New - ?

14 Solutions of Trigonometric Equations

Trigonometric Equations :- The equations involving atleast one trigonometric function, are called Trigonometric Equations. For example $\sin^2 x = \frac{1}{4}$, $\sec x = \tan x$; $\sin^2 x - \sec x + 1 = 0$ etc.

EXERCISE 14

① Find the Solutions of the following equations which lie in $[0, 2\pi]$

(i) $\sin x = -\frac{\sqrt{3}}{2}$

Sol:- Given that $\sin x = -\frac{\sqrt{3}}{2}$

$\because \sin x$ is $-ve$ in III and IV quad. with reference angle $= \frac{\pi}{3}$

$\therefore x = \pi + \frac{\pi}{3}$ and $x = 2\pi - \frac{\pi}{3}$ $\because x \in [0, 2\pi]$

$\Rightarrow x = \frac{4\pi}{3}$, $x = \frac{5\pi}{3}$

$\therefore S.S. = \{\frac{4\pi}{3}, \frac{5\pi}{3}\}$

(ii) $\operatorname{cosec}\theta = 2$

Sol:- Given that $\operatorname{cosec}\theta = 2$

$\Rightarrow \sin\theta = \frac{1}{\operatorname{cosec}\theta} = \frac{1}{2}$

$\because \sin\theta$ is $+ve$ in I and II quad. with reference angle $= \frac{\pi}{6}$

$\therefore \theta = \frac{\pi}{6}$ and $\theta = \pi - \frac{\pi}{6}$ $\because \theta \in [0, 2\pi]$

$\Rightarrow \theta = \pi/6$ and $\theta = \frac{5\pi}{6}$

$\therefore S.S. = \{\frac{\pi}{6}, \frac{5\pi}{6}\}$

(iii) $\sec x = -2$

Sol:- Given that $\sec x = -2$

$\Rightarrow \cos x = \frac{1}{\sec x} = \frac{1}{-2} = -\frac{1}{2}$

$\because \cos x$ is $-ve$ in II and III quad. with reference angle $= \frac{\pi}{3}$

$\therefore x = \pi - \frac{\pi}{3}$ and $x = \pi + \frac{\pi}{3}$ $\because x \in [0, 2\pi]$

$\Rightarrow x = \frac{2\pi}{3}$ and $x = \frac{4\pi}{3}$

$\therefore S.S. = \{\frac{2\pi}{3}, \frac{4\pi}{3}\}$

(iv) $\cot\theta = \frac{1}{\sqrt{3}}$

Sol:- $\cot\theta = \frac{1}{\sqrt{3}}$ $\Rightarrow \tan\theta = \frac{1}{\cot\theta} = \frac{1}{1/\sqrt{3}}$

$\Rightarrow \tan\theta = \sqrt{3}$

$\because \tan\theta$ is $+ve$ in I and III quad. with reference angle $= \frac{\pi}{3}$

$\therefore \theta = \frac{\pi}{3}$ and $\theta = \pi + \frac{\pi}{3}$, $\theta \in [0, 2\pi]$

$\Rightarrow \theta = \frac{\pi}{3}$ and $\theta = \frac{4\pi}{3}$

$\therefore S.S. = \{\frac{\pi}{3}, \frac{4\pi}{3}\}$

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② Solve the following trigonometric equations.

(i) $\tan^2\theta = \frac{1}{3}$

Sol:- Given that $\tan^2\theta = \frac{1}{3}$

$\Rightarrow \tan\theta = \pm\sqrt{\frac{1}{3}}$

$\Rightarrow \tan\theta = \pm\frac{1}{\sqrt{3}}$

$\Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$

$\because \tan\theta$ is $+ve$ in I and III quad. with reference angle $= \frac{\pi}{6}$

$\therefore \theta = \frac{\pi}{6}$ & $\theta = \pi + \frac{\pi}{6}$

$\Rightarrow \theta = \frac{\pi}{6}$ & $\theta = \frac{7\pi}{6}$ where $\theta \in [0, 2\pi]$

$\because \pi$ is the period of $\tan\theta$

$\therefore$ general values of θ are $\frac{\pi}{6} + n\pi$, $n \in Z$

$\tan\theta = -\frac{1}{\sqrt{3}}$

$\because \tan\theta$ is $-ve$ in II & IV quad. with reference angle $= \frac{\pi}{6}$

