

Question # 1

Find the solution set of the following equation which lies in $[0, 2\pi]$

(i) $\sin x = -\frac{\sqrt{3}}{2}$

(ii) $\operatorname{cosec} \theta = 2$

(iv) $\cot \theta = \frac{1}{\sqrt{3}}$

Solution

(i) Since $\sin x = -\frac{\sqrt{3}}{2} \Rightarrow x = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

$$\Rightarrow x = \frac{5\pi}{3}, \frac{4\pi}{3} \quad \text{where } x \in [0, 2\pi]$$

(ii) Since $\operatorname{cosec} \theta = 2$

$$\Rightarrow \frac{1}{\sin \theta} = 2 \Rightarrow \sin \theta = \frac{1}{2}$$

$$\Rightarrow \theta = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}, \frac{5\pi}{6} \quad \text{where } \theta \in [0, 2\pi]$$

(iii) *Do yourself*

(iv) Since $\cot \theta = \frac{1}{\sqrt{3}}$

$$\Rightarrow \frac{1}{\tan \theta} = \frac{1}{\sqrt{3}} \Rightarrow \tan \theta = \sqrt{3}$$

$$\Rightarrow \theta = \tan^{-1}(\sqrt{3})$$

$$\Rightarrow \theta = \frac{\pi}{3}, \frac{4\pi}{3} \quad \text{where } \theta \in [0, 2\pi]$$

Question # 2

Solve the following trigonometric equations:

(i) $\tan^2 \theta = \frac{1}{3}$

(ii) $\operatorname{cosec}^2 \theta = \frac{4}{3}$

(iii) $\sec^2 \theta = \frac{4}{3}$

(iv) $\cot^2 \theta = \frac{1}{3}$

Solution

(i) Since $\tan^2 \theta = \frac{1}{3} \Rightarrow \tan \theta = \pm \frac{1}{\sqrt{3}}$

$$\Rightarrow \tan \theta = \frac{1}{\sqrt{3}} \quad \text{or} \quad \tan \theta = -\frac{1}{\sqrt{3}}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) \quad \text{or} \quad \theta = \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$$

$$\Rightarrow \theta = \frac{\pi}{6} \quad \text{or} \quad \theta = \frac{5\pi}{6}$$

Since period of $\tan \theta$ is π

Therefore general value of $\theta = \frac{\pi}{6} + n\pi, \frac{5\pi}{6} + n\pi$

So Solution Set = $\left\{ \frac{\pi}{6} + n\pi \right\} \cup \left\{ \frac{5\pi}{6} + n\pi \right\}$ where $n \in \mathbb{Z}$

(ii) Since $\operatorname{cosec}^2 \theta = \frac{4}{3}$

$$\Rightarrow \operatorname{cosec} \theta = \pm \frac{2}{\sqrt{3}}$$

$$\Rightarrow \operatorname{cosec} \theta = \frac{2}{\sqrt{3}} \quad \text{or} \quad \operatorname{cosec} \theta = -\frac{2}{\sqrt{3}}$$

$$\Rightarrow \sin \theta = \frac{\sqrt{3}}{2} \quad \text{or} \quad \sin \theta = -\frac{\sqrt{3}}{2}$$

$$\Rightarrow \theta = \sin^{-1} \left(\frac{\sqrt{3}}{2} \right) \quad \text{or} \quad \theta = \sin^{-1} \left(-\frac{\sqrt{3}}{2} \right)$$

$$\Rightarrow \theta = \frac{\pi}{3}, \frac{2\pi}{3} \quad \text{or} \quad \theta = \frac{4\pi}{3}, \frac{5\pi}{3}$$

Since period of $\operatorname{cosec} \theta$ is 2π

Therefore general value of $\theta = \frac{\pi}{3} + 2n\pi, \frac{2\pi}{3} + 2n\pi, \frac{4\pi}{3} + 2n\pi, \frac{5\pi}{3} + 2n\pi$

Solution set = $\left\{ \frac{\pi}{3} + 2n\pi \right\} \cup \left\{ \frac{2\pi}{3} + 2n\pi \right\} \cup \left\{ \frac{4\pi}{3} + 2n\pi \right\} \cup \left\{ \frac{5\pi}{3} + 2n\pi \right\}$ where $n \in \mathbb{Z}$.

(iii) Since $\sec^2 \theta = \frac{4}{3}$

$$\Rightarrow \sec \theta = \pm \frac{2}{\sqrt{3}}$$

$$\Rightarrow \sec \theta = \frac{2}{\sqrt{3}} \quad \text{or} \quad \sec \theta = -\frac{2}{\sqrt{3}}$$

$$\Rightarrow \cos \theta = \frac{\sqrt{3}}{2} \quad \text{or} \quad \cos \theta = -\frac{\sqrt{3}}{2}$$

$$\Rightarrow \theta = \cos^{-1} \left(\frac{\sqrt{3}}{2} \right) \quad \text{or} \quad \theta = \cos^{-1} \left(-\frac{\sqrt{3}}{2} \right)$$

$$\Rightarrow \theta = \frac{\pi}{6}, \frac{11\pi}{6} \quad \text{or} \quad \theta = \frac{5\pi}{6}, \frac{7\pi}{6}$$

\therefore period of $\sec \theta$ is 2π

\therefore general values of $\theta = \frac{\pi}{6} + 2n\pi, \frac{5\pi}{6} + 2n\pi, \frac{7\pi}{6} + 2n\pi, \frac{11\pi}{6} + 2n\pi$

S.Set = $\left\{ \frac{\pi}{6} + 2n\pi \right\} \cup \left\{ \frac{5\pi}{6} + 2n\pi \right\} \cup \left\{ \frac{7\pi}{6} + 2n\pi \right\} \cup \left\{ \frac{11\pi}{6} + 2n\pi \right\}$ where $n \in \mathbb{Z}$.

