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**EXERCISE 3.1**

1. If  $A = \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}$ , and  $B = \begin{bmatrix} 1 & 7 \\ 6 & 4 \end{bmatrix}$ , then show that

(i)  $4A - 3A = A$  (ii)  $3B - 3A = 3(B - A)$ .

**Solution.**  $A = \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix} \Rightarrow 4A = 4 \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 2(4) & 3(4) \\ 1(4) & 5(4) \end{bmatrix} = \begin{bmatrix} 8 & 12 \\ 4 & 20 \end{bmatrix}$

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix} \Rightarrow 3A = 3 \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 2(3) & 3(3) \\ 1(3) & 5(3) \end{bmatrix} = \begin{bmatrix} 6 & 9 \\ 3 & 15 \end{bmatrix}$$

$$4A - 3A = \begin{bmatrix} 8 & 12 \\ 4 & 20 \end{bmatrix} - \begin{bmatrix} 6 & 9 \\ 3 & 15 \end{bmatrix} = \begin{bmatrix} 8-6 & 12-9 \\ 4-3 & 20-15 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix} = A$$

(ii) Now  $3B = 3 \begin{bmatrix} 1 & 7 \\ 6 & 4 \end{bmatrix} = \begin{bmatrix} 1(3) & 7(3) \\ 6(3) & 4(3) \end{bmatrix} = \begin{bmatrix} 3 & 21 \\ 18 & 12 \end{bmatrix}$

$$\therefore 3B - 3A = \begin{bmatrix} 3 & 21 \\ 18 & 12 \end{bmatrix} - \begin{bmatrix} 6 & 9 \\ 3 & 15 \end{bmatrix} = \begin{bmatrix} 3-6 & 21-9 \\ 18-3 & 12-15 \end{bmatrix} = \begin{bmatrix} -3 & 12 \\ 15 & -3 \end{bmatrix} \dots (1)$$

$$\begin{aligned} 3(B-A) &= 3 \left\{ \begin{bmatrix} 1 & 7 \\ 6 & 4 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix} \right\} = 3 \begin{bmatrix} 1-2 & 7-3 \\ 6-1 & 4-5 \end{bmatrix} = 3 \begin{bmatrix} -1 & 4 \\ 5 & -1 \end{bmatrix} \\ &= \begin{bmatrix} -1(3) & 4(3) \\ 5(3) & -1(3) \end{bmatrix} = \begin{bmatrix} -3 & 12 \\ 15 & -3 \end{bmatrix} \dots (2) \end{aligned}$$

From (1) and (2), we get :  $3B - 3A = 3(B - A)$ .

2. If  $A = \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix}$ , show that  $A^4 = I_2$ .

$$\begin{aligned} \text{Solution. } A &= \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix} \Rightarrow A^2 = A.A = \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix} \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix} \\ &= \begin{bmatrix} i(i)+0(1) & i(0)+0(-i) \\ 1(i)+(-i)(1) & 1(0)+(-i)(-i) \end{bmatrix} = \begin{bmatrix} i^2 & 0 \\ 0 & i^2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} A^4 &= A^2 . A^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \\ &= \begin{bmatrix} (-1)(-1)+0(0) & (-1)(0)+0(-1) \\ 0(-1)+(-1)(0) & 0(0)+(-1)(-1) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2 \end{aligned}$$

3. Find  $x$  and  $y$  if

$$\text{Solution. (i)} \quad \begin{bmatrix} x+3 & 1 \\ -3 & 3y-4 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$$

Using definition of equality of two matrices, we have

$$x+3 = 2 \Rightarrow x = 2-3 = -1$$

$$\text{and } 3y-4 = 2 \Rightarrow 3y = 6 \Rightarrow y = 2 \quad \therefore \boxed{x = -1, y = 2}$$

$$\text{Solution. (ii)} \quad \begin{bmatrix} x+3 & 1 \\ -3 & 3y-4 \end{bmatrix} = \begin{bmatrix} y & 1 \\ -3 & 2x \end{bmatrix}$$