$\therefore \theta = \pi - \frac{\pi}{6}$ & $\theta = 2\pi - \frac{\pi}{6}$

$\theta = \frac{5\pi}{6}$ & $\theta = \frac{11\pi}{6}$ where $\theta \in [0, 2\pi]$

$\because \pi$ is the period of $\tan\theta$

$\therefore$ general values of θ are $\frac{5\pi}{6} + n\pi$, $n \in Z$

$\therefore S.S. = \{\frac{\pi}{6} + n\pi\} \cup \{\frac{5\pi}{6} + n\pi\}, n \in Z$

(ii) Given that $\operatorname{cosec}^2\theta = \frac{4}{3}$

$\Rightarrow \operatorname{cosec}\theta = \pm\frac{2}{\sqrt{3}}$

$\Rightarrow \operatorname{cosec}\theta = \frac{2}{\sqrt{3}}$

$\Rightarrow \sin\theta = \frac{1}{\operatorname{cosec}\theta} = \frac{1}{2/\sqrt{3}}$

$\Rightarrow \sin\theta = \frac{\sqrt{3}}{2}$

$\because \sin\theta$ is $+ve$ in I and II quad. with reference angle $= \frac{\pi}{3}$

$\therefore \theta = \frac{\pi}{3}$ & $\theta = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$ where $\theta \in [0, 2\pi]$

$\because 2\pi$ is the period of $\sin\theta$

$\therefore$ general values of θ are $\frac{\pi}{3} + 2n\pi$

$\operatorname{cosec}\theta = -\frac{2}{\sqrt{3}}$

$\Rightarrow \sin\theta = \frac{1}{\operatorname{cosec}\theta} = \frac{1}{-\frac{2}{\sqrt{3}}}$

$\Rightarrow \sin\theta = -\frac{\sqrt{3}}{2}$

$\because \sin\theta$ is $-ve$ in III and IV quad with reference angle $= \frac{\pi}{3}$

$\therefore \theta = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$ & $\theta = 2\pi - \frac{\pi}{3}$

$\theta = \frac{4\pi}{3}$ & $\theta = \frac{5\pi}{3}$

where $\theta \in [0, 2\pi]$

$\because 2\pi$ is the period of $\sin\theta$

and $\frac{2\pi}{3}+2n\pi$, $n\in\mathbb{Z}$

$\therefore$ general values of θ are $\frac{4\pi}{3}+2n\pi$ and $\frac{5\pi}{3}+2n\pi$, $n\in\mathbb{Z}$

$\therefore S.S. = \left\{\frac{\pi}{3}+2n\pi\right\} \cup \left\{\frac{2\pi}{3}+2n\pi\right\} \cup \left\{\frac{4\pi}{3}+2n\pi\right\} \cup \left\{\frac{5\pi}{3}+2n\pi\right\}$, $n\in\mathbb{Z}$

(iii) Given that $\sec^2\theta = \frac{4}{3}$

$\Rightarrow \sec\theta = \pm\frac{2}{\sqrt{3}}$

$\Rightarrow \sec\theta = \frac{2}{\sqrt{3}}$

$\Rightarrow \cos\theta = \frac{1}{\sec\theta} = \frac{1}{2/\sqrt{3}} = \frac{\sqrt{3}}{2}$

$\because \cos\theta$ is +ve in I & IV quad. with reference angle $= \frac{\pi}{6}$

$\therefore \theta = \frac{\pi}{6}$ & $\theta = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$ where $\theta \in [0, 2\pi]$

$\because 2\pi$ is the period of $\cos\theta$

$\therefore$ general values of θ are $\frac{\pi}{6}+2n\pi$ and $\frac{11\pi}{6}+2n\pi$, $n\in\mathbb{Z}$

$\sec\theta = -\frac{2}{\sqrt{3}}$

$\cos\theta = \frac{1}{\sec\theta} = \frac{1}{-2/\sqrt{3}} = -\frac{\sqrt{3}}{2}$

$\because \cos\theta$ is −ve in II & III quad. with ref. angle $= \frac{\pi}{6}$

$\therefore \theta = \pi - \pi/6 = \frac{5\pi}{6}$ and $\theta = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$ where $\theta\in[0,2\pi]$