(iv)

Do yourself

Question # 3

Find the value of θ satisfying the following equation:

$$3 \tan^2 \theta + 2\sqrt{3} \tan \theta + 1 = 0$$

Solution $3 \tan^2 \theta + 2\sqrt{3} \tan \theta + 1 = 0$

$$\Rightarrow (\sqrt{3} \tan \theta)^2 + 2(\sqrt{3} \tan \theta)(1) + (1)^2 = 0$$

$$\Rightarrow (\sqrt{3} \tan \theta + 1)^2 = 0$$

$$\Rightarrow (\sqrt{3} \tan \theta + 1) = 0$$

$$\Rightarrow \sqrt{3} \tan \theta = -1$$

$$\Rightarrow \tan \theta = -\frac{1}{\sqrt{3}}$$

$$\Rightarrow \theta = \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$$

$$\Rightarrow \theta = \frac{5\pi}{6}$$

\therefore period of $\tan \theta$ is π

\therefore general value of $\theta = \frac{5\pi}{6} + n\pi$, $n \in \mathbb{Z}$

Question # 4

Find the value of θ satisfying the following equation:

$$\tan^2 \theta - \sec \theta - 1 = 0$$

Solution $\tan^2 \theta - \sec \theta - 1 = 0$

$$\Rightarrow (\sec^2 \theta - 1) - \sec \theta - 1 = 0$$

$$\Rightarrow \sec^2 \theta - 1 - \sec \theta - 1 = 0$$

$$\Rightarrow \sec^2 \theta - \sec \theta - 2 = 0$$

$$\Rightarrow \sec^2 \theta - 2\sec \theta + \sec \theta - 2 = 0$$

$$\Rightarrow \sec \theta(\sec \theta - 2) + 1(\sec \theta - 2) = 0$$

$$\Rightarrow (\sec \theta + 1)(\sec \theta - 2) = 0$$

$$\Rightarrow (\sec \theta + 1) = 0 \quad \text{or} \quad (\sec \theta - 2) = 0$$

$$\Rightarrow \sec \theta = -1 \quad \text{or} \quad \sec \theta = +2$$

$$\Rightarrow \cos \theta = -1 \quad \text{or} \quad \cos \theta = \frac{1}{2}$$

$$\Rightarrow \theta = \cos^{-1}(-1) \quad \text{or} \quad \theta = \cos^{-1}\left(\frac{1}{2}\right)$$

$$\Rightarrow \theta = \frac{3\pi}{2} \quad \text{or} \quad \theta = \frac{\pi}{3}, \frac{5\pi}{3}$$

\therefore period of $\cos\theta$ is 2π

$$\therefore \text{general value of } \theta = \frac{3\pi}{2} + 2n\pi, \frac{\pi}{3} + 2n\pi, \frac{5\pi}{3} + 2n\pi \quad \text{where } n \in \mathbb{Z}$$

Question # 5

Find the value of θ satisfying the following equation:

$$2\sin\theta + \cos^2\theta - 1 = 0$$

Solution

$$2\sin\theta + \cos^2\theta - 1 = 0$$

$$\Rightarrow 2\sin\theta + 1 - \sin^2\theta - 1 = 0$$

$$\Rightarrow -\sin^2\theta + 2\sin\theta = 0$$

$$\Rightarrow -\sin\theta(\sin\theta - 2) = 0$$

$$\Rightarrow -\sin\theta = 0 \quad \text{or} \quad \sin\theta - 2 = 0$$

$$\Rightarrow \sin\theta = 0 \quad \text{or} \quad \sin\theta = 2$$

$$\Rightarrow \theta = \sin^{-1}(0) \quad \text{Which does not hold as } \sin\theta \in [-1, 1]$$

$$\Rightarrow \theta = 0, \pi$$

\therefore period of $\sin\theta$ is 2π

$$\therefore \text{general value of } \theta = 0 + 2n\pi, \pi + 2n\pi$$

$$= 2n\pi, \pi + 2n\pi \quad \text{where } n \in \mathbb{Z}$$

Question # 6

Find the value of θ satisfying the following equation:

$$2\sin^2\theta - \sin\theta = 0$$

Solution

$$2\sin^2\theta - \sin\theta = 0$$

$$\Rightarrow \sin\theta(2\sin\theta - 1) = 0 \Rightarrow \sin\theta = 0 \quad \text{or} \quad 2\sin\theta - 1 = 0$$