Using definition of equality of two matrices, we have

$$\begin{bmatrix} x+3 & 1 \\ -3 & 3y-4 \end{bmatrix} = \begin{bmatrix} y & 1 \\ -3 & 2x \end{bmatrix} \text{ implies}$$

$$x+3 = y \qquad 3y-4 = 2x$$

$$\text{or } x - y = -3 \quad \dots (1), \qquad 2x - 3y = -4 \quad \dots (2)$$

Multiplying (1) by 3 gives  $3x - 3y = -9 \dots (3)$

Subtracting (2) from (3), we get :  $x = -5$ ,

put in (1) then  $y = x + 3 = -5 + 3 = -2$

$$\therefore \boxed{x = -5, y = -2}$$

4. If  $A = \begin{bmatrix} -1 & 2 & 3 \\ 1 & 0 & 2 \end{bmatrix}$  and  $B = \begin{bmatrix} 0 & 3 & 2 \\ 1 & -1 & 2 \end{bmatrix}$ , find the following :

(i)  $4A - 3B$       (ii)  $A + 3(B - A)$ .

**Solution.** (i)  $4A = 4 \begin{bmatrix} -1 & 2 & 3 \\ 1 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 4(-1) & 4(2) & 4(3) \\ 4(1) & 4(0) & 4(2) \end{bmatrix} = \begin{bmatrix} -4 & 8 & 12 \\ 4 & 0 & 8 \end{bmatrix}$

$$3B = 3 \begin{bmatrix} 0 & 3 & 2 \\ 1 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 3(0) & 3(3) & 3(2) \\ 3(1) & 3(-1) & 3(2) \end{bmatrix} = \begin{bmatrix} 0 & 9 & 6 \\ 3 & -3 & 6 \end{bmatrix}$$

$$\therefore 4A - 3B = \begin{bmatrix} -4 & 8 & 12 \\ 4 & 0 & 8 \end{bmatrix} - \begin{bmatrix} 0 & 9 & 6 \\ 3 & -3 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} -4-0 & 8-9 & 12-6 \\ 4-3 & 0-(-3) & 8-6 \end{bmatrix} = \begin{bmatrix} -4 & -1 & 6 \\ 1 & 3 & 2 \end{bmatrix}$$

(ii)  $B - A = \begin{bmatrix} 0 & 3 & 2 \\ 1 & -1 & 2 \end{bmatrix} - \begin{bmatrix} -1 & 2 & 3 \\ 1 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 0-(-1) & 3-2 & 2-3 \\ 1-1 & -1-0 & 2-2 \end{bmatrix}$

$$= \begin{bmatrix} 1 & 1 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$A + 3(B - A) = \begin{bmatrix} -1 & 2 & 3 \\ 1 & 0 & 2 \end{bmatrix} + 3 \begin{bmatrix} 1 & 1 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 2 & 3 \\ 1 & 0 & 2 \end{bmatrix} + \begin{bmatrix} 3(1) & 3(1) & 3(-1) \\ 3(0) & 3(-1) & 3(0) \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 2 & 3 \\ 1 & 0 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 3 & -3 \\ 0 & -3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1+3 & 2+3 & 3-3 \\ 1+0 & 0-3 & 2+0 \end{bmatrix} = \begin{bmatrix} 2 & 5 & 0 \\ 1 & -3 & 2 \end{bmatrix}$$

5. Find  $x$  and  $y$  if  $\begin{bmatrix} 2 & 0 & x \\ 1 & y & 3 \end{bmatrix} + 2 \begin{bmatrix} 1 & x & y \\ 0 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 4 & -2 & 3 \\ 1 & 6 & 1 \end{bmatrix}$ .

**Solution.**  $\begin{bmatrix} 2 & 0 & x \\ 1 & y & 3 \end{bmatrix} + \begin{bmatrix} 2 & 2x & 2y \\ 0 & 4 & -2 \end{bmatrix} = \begin{bmatrix} 4 & -2 & 3 \\ 1 & 6 & 1 \end{bmatrix}$

$$\begin{bmatrix} 2+2 & 0+2x & x+2y \\ 1+0 & y+4 & 3-2 \end{bmatrix} = \begin{bmatrix} 4 & -2 & 3 \\ 1 & 6 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 2x & x+2y \\ 1 & y+4 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -2 & 3 \\ 1 & 6 & 1 \end{bmatrix}$$