$\because 2\pi$ is the period of $\cos\theta$

$\therefore$ general values of θ are $\frac{5\pi}{6}+2n\pi$ & $\frac{7\pi}{6}+2n\pi$, $n\in\mathbb{Z}$

$\therefore S.S. = \left\{\frac{\pi}{6}+2n\pi\right\} \cup \left\{\frac{11\pi}{6}+2n\pi\right\} \cup \left\{\frac{5\pi}{6}+2n\pi\right\} \cup \left\{\frac{7\pi}{6}+2n\pi\right\}$, $n\in\mathbb{Z}$

(iv) Given that $\cot^2\theta = \frac{1}{3}$

$\Rightarrow \cot\theta = \pm\frac{1}{\sqrt{3}}$

$\Rightarrow \cot\theta = \frac{1}{\sqrt{3}}$

$\Rightarrow \tan\theta = \frac{1}{\cot\theta} = \sqrt{3}$

$\because \tan\theta$ is +ve in I & III quad. with ref. angle $= \frac{\pi}{3}$

$\therefore \theta = \frac{\pi}{3}$ & $\theta = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$ where $\theta\in[0,2\pi]$

$\because \pi$ is the period of $\tan\theta$

$\therefore$ general values of θ are $\frac{\pi}{3}+n\pi$, $n\in\mathbb{Z}$

$\cot\theta = -\frac{1}{\sqrt{3}}$

$\Rightarrow \tan\theta = \frac{1}{\cot\theta} = -\sqrt{3}$

$\because \tan\theta$ is −ve in II & IV quad. with ref. angle $= \frac{\pi}{3}$

$\therefore \theta = \pi - \pi/3 = \frac{2\pi}{3}$ and $\theta = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$ where $\theta\in[0,2\pi]$

$\because \pi$ is the period of $\tan\theta$

$\therefore$ general values of θ are $\frac{2\pi}{3}+n\pi$, $n\in\mathbb{Z}$

$\therefore S.S. = \left\{\frac{\pi}{3}+n\pi\right\} \cup \left\{\frac{2\pi}{3}+n\pi\right\}$

③ $3\tan^2\theta + 2\sqrt{3}\tan\theta + 1 = 0$

$\Rightarrow (\sqrt{3}\tan\theta + 1)^2 = 0$

$\Rightarrow \sqrt{3}\tan\theta + 1 = 0$

$\Rightarrow \sqrt{3}\tan\theta = -1$

$\Rightarrow \tan\theta = -\frac{1}{\sqrt{3}}$ (−ve)

$\because \tan\theta$ is −ve in II and IV quad. with reference angle $= \frac{\pi}{6}$

$\therefore \theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$ & $\theta = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$ where $\theta\in[0,2\pi]$

$\because \pi$ is the period of $\tan\theta$

$\therefore$ general values of θ are $\frac{5\pi}{6}+n\pi$, $n\in\mathbb{Z}$

$\therefore S.S. = \left\{\frac{5\pi}{6}+n\pi\right\}$, $n\in\mathbb{Z}$

④ $\tan^2\theta - \sec\theta - 1 = 0$

$\Rightarrow \sec^2\theta - 1 - \sec\theta - 1 = 0$

$\Rightarrow \sec^2\theta - \sec\theta - 2 = 0$

$\Rightarrow \sec^2\theta - 2\sec\theta + \sec\theta - 2 = 0$

$\Rightarrow \sec\theta(\sec\theta - 2) + 1(\sec\theta - 2) = 0$

$\Rightarrow (\sec\theta - 2)(\sec\theta + 1) = 0$

$\Rightarrow \sec\theta - 2 = 0$

$\Rightarrow \sec\theta = 2$

$\Rightarrow \cos\theta = \frac{1}{\sec\theta} = \frac{1}{2}$

$\because \cos\theta$ is +ve in I & IV quad. with ref. angle $= \frac{\pi}{3}$

$\therefore \theta = \frac{\pi}{3}$ & $\theta = 2\pi - \frac{\pi}{3}$, $\theta = \frac{5\pi}{3}$ where $\theta\in[0,2\pi]$