Now do yourself

Question # 7

Find the value of θ satisfying the following equation:

$$3\cos^2\theta - 2\sqrt{3}\sin\theta\cos\theta - 3\sin^2\theta = 0$$

Solution

$$3\cos^2\theta - 2\sqrt{3}\sin\theta\cos\theta - 3\sin^2\theta = 0$$

Dividing throughout by $\cos^2\theta$

$$\frac{3\cos^2\theta}{\cos^2\theta} - \frac{2\sqrt{3}\sin\theta\cos\theta}{\cos^2\theta} - \frac{3\sin^2\theta}{\cos^2\theta} = 0$$

$$\Rightarrow 3 - 2\sqrt{3}\tan\theta - 3\tan^2\theta = 0$$

$$\Rightarrow -3\tan^2\theta - 2\sqrt{3}\tan\theta + 3 = 0$$

$$\Rightarrow 3\tan^2\theta + 2\sqrt{3}\tan\theta - 3 = 0$$

×ing by -1

$$\Rightarrow \tan\theta = \frac{-2\sqrt{3} \pm \sqrt{(2\sqrt{3})^2 - 4(3)(-3)}}{2(3)}$$

$$\Rightarrow \tan \theta = \frac{-2\sqrt{3} \pm \sqrt{12+36}}{6} = \frac{-2\sqrt{3} \pm \sqrt{48}}{6}$$

$$= \frac{-2\sqrt{3} \pm \sqrt{16 \times 3}}{6} = \frac{-2\sqrt{3} \pm 4\sqrt{3}}{6}$$

$$\Rightarrow \tan \theta = \frac{-2\sqrt{3} + 4\sqrt{3}}{6} = \frac{2\sqrt{3}}{6} \quad \text{or} \quad \tan \theta = \frac{-2\sqrt{3} - 4\sqrt{3}}{6} = -\frac{6\sqrt{3}}{6}$$

$$\Rightarrow \tan \theta = \frac{\sqrt{3}}{3} = \frac{\sqrt{3}}{(\sqrt{3})^2} = \frac{1}{\sqrt{3}} \quad \text{or} \quad \Rightarrow \tan \theta = -\sqrt{3}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) \quad \text{or} \quad \theta = \tan^{-1}(-\sqrt{3})$$

$$\Rightarrow \theta = \frac{\pi}{6} \quad \text{or} \quad \theta = \frac{11\pi}{6}$$

\therefore period of $\tan \theta$ is π

\therefore general value of $\theta = \frac{\pi}{6} + n\pi, \frac{11\pi}{6} + n\pi$ where $n \in \mathbb{Z}$.

Question # 8

Find the value of θ satisfying the following equation:

$$4\sin^2 \theta - 8\cos \theta + 1 = 0$$

Solution $4\sin^2 \theta - 8\cos \theta + 1 = 0$

$$\Rightarrow 4(1 - \cos^2 \theta) - 8\cos \theta + 1 = 0$$

$$\Rightarrow 4 - 4\cos^2 \theta - 8\cos \theta + 1 = 0$$

$$\Rightarrow -4\cos^2 \theta - 8\cos \theta + 5 = 0$$

$$\Rightarrow 4\cos^2 \theta + 8\cos \theta - 5 = 0$$

$$\Rightarrow 4\cos^2 \theta + 10\cos \theta - 2\cos \theta - 5 = 0$$

$$\Rightarrow 2\cos \theta(2\cos \theta + 5) - 1(2\cos \theta + 5) = 0$$

$$\Rightarrow (2\cos \theta + 5)(2\cos \theta - 1) = 0$$

$$\Rightarrow 2\cos \theta + 5 = 0 \quad \text{or} \quad 2\cos \theta - 1 = 0$$

$$\Rightarrow 2\cos \theta = -5 \quad \text{or} \quad 2\cos \theta = 1$$

$$\Rightarrow \cos \theta = \frac{-5}{2} \quad \text{or} \quad \cos \theta = \frac{1}{2}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{-5}{2}\right) \quad \text{or} \quad \theta = \cos^{-1}\left(\frac{1}{2}\right)$$

Which is not possible as $\cos \theta \in [-1, 1]$ or $\theta = \frac{\pi}{3}, \frac{5\pi}{3}$

\therefore period of $\cos \theta$ is 2π

\therefore general value of $\theta = \frac{\pi}{3} + 2\pi n, \frac{5\pi}{3} + 2\pi n$ where $n \in \mathbb{Z}$.

Question # 9

Find the solution set; $\sqrt{3} \tan x - \sec x - 1 = 0$

Solution $\sqrt{3} \tan x - \sec x - 1 = 0 \dots\dots(i)$

$$\Rightarrow \sqrt{3} \frac{\sin x}{\cos x} - \frac{1}{\cos x} - 1 = 0$$

$$\Rightarrow \sqrt{3} \sin x - 1 - \cos x = 0 \quad \times \text{ing by } \cos \theta.$$

$$\Rightarrow \sqrt{3} \sin x - 1 = \cos x$$

On squaring both sides.