Now, using the equality of matrices, we have

$$\therefore 2x = -2 \Rightarrow x = -1$$

$$y + 4 = 6 \Rightarrow y = 6 - 4 = 2 \quad \therefore x = -1, y = 2$$

6. If  $A = [a_{ij}]_{3 \times 3}$ , show that

$$(i) \lambda(\mu A) = (\lambda\mu)A \quad (ii) (\lambda + \mu)A = \lambda A + \mu A \quad (iii) \lambda A - A = (\lambda - 1)A$$

**Solution.** (i)  $\lambda(\mu A) = \lambda(\mu[a_{ij}]_{3 \times 3}) = \lambda([\mu a_{ij}]_{3 \times 3})$

$$= [\lambda\mu a_{ij}]_{3 \times 3} = \lambda\mu[a_{ij}]_{3 \times 3} = (\lambda\mu)A$$

(ii)  $(\lambda + \mu)A = (\lambda + \mu)[a_{ij}]_{3 \times 3} = [(\lambda + \mu)a_{ij}]_{3 \times 3} = [\lambda a_{ij} + \mu a_{ij}]_{3 \times 3}$

$$= [\lambda a_{ij}]_{3 \times 3} + [\mu a_{ij}]_{3 \times 3} = \lambda[a_{ij}]_{3 \times 3} + \mu[a_{ij}]_{3 \times 3} = \lambda A + \mu A$$

(iii)  $\lambda A - A = \lambda[a_{ij}]_{3 \times 3} - 1[a_{ij}]_{3 \times 3}$ . Taking  $[a_{ij}]_{3 \times 3}$  common, we get

$$= (\lambda - 1)[a_{ij}]_{3 \times 3} = (\lambda - 1)A$$

7. If  $A = [a_{ij}]_{2 \times 3}$ ,  $B = [b_{ij}]_{2 \times 3}$ , show that  $\lambda(A + B) = \lambda A + \lambda B$ .

**Solution.**  $\lambda(A + B) = \lambda([a_{ij}]_{2 \times 3} + [b_{ij}]_{2 \times 3})$

$$= \lambda[a_{ij}]_{2 \times 3} + \lambda[b_{ij}]_{2 \times 3} = \lambda A + \lambda B.$$

8. If  $A = \begin{bmatrix} 1 & 2 \\ a & b \end{bmatrix}$  and  $A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ , find the values of  $a$  and  $b$ .

**Solution.**  $A = \begin{bmatrix} 1 & 2 \\ a & b \end{bmatrix}$ , then

$$A^2 = A.A = \begin{bmatrix} 1 & 2 \\ a & b \end{bmatrix} \begin{bmatrix} 1 & 2 \\ a & b \end{bmatrix} = \begin{bmatrix} 1+2a & 2+2b \\ a+ab & 2a+b^2 \end{bmatrix}$$

Given that  $A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1+2a & 2+2b \\ a+ab & 2a+b^2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Equality of matrices  $\Rightarrow 1 + 2a = 0 \Rightarrow a = -\frac{1}{2}$

and  $2 + 2b = 0 \Rightarrow b = -1 \quad \therefore \boxed{a = -1/2, b = -1}$

9. If  $A = \begin{bmatrix} 1 & -1 \\ a & b \end{bmatrix}$  and  $A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ , find the values of  $a$  and  $b$ .

**Solution.**  $A = \begin{bmatrix} 1 & -1 \\ a & b \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} 1 & -1 \\ a & b \end{bmatrix} \begin{bmatrix} 1 & -1 \\ a & b \end{bmatrix} = \begin{bmatrix} 1-a & -1-b \\ a+ab & -a+b^2 \end{bmatrix}$

Given :  $A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1-a & -1-b \\ a+ab & -a+b^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Equality of matrices  $\Rightarrow 1-a = 1 \Rightarrow a = 0$

and  $-1-b = 0 \Rightarrow b = -1 \therefore \boxed{a = 0, b = -1}$

**Definition.** A matrix obtained by interchanging rows and columns is said to be the *transpose* of the original matrix and it is denoted by  $A^t$ . In other words, let  $A$  be any matrix. If rows and columns of  $A$  are interchanged then the resulting matrix is called the transpose of the matrix  $A$ .

For example,  $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & 1 \end{bmatrix}$  then  $A^t = \begin{bmatrix} 1 & 0 \\ -1 & 3 \\ 2 & 1 \end{bmatrix}$ .