$\because 2\pi$ is the period of $\cos\theta$

$\therefore$ general values of θ are $\frac{\pi}{3}+2n\pi$, $\frac{5\pi}{3}+2n\pi$, $n\in\mathbb{Z}$

$\sec\theta + 1 = 0$

$\Rightarrow \sec\theta = -1$

$\Rightarrow \cos\theta = \frac{1}{\sec\theta} = \frac{1}{-1} = -1$

$\because \cos\theta$ is −ve in II & III quad. with ref. angle $= 0$

$\therefore \theta = \pi - 0$ & $\theta = \pi + 0$

$\Rightarrow \theta = \pi$, $\theta\in[0,2\pi]$

$\because 2\pi$ is the period of $\cos\theta$

$\therefore$ general values of θ are $\pi + 2n\pi$ where $n\in\mathbb{Z}$

$\therefore S.S. = \left\{\frac{\pi}{3}+2n\pi\right\} \cup \left\{\frac{5\pi}{3}+2n\pi\right\} \cup \left\{\pi+2n\pi\right\}$ $n\in\mathbb{Z}$.

⑤ $2\sin\theta + \cos^2\theta - 1 = 0$

$\Rightarrow 2\sin\theta + \cancel{1} - \sin^2\theta - \cancel{1} = 0$

$\Rightarrow 2\sin\theta - \sin^2\theta = 0$

$\Rightarrow \sin\theta(2 - \sin\theta) = 0$

$\Rightarrow \sin\theta = 0$

$\Rightarrow \theta = n\pi$, $n\in\mathbb{Z}$

$2 - \sin\theta = 0$

$\Rightarrow -\sin\theta = -2$

$\Rightarrow \sin\theta = 2$

which is impossible $\because \sin\theta \in [-1, 1]$

$\therefore S.S. = \{n\pi\}$, $n\in\mathbb{Z}$

⑥ $2\sin^2\theta - \sin\theta = 0$

$\Rightarrow \sin\theta(2\sin\theta - 1) = 0$

$\Rightarrow \sin\theta = 0$

$\Rightarrow \theta = n\pi$, $n\in\mathbb{Z}$

$2\sin\theta - 1 = 0$

$\Rightarrow 2\sin\theta = 1$

$\Rightarrow \sin\theta = \frac{1}{2}$

$\because \sin\theta$ is +ve in I & II quad. with ref. angle $= \frac{\pi}{6}$

$\therefore \theta = \frac{\pi}{6}$ & $\theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$ where $\theta \in [0, 2\pi]$

$\because 2\pi$ is the period of $\sin\theta$

$\therefore$ general values of θ are $\frac{\pi}{6} + 2n\pi$ and $\frac{5\pi}{6} + 2n\pi$, $n \in Z$

$\therefore S.S. = \{n\pi\} \cup \{\frac{\pi}{6} + 2n\pi\} \cup \{\frac{5\pi}{6} + 2n\pi\}$, $n \in Z$.

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(7) $3\cos^2\theta - 2\sqrt{3}\cos\theta\sin\theta - 3\sin^2\theta = 0$

Dividing by $\sin^2\theta$, we get

$3\cot^2\theta - 2\sqrt{3}\cot\theta - 3 = 0$

Using quadratic formula,

$\cot\theta = \frac{-(-2\sqrt{3}) \pm \sqrt{(-2\sqrt{3})^2 - 4(3)(-3)}}{2(3)}$

$= \frac{2\sqrt{3} \pm \sqrt{12+36}}{6} = \frac{2\sqrt{3} \pm \sqrt{48}}{6}$

$= \frac{2\sqrt{3} \pm 4\sqrt{3}}{6} = \frac{2\sqrt{3}+4\sqrt{3}}{6}, \frac{2\sqrt{3}-4\sqrt{3}}{6}$

$= \frac{6\sqrt{3}}{6}, \frac{-2\sqrt{3}}{6} = \sqrt{3}, -\frac{\sqrt{3}}{3}$

$\because \cot\theta = \sqrt{3}, -\frac{1}{\sqrt{3}}$

$\Rightarrow \cot\theta = \sqrt{3}$

$\Rightarrow \tan\theta = \frac{1}{\cot\theta} = \frac{1}{\sqrt{3}}$

$\because \tan\theta$ is +ve in I & III quad. with ref. angle $= \frac{\pi}{6}$

$\therefore \theta = \frac{\pi}{6}$ & $\theta = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$ where $\theta \in [0, 2\pi]$