$$(\sqrt{3} \sin x - 1)^2 = (\cos x)^2$$

$$\Rightarrow 3 \sin^2 x - 2\sqrt{3} \sin x + 1 = \cos^2 x$$

$$\Rightarrow 3 \sin^2 x - 2\sqrt{3} \sin x + 1 = 1 - \sin^2 x$$

$$\Rightarrow 3 \sin^2 x - 2\sqrt{3} \sin x + 1 - 1 + \sin^2 x = 0$$

$$\Rightarrow 4 \sin^2 x - 2\sqrt{3} \sin x = 0$$

$$\Rightarrow 2 \sin x (2 \sin x - \sqrt{3}) = 0$$

$$\Rightarrow 2 \sin x = 0 \quad \text{or} \quad 2 \sin x = \sqrt{3}$$

$$\Rightarrow \sin x = 0 \quad \text{or} \quad \sin x = \frac{\sqrt{3}}{2}$$

$$\Rightarrow x = \sin^{-1}(0) \quad \text{or} \quad x = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$\Rightarrow x = 0, \pi \quad \text{or} \quad x = \frac{\pi}{3}, \frac{2\pi}{3}$$

Now to check extraneous roots put $x = 0$ in (i)

$$\text{L.H.S} = \sqrt{3} \tan(0) - \sec(0) - 1 = 0 - 1 - 1 = -2 \neq 0 = \text{R.H.S}$$

Implies that $x = 0$ is an extraneous root of given equation.

Now put $x = \pi$ in (i)

$$\text{L.H.S} = \sqrt{3} \tan(\pi) - \sec(\pi) - 1 = 0 - (-1) - 1 = 0 = \text{R.H.S}$$

Implies that $x = \pi$ is a root of the equation.

Now put $x = \frac{\pi}{3}$ in (i)

$$\text{L.H.S} = \sqrt{3} \tan\left(\frac{\pi}{3}\right) - \sec\left(\frac{\pi}{3}\right) - 1 = \sqrt{3}(\sqrt{3}) - 2 - 1 = 1 - 2 - 1 = 0 = \text{R.H.S}$$

Implies that $x = \frac{\pi}{3}$ is a root of given equation.. Since period of tan is π .

Now put $x = \frac{2\pi}{3}$ in (i)

$$\begin{aligned} \text{L.H.S} &= \sqrt{3} \tan\left(\frac{2\pi}{3}\right) - \sec\left(\frac{2\pi}{3}\right) - 1 \\ &= \sqrt{3}(-\sqrt{3}) - (-2) - 1 = -3 + 2 - 1 = -2 = \text{R.H.S} \end{aligned}$$

Implies $x = \frac{2\pi}{3}$ is an extraneous root of given equation.

\therefore period of $\sin x$ is 2π

\therefore general values of $x = \pi + 2n\pi, \frac{\pi}{3} + 2n\pi$

$$\text{Solution Set} = \{\pi + 2n\pi\} \cup \left\{ \frac{\pi}{3} + 2n\pi \right\} \quad \text{where } n \in \mathbb{Z}.$$

Question # 10

Find the solution set; $\cos 2x = \sin 3x$

Solution

$$\cos 2x = \sin 3x$$

$$\Rightarrow \cos^2 x - \sin^2 x = 3\sin x - 4\sin^3 x \quad \because \cos 2x = \cos^2 x - \sin^2 x$$

$$\Rightarrow \cos^2 x - \sin^2 x - 3\sin x + 4\sin^3 x = 0 \quad \sin 3x = 3\sin x - 4\sin^3 x$$

$$\Rightarrow (1 - \sin^2 x) - \sin^2 x - 3\sin x + 4\sin^3 x = 0$$

$$\Rightarrow 1 - 2\sin^2 x - 3\sin x + 4\sin^3 x = 0$$

$$\Rightarrow 4\sin^3 x - 2\sin^2 x - 3\sin x + 1 = 0$$

Take $\sin x = 1$ as a root then by synthetic division

$$\begin{array}{r|rrrr} 1 & 4 & -2 & -3 & 1 \\ & \downarrow & & & \\ \hline & 4 & 2 & -1 & \underline{0} \end{array}$$

$$\Rightarrow (\sin x - 1)(4\sin^2 x + 2\sin x - 1) = 0$$

$$\Rightarrow \sin x - 1 = 0 \quad \text{or} \quad 4\sin^2 x + 2\sin x - 1 = 0$$

$$\Rightarrow \sin x = 1 \quad \text{or} \quad \sin x = \frac{-2 \pm \sqrt{(2)^2 - 4(4)(-1)}}{2(4)}$$

$$\Rightarrow x = \sin^{-1}(1) \quad \text{or} \quad \sin x = \frac{-2 \pm \sqrt{4+16}}{8} = \frac{-2 \pm \sqrt{20}}{8}$$

$$\Rightarrow x = \frac{\pi}{2} \quad \text{or} \quad \sin x = \frac{-2 + \sqrt{20}}{8} \quad \text{or} \quad \sin x = \frac{-2 - \sqrt{20}}{8}$$

$$\sin x = 0.309 \quad \text{or} \quad \sin x = -0.809$$

$$\Rightarrow x = \sin^{-1}(0.309) \quad \text{or} \quad x = \sin^{-1}(-0.809)$$

$$\approx 18, 162 \quad \text{or} \quad \approx 234, 306$$

$$\Rightarrow x = 18 \cdot \frac{\pi}{180}, 162 \cdot \frac{\pi}{180} \quad \text{or} \quad x = 234 \cdot \frac{\pi}{180}, 306 \cdot \frac{\pi}{180}$$

$$= \frac{\pi}{10}, \frac{9\pi}{10} \quad \text{or} \quad = \frac{13\pi}{10}, \frac{17\pi}{10}$$