Note the formula :  $(A+B)^t = A^t + B^t$  &  $(AB)^t = B^t A^t$ .

10. If  $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & 1 \end{bmatrix}$ ,  $B = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 2 & -1 \end{bmatrix}$ , show that  $(A+B)^t = A^t + B^t$ .

**Solution.**  $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & 1 \end{bmatrix} \Rightarrow A^t = \begin{bmatrix} 1 & 0 \\ -1 & 3 \\ 2 & 1 \end{bmatrix}$ ,

$B = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 2 & -1 \end{bmatrix} \Rightarrow B^t = \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ 0 & -1 \end{bmatrix}$

Adding:  $A+B = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 3 & 0 \\ 1 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 2 \\ 1 & 5 & 0 \end{bmatrix}$

$A+B = \begin{bmatrix} 3 & 2 & 2 \\ 1 & 5 & 0 \end{bmatrix} \Rightarrow (A+B)^t = \begin{bmatrix} 3 & 1 \\ 2 & 5 \\ 2 & 0 \end{bmatrix} \dots (1)$

Also  $A^t + B^t = \begin{bmatrix} 1 & 0 \\ -1 & 3 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 2 & 5 \\ 2 & 0 \end{bmatrix} \dots (2)$

From (1) and (2)  $(A+B)^t = A^t + B^t$ .

11. Find  $A^3$  if  $A = \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$ .

**Solution.** We have  $A^2 = AA = \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix} \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$

$$= \begin{bmatrix} 1(1)+1(5)+3(-2) & 1(1)+1(2)+3(-1) & 1(3)+1(6)+3(-3) \\ 5(1)+2(5)+6(-2) & 5(1)+2(2)+6(-1) & 5(3)+2(6)+6(-3) \\ (-2)1+(-1)5+(-3)(-2) & (-2)1+(-1)2+(-3)(-1) & (-2)3+(-1)6+(-3)(-3) \end{bmatrix}$$

$$= \begin{bmatrix} 1+5-6 & 1+2-3 & 3+6-9 \\ 5+10-12 & 5+4-6 & 15+12-18 \\ -2-5+6 & -2-2+3 & -6-6+9 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 3 & 3 & 9 \\ -1 & -1 & -3 \end{bmatrix}$$

$$\therefore A^3 = A^2 \cdot A = \begin{bmatrix} 0 & 0 & 0 \\ 3 & 3 & 9 \\ -1 & -1 & -3 \end{bmatrix} \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 0(1)+0(5)+0(-2) & 0(1)+0(2)+0(-1) & 0(3)+0(6)+0(-3) \\ 3(1)+3(5)+9(-2) & 3(1)+3(2)+9(-1) & 3(3)+3(6)+9(-3) \\ (-1)1+(-1)5+(-3)(-2) & (-1)1+(-1)2+(-3)(-1) & (-1)3+(-1)6+(-3)(-3) \end{bmatrix}$$

$$= \begin{bmatrix} 0+0+0 & 0+0+0 & 0+0+0 \\ 3+15-18 & 3+6-9 & 9+18-27 \\ -1-5+6 & -1-2+3 & -3-6+9 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O_3$$

12. Find the matrix  $X$  if (i)  $X \begin{bmatrix} 5 & 2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 12 & 3 \end{bmatrix}$ ,

(ii)  $\begin{bmatrix} 5 & 2 \\ -2 & 1 \end{bmatrix} X = \begin{bmatrix} 2 & 1 \\ 5 & 10 \end{bmatrix}$ .

**Solution.** (i)  $X \begin{bmatrix} 5 & 2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 12 & 3 \end{bmatrix}$

Let  $X = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ , then

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 5 & 2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 12 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 5a-2b & 2a+b \\ 5c-2d & 2c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 12 & 3 \end{bmatrix}$$

Comparing the corresponding elements, we get

$$5a-2b = -1 \dots (i), \quad 2a+b = 5 \dots (ii)$$

$$5c-2d = 12 \dots (iii), \quad 2c+d = 3 \dots (iv)$$

To solve (i) and (ii), put  $b = 5 - 2a$  from (ii) in (i), then

$$5a - 2(5 - 2a) = -1 \Rightarrow 5a - 10 + 4a = -1 \Rightarrow 9a = 9 \Rightarrow \boxed{a = 1}$$