$\because \pi$ is the period of $\tan\theta$

$\therefore$ general values of θ are $\frac{\pi}{6} + n\pi$, $n \in Z$

$\cot\theta = -\frac{1}{\sqrt{3}}$

$\Rightarrow \tan\theta = \frac{1}{\cot\theta} = -\sqrt{3}$

$\because \tan\theta$ is -ve in II and IV quad. with ref. angle $= \frac{\pi}{3}$

$\therefore \theta = \pi - \frac{\pi}{3}, 2\pi - \pi/3$

$\theta = \frac{2\pi}{3}$, $\theta = \frac{5\pi}{3}$ where $\theta \in [0, 2\pi]$

$\because \pi$ is the period of $\tan\theta$

$\therefore$ general values of θ are $\frac{2\pi}{3} + n\pi$, $n \in Z$

$\therefore S.S. = \{\frac{\pi}{6} + n\pi\} \cup \{\frac{2\pi}{3} + n\pi\}$ where $n \in Z$.

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(8) $4\sin^2\theta - 8\cos\theta + 1 = 0$

$\Rightarrow 4(1 - \cos^2\theta) - 8\cos\theta + 1 = 0$

$\Rightarrow 4 - 4\cos^2\theta - 8\cos\theta + 1 = 0$

$\Rightarrow -4\cos^2\theta - 8\cos\theta + 5 = 0$

$\Rightarrow -1(4\cos^2\theta + 8\cos\theta - 5) = 0$

$\Rightarrow 4\cos^2\theta + 8\cos\theta - 5 = 0 \quad \because -1 \neq 0$

$\Rightarrow 4\cos^2\theta + 10\cos\theta - 2\cos\theta - 5 = 0$

$\Rightarrow 2\cos\theta(2\cos\theta + 5) - 1(2\cos\theta + 5) = 0$

$\Rightarrow (2\cos\theta + 5)(2\cos\theta - 1) = 0$

$\Rightarrow 2\cos\theta + 5 = 0$

$\Rightarrow 2\cos\theta = -5$

$\Rightarrow \cos\theta = -\frac{5}{2}$

$\Rightarrow \cos\theta = -2.5$

which is impossible $\because \cos\theta \in [-1, 1]$

$2\cos\theta - 1 = 0$

$\Rightarrow 2\cos\theta = 1$

$\Rightarrow \cos\theta = \frac{1}{2}$

$\because \cos\theta$ is +ve in I & IV quad. with ref. angle $= \frac{\pi}{3}$

$\therefore \theta = \frac{\pi}{3}$ and $\theta = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$ where $\theta \in [0, 2\pi]$

$\because 2\pi$ is the period of $\cos\theta$

$\therefore$ general values of θ are $\frac{\pi}{3} + 2n\pi$ and $\frac{5\pi}{3} + 2n\pi$, $n \in Z$

$\therefore S.S. = \{\frac{\pi}{3} + 2n\pi\} \cup \{\frac{5\pi}{3} + 2n\pi\}$, $n \in Z$

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(9) $\sqrt{3}\tan x - \sec x - 1 = 0$ ——— (1)