\therefore period of $\sin x$ is 2π

$$\therefore \text{ general value of } x = \frac{\pi}{10} + 2n\pi, \frac{9\pi}{10} + 2n\pi, \frac{13\pi}{10} + 2n\pi, \frac{17\pi}{10} + 2n\pi, \frac{\pi}{2} + 2n\pi$$

$$\text{S.Set} = \left\{ \frac{\pi}{10} + 2n\pi \right\} \cup \left\{ \frac{9\pi}{10} + 2n\pi \right\} \cup \left\{ \frac{13\pi}{10} + 2n\pi \right\} \cup \left\{ \frac{17\pi}{10} + 2n\pi \right\} \cup \left\{ \frac{\pi}{2} + 2n\pi \right\}$$

Question # 11

Find the solution set; $\sec 3\theta = \sec \theta$

Solution

$$\begin{aligned} \sec 3\theta &= \sec \theta \\ \Rightarrow \frac{1}{\cos 3\theta} &= \frac{1}{\cos \theta} \\ \Rightarrow \cos 3\theta &= \cos \theta \\ \Rightarrow 4\cos^3 \theta - 3\cos \theta &= \cos \theta & \because \cos 3\theta = 4\cos^3 \theta - 3\cos \theta \\ \Rightarrow 4\cos^3 \theta - 3\cos \theta - \cos \theta &= 0 \\ \Rightarrow 4\cos^3 \theta - 4\cos \theta &= 0 \\ \Rightarrow 4\cos \theta (\cos^2 \theta - 1) &= 0 \\ \Rightarrow 4\cos \theta = 0 \text{ or } \cos^2 \theta - 1 &= 0 \\ \Rightarrow \cos \theta = 0 \text{ or } \cos^2 \theta = 1 \\ \Rightarrow \theta = \cos^{-1}(0) \text{ or } \cos \theta = \pm 1 \\ \Rightarrow \theta = \cos^{-1}(1), \theta = \cos^{-1}(-1) \\ \Rightarrow \theta = \frac{\pi}{2}, \frac{3\pi}{2} \text{ or } \theta = 0, \pi \end{aligned}$$

\therefore period of $\cos \theta$ is 2π

$$\begin{aligned} \therefore \text{ general values of } \theta &= \frac{\pi}{2} + 2n\pi, \frac{3\pi}{2} + 2n\pi, 0 + 2n\pi, \pi + 2n\pi \\ &= \frac{\pi}{2} + 2n\pi, \frac{3\pi}{2} + 2n\pi, n\pi \end{aligned}$$

$$\text{S. Set} = \left\{ \frac{\pi}{2} + 2n\pi \right\} \cup \left\{ \frac{3\pi}{2} + 2n\pi \right\} \cup \{n\pi\} \quad \text{where } n \in \mathbb{Z}.$$

Question # 12

Find the solution set; $\tan 2\theta + \cot \theta = 0$

Solution

$$\begin{aligned} \tan 2\theta + \cot \theta &= 0 \\ \Rightarrow \tan 2\theta &= -\cot \theta \\ \Rightarrow \frac{\sin 2\theta}{\cos 2\theta} &= -\frac{\cos \theta}{\sin \theta} \\ \Rightarrow \frac{2\sin \theta \cos \theta}{\cos^2 \theta - \sin^2 \theta} &= -\frac{\cos \theta}{\sin \theta} \\ \Rightarrow (2\sin \theta \cos \theta)(\sin \theta) &= (-\cos \theta)(\cos^2 \theta - \sin^2 \theta) \\ \Rightarrow 2\sin^2 \theta \cos \theta &= -\cos^3 \theta + \sin^2 \theta \cos \theta \\ \Rightarrow 2\sin^2 \theta \cos \theta + \cos^3 \theta - \sin^2 \theta \cos \theta &= 0 \\ \Rightarrow \sin^2 \theta \cos \theta + \cos^3 \theta &= 0 \end{aligned}$$

$$\Rightarrow \cos \theta (\sin^2 \theta + \cos^2 \theta) = 0 \Rightarrow \cos \theta (1) = 0$$

$$\Rightarrow \theta = \cos^{-1}(0)$$

$$= \frac{\pi}{2}, \frac{3\pi}{2}$$

\therefore period of $\cos \theta$ is 2π

\therefore general values of $\theta = \frac{\pi}{2} + 2n\pi, \frac{3\pi}{2} + 2n\pi$

$$\text{S. Set} = \left\{ \frac{\pi}{2} + 2n\pi \right\} \cup \left\{ \frac{3\pi}{2} + 2n\pi \right\} \quad \text{where } n \in \mathbb{Z}.$$