Put  $a = 1$  in (ii), then  $b = 5 - 2a = 5 - 2(1) = 5 - 2 = 3 \Rightarrow \boxed{b = 3}$

To solve (iii) and (iv), put  $d = 3 - 2c$  from (iv) in (iii), then

$$5c - 2(3 - 2c) = 12 \Rightarrow 5c - 6 + 4c = 12 \Rightarrow 9c = 18 \Rightarrow \boxed{c = 2}$$

Put  $c = 2$  in (iv), then  $d = 3 - 2c = 3 - 2(2) = 3 - 4 = -1 \Rightarrow \boxed{d = -1}$

Thus  $X = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$

Solution. (ii)  $\begin{bmatrix} 5 & 2 \\ -2 & 1 \end{bmatrix} X = \begin{bmatrix} 2 & 1 \\ 5 & 10 \end{bmatrix}$ .

Let  $X = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ , then

$$\begin{bmatrix} 5 & 2 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 5 & 10 \end{bmatrix}$$

$$\begin{bmatrix} 5a+2c & 5b+2d \\ -2a+c & -2b+d \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 5 & 10 \end{bmatrix}$$

Comparing the corresponding elements, we get

$$5a+2c = 2 \dots (i), \quad 5b+2d = 1 \dots (ii)$$

$$-2a+c = 5 \dots (iii), \quad -2b+d = 10 \dots (iv)$$

To solve (i) and (iii), put  $c = 5 + 2a$  from (iii) in (i), then

$$5a + 2(5 + 2a) = 2 \Rightarrow 5a + 10 + 4a = 2 \Rightarrow 9a = -8 \Rightarrow \boxed{a = -\frac{8}{9}}$$

Put  $a = -\frac{8}{9}$  in (iii), then  $c = 5 + 2a = 5 + 2\left(-\frac{8}{9}\right) = 5 - \frac{16}{9} = \frac{45-16}{9} = \frac{29}{9}$

$$\Rightarrow \boxed{c = \frac{29}{9}}$$

To solve (ii) and (iv), put  $d = 10 + 2b$  from (iv) in (ii), then

$$5b + 2(10 + 2b) = 1 \Rightarrow 5b + 20 + 4b = 1 \Rightarrow 9b = -19 \Rightarrow \boxed{b = -\frac{19}{9}}$$

Put  $b = -\frac{19}{9}$  in (ii), then

$$d = 10 + 2b = 10 + 2\left(-\frac{19}{9}\right) = 10 - \frac{38}{9} = \frac{90 - 38}{9} = \frac{52}{9} \Rightarrow \boxed{d = \frac{52}{9}}$$

$$\text{Thus } X = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} -\frac{8}{9} & -\frac{19}{9} \\ \frac{29}{9} & \frac{52}{9} \end{bmatrix}$$

13. Find the matrix  $A$  if (i)  $\begin{bmatrix} 5 & -1 \\ 0 & 0 \\ 3 & 1 \end{bmatrix} A = \begin{bmatrix} 3 & -7 \\ 0 & 0 \\ 7 & 2 \end{bmatrix}$ ,

(ii)  $\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} A = \begin{bmatrix} 0 & -3 & 8 \\ 3 & 3 & -7 \end{bmatrix}$ .

**Solution.** (i) Let  $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ , then

$$\begin{bmatrix} 5 & -1 \\ 0 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 3 & -7 \\ 0 & 0 \\ 7 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 5a - c & 5b - d \\ 0 + 0 & 0 + 0 \\ 3a + c & 3b + d \end{bmatrix} = \begin{bmatrix} 3 & -7 \\ 0 & 0 \\ 7 & 2 \end{bmatrix}$$

Comparing the corresponding elements, we get

$$5a - c = 3 \quad \dots \quad \text{(i)}, \quad 5b - d = -7 \quad \dots \quad \text{(ii)}$$

$$3a + c = 7 \quad \dots \quad \text{(iii)}, \quad 3b + d = 2 \quad \dots \quad \text{(iv)}$$

