a + 4b & a + 2b \\ 3c + 4d & c + 2d \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 6 \end{bmatrix}$$

$$\Rightarrow 3a + 4b = 1 \quad \dots (1), \quad a + 2b = 2 \quad \dots (2)$$

$$3c + 4d = 2 \quad \dots (3), \quad c + 2d = 6 \quad \dots (4)$$

To find a and b , put $a = 2 - 2b$ from (2) in (1), then

$$3(2 - 2b) + 4b = 1 \Rightarrow 6 - 6b + 4b = 1 \Rightarrow -2b = -5 \Rightarrow b = \frac{5}{2}$$

$$\text{Then } a = 2 - 2b = 2 - 2 \left(\frac{5}{2} \right) = 2 - 5 = -3$$

To find c and d , put $c = 6 - 2d$ from (4) in (3), then

$$3(6 - 2d) + 4d = 2 \Rightarrow 18 - 6d + 4d = 2 \Rightarrow -2d = -16 \Rightarrow d = 8$$

$$\text{Then } c = 6 - 2d = 6 - 2(8) = 6 - 16 = -10$$

$$\text{Hence } A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} -3 & 5/2 \\ -10 & 8 \end{bmatrix}$$

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