Question # 13

Find the solution set; $\sin 2x + \sin x = 0$

Solution

$$\sin 2x + \sin x = 0$$

$$\Rightarrow 2 \sin x \cos x + \sin x = 0$$

$$\because \sin 2\theta = 2 \sin \theta \cos \theta$$

$$\Rightarrow \sin x (2 \cos x + 1) = 0$$

$$\Rightarrow \sin x = 0 \quad \text{or} \quad 2 \cos x + 1 = 0$$

$$\Rightarrow x = \sin^{-1}(0) \quad \text{or} \quad \cos x = -\frac{1}{2}$$

$$\Rightarrow x = \cos^{-1}\left(-\frac{1}{2}\right)$$

$$\Rightarrow x = 0, \pi \quad \text{or} \quad x = \frac{2\pi}{3}, \frac{4\pi}{3}$$

\therefore period of $\sin x$ and $\cos x$ is 2π

\therefore general values of $x = 0 + 2n\pi, \pi + 2n\pi, \frac{2\pi}{3} + 2n\pi, \frac{4\pi}{3} + 2n\pi$

$$= n\pi, \frac{2\pi}{3} + 2n\pi, \frac{4\pi}{3} + 2n\pi$$

$$\text{So solution set} = \left\{ n\pi \right\} \cup \left\{ \frac{2\pi}{3} + 2n\pi \right\} \cup \left\{ \frac{4\pi}{3} + 2n\pi \right\} \quad \text{where } n \in \mathbb{Z}.$$

Question # 14

Find the solution set; $\sin 4x - \sin 2x = \cos 3x$

Solution

$$\sin 4x - \sin 2x = \cos 3x$$

$$\Rightarrow 2 \cos \left(\frac{4x+2x}{2} \right) \sin \left(\frac{4x-2x}{2} \right) = \cos 3x$$

$$\Rightarrow 2 \cos 3x \sin x - \cos 3x = 0$$

$$\Rightarrow \cos 3x (2 \sin x - 1) = 0$$

$$\Rightarrow \cos 3x = 0 \quad \text{or} \quad 2 \sin x - 1 = 0$$

$$\Rightarrow 3x = \cos^{-1}(0), \quad \sin x = \frac{1}{2}$$

$$\Rightarrow 3x = \frac{\pi}{2}, \frac{3\pi}{2}, \quad x = \sin^{-1}\left(\frac{1}{2}\right)$$

$$\Rightarrow x = \frac{\pi}{6}, \frac{\pi}{2}, \quad , \quad x = \frac{\pi}{6}, \frac{5\pi}{6}$$

Since period of $\cos 3x$ is $\frac{2\pi}{3}$ and period of $\sin x$ is 2π

$$\therefore \text{ general values of } x = \frac{\pi}{6} + \frac{2n\pi}{3}, \frac{\pi}{2} + \frac{2n\pi}{3}, \frac{\pi}{6} + 2n\pi, \frac{5\pi}{6} + 2n\pi$$

$$\text{So solution set} = \left\{ \frac{\pi}{6} + \frac{2n\pi}{3} \right\} \cup \left\{ \frac{\pi}{2} + \frac{2n\pi}{3} \right\} \cup \left\{ \frac{\pi}{6} + 2n\pi \right\} \cup \left\{ \frac{5\pi}{6} + 2n\pi \right\} \quad \text{where } n \in \mathbb{Z}.$$

Question # 15

Find the solution set; $\sin x + \cos 3x = \cos 5x$

Solution

$$\sin x + \cos 3x = \cos 5x$$

$$\Rightarrow \sin x = \cos 5x - \cos 3x$$

$$\Rightarrow \sin x = -2 \sin \left(\frac{5x+3x}{2} \right) \sin \left(\frac{5x-3x}{2} \right)$$

$$\Rightarrow \sin x = -2 \sin 4x \sin x$$

$$\Rightarrow \sin x + 2 \sin 4x \sin x = 0$$

$$\Rightarrow \sin x (1 + 2 \sin 4x) = 0$$

$$\Rightarrow \sin x = 0 \quad \text{or} \quad 1 + 2 \sin 4x = 0$$

$$\Rightarrow x = \sin^{-1}(0) \quad \text{or} \quad \sin 4x = -\frac{1}{2} \Rightarrow 4x = \sin^{-1} \left(-\frac{1}{2} \right)$$

$$\Rightarrow x = 0, \pi \quad \text{or} \quad 4x = \frac{7\pi}{6}, \frac{11\pi}{6} \Rightarrow x = \frac{7\pi}{24}, \frac{11\pi}{24}$$

Since period of $\sin x$ is 2π and period of $\sin 4x$ is $\frac{2\pi}{4} = \frac{\pi}{2}$

$$\therefore \text{ general values of } x = 0 + 2n\pi, \pi + 2n\pi, \frac{7\pi}{24} + \frac{n\pi}{2}, \frac{11\pi}{24} + \frac{n\pi}{2}$$

$$\text{So solution set} = \{2n\pi\} \cup \{\pi + 2n\pi\} \cup \left\{ \frac{7\pi}{24} + \frac{n\pi}{2} \right\} \cup \left\{ \frac{11\pi}{24} + \frac{n\pi}{2} \right\} \quad \text{where } n \in \mathbb{Z}.$$