Adding (i) and (iii), we get

$$8a = 10 \Rightarrow a = \frac{10}{8} = \frac{5}{4}$$



$$(iii) \text{ gives } c = 7 - 3a = 7 - \frac{15}{4} = \frac{28 - 15}{4} = \frac{13}{4}$$

$$\text{Adding (ii) and (iv), we get } 8b = -5 \Rightarrow b = -\frac{5}{8}$$

$$(iv) \text{ gives } d = 2 - 3b = 2 + \frac{15}{8} = \frac{16 + 15}{8} = \frac{31}{8}$$

$$\text{Hence required matrix is } A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 5/4 & -5/8 \\ 13/4 & 31/8 \end{bmatrix}$$

$$\text{Solution. (ii) } \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} A = \begin{bmatrix} 0 & -3 & 8 \\ 3 & 3 & -7 \end{bmatrix}$$

$$\text{Let } A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}, \text{ then}$$

$$\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \begin{bmatrix} 0 & -3 & 8 \\ 3 & 3 & -7 \end{bmatrix}$$

$$\begin{bmatrix} 2a-d & 2b-e & 2c-f \\ -a+2d & -b+2e & -c+2f \end{bmatrix} = \begin{bmatrix} 0 & -3 & 8 \\ 3 & 3 & -7 \end{bmatrix}$$

Comparing the corresponding elements, we get

$$2a - d = 0 \quad \dots \quad (i), \quad 2b - e = -3 \quad \dots \quad (ii), \quad 2c - f = 8 \quad \dots \quad (iii)$$

$$-a + 2d = 3 \quad \dots \quad (iv), \quad -b + 2e = 3 \quad \dots \quad (v), \quad -c + 2f = -7 \quad \dots \quad (vi)$$

To solve (i) and (iv), put  $d = 2a$  from (i) in (iv), then

$$-a + 2d = 3 \Rightarrow -a + 2(2a) = 3 \Rightarrow -a + 4a = 3 \Rightarrow 3a = 3 \Rightarrow \boxed{a = 1}$$

$$\text{Put } a = 1 \text{ in (iv), then } d = 2a = 2(1) = 2 \Rightarrow \boxed{d = 2}$$

To solve (ii) and (v), put  $e = 2b + 3$  from (ii) in (v), then

$$-b + 2(2b + 3) = 3 \Rightarrow -b + 4b + 6 = 3 \Rightarrow 3b = -3 \Rightarrow \boxed{b = -1}$$

$$\text{Put } b = -1 \text{ in (v), then } e = 2b + 3 = 2(-1) + 3 = -2 + 3 = 1 \Rightarrow \boxed{e = 1}$$

To solve (iii) and (vi), put  $f = 2c - 8$  from (iii) in (vi), then

$$-c + 2f = -7 \Rightarrow -c + 2(2c - 8) = -7$$

$$\Rightarrow -c + 4c - 16 = -7 \Rightarrow 3c = 9 \Rightarrow \boxed{c = 3}$$

$$\text{Put } c = 3 \text{ in (vi), then } f = 2c - 8 = 2(3) - 8 = 6 - 8 = -2 \Rightarrow \boxed{f = -2}$$

$$\text{Hence, } A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \begin{bmatrix} 1 & -1 & 3 \\ 2 & 1 & -2 \end{bmatrix}$$

14 Show that 
$$\begin{bmatrix} r \cos \phi & 0 & -\sin \phi \\ 0 & r & 0 \\ r \sin \phi & 0 & \cos \phi \end{bmatrix} \begin{bmatrix} \cos \phi & 0 & \sin \phi \\ 0 & 1 & 0 \\ -\sin \phi & 0 & r \cos \phi \end{bmatrix} = r I_3.$$

Solution. 
$$\begin{bmatrix} r \cos^2 \phi + 0 + r \sin^2 \phi & 0 + 0 + 0 & r \cos \phi \sin \phi + 0 - r \cos \phi \sin \phi \\ 0 + 0 + 0 & 0 + r + 0 & 0 + 0 + 0 \\ r \cos \phi \sin \phi + 0 - r \cos \phi \sin \phi & 0 + 0 + 0 & r \sin^2 \phi + 0 + r \cos^2 \phi \end{bmatrix}$$

$$= \begin{bmatrix} r & 0 & 0 \\ 0 & r & 0 \\ 0 & 0 & r \end{bmatrix} = r \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = r I_3$$

because  $r \sin^2 \phi + r \cos^2 \phi = r (\sin^2 \phi + \cos^2 \phi) = r (1) = r.$

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