Question # 16

Find the solution set; $\sin 3x + \sin 2x + \sin x = 0$

Solution

$$\sin 3x + \sin 2x + \sin x = 0$$

$$\Rightarrow (\sin 3x + \sin x) + \sin 2x = 0$$

$$\Rightarrow 2 \sin \left(\frac{3x+x}{2} \right) \cos \left(\frac{3x-x}{2} \right) + \sin 2x = 0$$

$$\Rightarrow 2 \sin 2x \cos x + \sin 2x = 0$$

$$\Rightarrow \sin 2x (2 \cos x + 1) = 0$$

$$\Rightarrow \sin 2x = 0 \quad \text{or} \quad 2 \cos x + 1 = 0$$

$$\Rightarrow 2x = \sin^{-1}(0) \quad \text{or} \quad \cos x = -\frac{1}{2}$$

$$\Rightarrow 2x = 0, \pi \quad \text{or} \quad x = \cos^{-1}\left(-\frac{1}{2}\right)$$

$$\Rightarrow x = 0, \frac{\pi}{2} \quad \text{or} \quad x = \frac{2\pi}{3}, \frac{4\pi}{3}$$

Since period of $\sin 2x$ is $\frac{2\pi}{2} = \pi$ and period of $\cos x$ is 2π

\therefore general values of $x = 0 + n\pi, \frac{\pi}{2} + n\pi, \frac{2\pi}{3} + 2n\pi, \frac{4\pi}{3} + 2n\pi$

$$\text{S. Set} = \{n\pi\} \cup \left\{\frac{\pi}{2} + n\pi\right\} \cup \left\{\frac{2\pi}{3} + 2n\pi\right\} \cup \left\{\frac{4\pi}{3} + 2n\pi\right\} \quad \text{where } n \in \mathbb{Z}.$$

Question # 17

Find the solution set; $\sin 7x - \sin x = \sin 3x$

Solution

$$\sin 7x - \sin x = \sin 3x$$

$$\Rightarrow 2\cos\left(\frac{7x+x}{2}\right)\sin\left(\frac{7x-x}{2}\right) = \sin 3x$$

$$\Rightarrow 2\cos 4x \sin 3x - \sin 3x = 0$$

$$\Rightarrow \sin 3x(2\cos 4x - 1) = 0$$

$$\Rightarrow \sin 3x = 0 \quad \text{or} \quad 2\cos 4x - 1 = 0$$

$$\Rightarrow 3x = \sin^{-1}(0) \quad \text{or} \quad \cos 4x = \frac{1}{2}$$

$$\Rightarrow 3x = 0, \pi \quad \text{or} \quad 4x = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}, \frac{5\pi}{3}$$

$$\Rightarrow x = 0, \frac{\pi}{3} \quad \text{or} \quad x = \frac{\pi}{12}, \frac{5\pi}{12}$$

Since period of $\sin 3x$ is $\frac{2\pi}{3}$ and period of $\cos 4x$ is $\frac{2\pi}{4} = \frac{\pi}{2}$

\therefore general values of $x = 0 + \frac{2n\pi}{3}, \frac{\pi}{3} + \frac{2n\pi}{3}, \frac{\pi}{12} + \frac{n\pi}{2}, \frac{5\pi}{12} + \frac{n\pi}{2}$

$$\text{So S. set} = \left\{\frac{2n\pi}{3}\right\} \cup \left\{\frac{\pi}{3} + \frac{2n\pi}{3}\right\} \cup \left\{\frac{\pi}{12} + \frac{n\pi}{2}\right\} \cup \left\{\frac{5\pi}{12} + \frac{n\pi}{2}\right\} \quad \text{where } n \in \mathbb{Z}.$$

Question # 18

Find the solution set; $\sin x + \sin 3x + \sin 5x = 0$

Solution

$$\sin x + \sin 3x + \sin 5x = 0$$

$$\Rightarrow (\sin 5x + \sin x) + \sin 3x = 0$$

$$\Rightarrow 2\sin\left(\frac{5x+x}{2}\right) \cdot \cos\left(\frac{5x-x}{2}\right) + \sin 3x = 0$$

$$\Rightarrow 2\sin 3x \cos 2x + \sin 3x = 0$$

$$\Rightarrow \sin 3x(2\cos 2x + 1) = 0$$

$$\Rightarrow \sin 3x = 0 \quad \text{or} \quad 2\cos 2x + 1 = 0$$

$$\Rightarrow 3x = \sin^{-1}(0) \quad \text{or} \quad 2\cos 2x = -1 \Rightarrow 2x = \cos^{-1}\left(-\frac{1}{2}\right)$$

$$\Rightarrow 3x = 0, \pi \quad \text{or} \quad 2x = \frac{2\pi}{3}, \frac{4\pi}{3}$$

$$\Rightarrow x = 0, \frac{\pi}{3} \quad \text{or} \quad x = \frac{\pi}{3}, \frac{2\pi}{3}$$

Since period of $\sin 3x$ is $\frac{2\pi}{3}$ and period of $\cos 2x$ is $\frac{2\pi}{2} = \pi$

\therefore general values of $x = 0 + \frac{2n\pi}{3}, \frac{\pi}{3} + \frac{2n\pi}{3}, \frac{\pi}{3} + n\pi, \frac{2\pi}{3} + n\pi$

$$\text{S.Set} = \left\{ \frac{2n\pi}{3} \right\} \cup \left\{ \frac{\pi}{3} + \frac{2n\pi}{3} \right\} \cup \left\{ \frac{\pi}{3} + n\pi \right\} \cup \left\{ \frac{2\pi}{3} + n\pi \right\} \quad \text{where } n \in \mathbb{Z}$$

Question # 19

Find the solution set; $\sin \theta + \sin 3\theta + \sin 5\theta + \sin 7\theta = 0$

Solution

$$\sin \theta + \sin 3\theta + \sin 5\theta + \sin 7\theta = 0$$

$$\Rightarrow (\sin 7\theta + \sin \theta) + (\sin 5\theta + \sin 3\theta) = 0$$

$$\Rightarrow 2\sin\left(\frac{7\theta + \theta}{2}\right)\cos\left(\frac{7\theta - \theta}{2}\right) + 2\sin\left(\frac{5\theta + 3\theta}{2}\right)\cos\left(\frac{5\theta - 3\theta}{2}\right) = 0$$

$$\Rightarrow 2\sin 4\theta \cos 3\theta + 2\sin 4\theta \cos \theta = 0$$

$$\Rightarrow 2\sin 4\theta (\cos 3\theta + \cos \theta) = 0$$

$$\Rightarrow 2\sin 4\theta \left(2\cos\left(\frac{3\theta + \theta}{2}\right)\cos\left(\frac{3\theta - \theta}{2}\right) \right) = 0$$

$$\Rightarrow 4\sin 4\theta \cos 2\theta \cos \theta = 0$$

$$\Rightarrow \sin 4\theta = 0 \quad \text{or} \quad \cos 2\theta = 0 \quad \text{or} \quad \cos \theta = 0$$

$$\Rightarrow 4\theta = \sin^{-1}(0), \quad 2\theta = \cos^{-1}(0), \quad \theta = \cos^{-1}(0)$$

$$\Rightarrow 4\theta = 0, \pi \quad \text{or} \quad 2\theta = \frac{\pi}{2}, \frac{3\pi}{2} \quad \text{or} \quad \theta = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\Rightarrow \theta = 0, \frac{\pi}{4} \quad \text{or} \quad \theta = \frac{\pi}{4}, \frac{3\pi}{4}$$

Since period of $\sin 4\theta$ is $\frac{2\pi}{4} = \frac{\pi}{2}$, $\cos 2\theta$ is $\frac{2\pi}{2} = \pi$ and $\cos \theta$ is 2π

\therefore general values of $\theta = 0 + \frac{n\pi}{2}, \frac{\pi}{4} + \frac{n\pi}{2}, \frac{\pi}{4} + n\pi, \frac{3\pi}{4} + n\pi, \frac{\pi}{2} + 2n\pi, \frac{3\pi}{2} + 2n\pi$

$$\text{S. Set} = \left\{ \frac{n\pi}{2} \right\} \cup \left\{ \frac{\pi}{4} + \frac{n\pi}{2} \right\} \cup \left\{ \frac{\pi}{4} + n\pi \right\} \cup \left\{ \frac{3\pi}{4} + n\pi \right\} \cup \left\{ \frac{\pi}{2} + 2n\pi \right\} \cup \left\{ \frac{3\pi}{2} + 2n\pi \right\}.$$

Question # 20

Find the solution set; $\cos \theta + \cos 3\theta + \cos 5\theta + \cos 7\theta = 0$

Solution

$$\cos \theta + \cos 3\theta + \cos 5\theta + \cos 7\theta = 0$$

$$\Rightarrow (\cos 7\theta + \cos \theta) + (\cos 5\theta + \cos 3\theta) = 0$$

$$\begin{aligned}
\Rightarrow 2\cos\left(\frac{7\theta + \theta}{2}\right)\cos\left(\frac{7\theta - \theta}{2}\right) + 2\cos\left(\frac{5\theta + 3\theta}{2}\right)\cos\left(\frac{5\theta - 3\theta}{2}\right) &= 0 \\
\Rightarrow 2\cos 4\theta \cos 3\theta + 2\cos 4\theta \cos \theta &= 0 \\
\Rightarrow 2\cos 4\theta (\cos 3\theta + \cos \theta) &= 0 \\
\Rightarrow 2\cos 4\theta \left(2\cos\left(\frac{3\theta + \theta}{2}\right)\cos\left(\frac{3\theta - \theta}{2}\right)\right) &= 0 \\
\Rightarrow 4\cos 4\theta \cos 2\theta \cos \theta &= 0
\end{aligned}$$

Now do yourself as above question.

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Error Analyst	
Halima Shafiq Ali Mazhar Gondal Junaid	Punjab College

Book: **Exercise 14**
Text Book of Algebra and Trigonometry Class XI

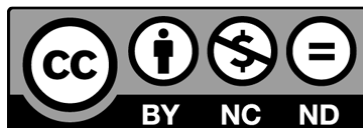
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