$\Rightarrow \sqrt{3}\tan x = \sec x + 1$

Squaring both sides

$\Rightarrow (\sqrt{3}\tan x)^2 = (\sec x + 1)^2$

$\Rightarrow 3\tan^2 x = \sec^2 x + 2\sec x + 1$

$\Rightarrow 3\tan^2 x - \sec^2 x - 2\sec x - 1 = 0$

$\Rightarrow 3(\sec^2 x - 1) - \sec^2 x - 2\sec x - 1 = 0$

$\Rightarrow 3\sec^2 x - 3 - \sec^2 x - 2\sec x - 1 = 0$

$\Rightarrow 2\sec^2 x - 2\sec x - 4 = 0$

$\Rightarrow 2(\sec^2 x - \sec x - 2) = 0$

$\Rightarrow \sec^2 x - \sec x - 2 = 0 \quad \because 2 \neq 0$

$\Rightarrow \sec^2 x - 2\sec x + \sec x - 2 = 0$

$\Rightarrow \sec x(\sec x - 2) + 1(\sec x - 2) = 0$

$\Rightarrow (\sec x - 2)(\sec x + 1) = 0$

$\Rightarrow \sec x - 2 = 0$

$\Rightarrow \sec x = 2$

$\Rightarrow \cos x = \frac{1}{\sec x} = \frac{1}{2}$

$\because \cos x$ is +ve in I & IV quad. with ref. angle $= \frac{\pi}{3}$

$\sec x + 1 = 0$

$\Rightarrow \sec x = -1$

$\Rightarrow \cos x = \frac{1}{\sec x} = \frac{1}{-1}$

$\Rightarrow \cos x = -1$

$\because \cos x$ is -ve in II & III quad. with ref. angle $= 0$

$\therefore x = \frac{\pi}{3}$ & $x = 2\pi - \frac{\pi}{3}$
$\Rightarrow x = \frac{\pi}{3}$ & $x = \frac{5\pi}{3}$
where $x \in [0, 2\pi]$

$\therefore x = \pi - 0$ & $x = \pi + 0$
$\Rightarrow x = \pi$ where $x \in [0, 2\pi]$

Putting $x = \frac{\pi}{3}$ in ①, we get

$\sqrt{3}\tan\frac{\pi}{3} - \sec\frac{\pi}{3} - 1 = 0$
$\Rightarrow \sqrt{3}\cdot\sqrt{3} - 2 - 1 = 0$
$\Rightarrow 3 - 2 - 1 = 0$
$\Rightarrow 3 - 3 = 0$
$\Rightarrow 0 = 0$ (Satisfied)

$\therefore x = \frac{\pi}{3}$ is a solution of ①
$\because 2\pi$ is the period of $\cos x$
$\therefore$ general values of x are
$\frac{\pi}{3} + 2n\pi,\ n \in Z$

Putting $x = \frac{5\pi}{3}$ in ①, we get

$\sqrt{3}\tan\frac{5\pi}{3} - \sec\frac{5\pi}{3} - 1 = 0$
$\Rightarrow \sqrt{3}(-\sqrt{3}) - 2 - 1 = 0$
$\Rightarrow -3 - 2 - 1 = 0$
$\Rightarrow -6 = 0$ (not satisfied)

$\therefore x = \frac{5\pi}{3}$ is not a solution of ①

Putting $x = \pi$ in ①, we get

$\sqrt{3}\tan\pi - \sec\pi - 1 = 0$
$\Rightarrow \sqrt{3}(0) - (-1) - 1 = 0$
$\Rightarrow 0 + 1 - 1 = 0$
$\Rightarrow 0 = 0$ (Satisfied)

$\therefore x = \pi$ is a solution of ①
$\because 2\pi$ is the period of $\cos x$
$\therefore$ general values of x are $\pi + 2n\pi,\ n \in Z$

Thus $S.S. = \{\frac{\pi}{3} + 2n\pi\} \cup \{\pi + 2n\pi\},\ n \in Z$

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⑩ $\cos 2x = \sin 3x$

$\Rightarrow \cos^2 x - \sin^2 x = 3\sin x - 4\sin^3 x$
$\Rightarrow \cos^2 x - \sin^2 x - 3\sin x + 4\sin^3 x = 0$
$\Rightarrow 1 - \sin^2 x - \sin^2 x - 3\sin x + 4\sin^3 x = 0$
$\Rightarrow 4\sin^3 x - 2\sin^2 x - 3\sin x + 1 = 0$ —①

$\because \sin x = 1$ satisfies eq. ①
$\therefore \sin x = 1$ is a root of ①

To find other roots
Using Synthetic Division

$$\begin{array}{r|rrrr} 1 & 4 & -2 & -3 & 1 \\ & \downarrow & 4 & 2 & -1 \\ \hline & 4 & 2 & -1 & \boxed{0} \end{array}$$

The depressed equation is
$4\sin^2 x + 2\sin x - 1 = 0$
Using quadratic formula

$$\sin x = \frac{-2 \pm \sqrt{(2)^2 - 4(4)(-1)}}{2(4)}$$

$\Rightarrow \sin x = \frac{-2 \pm \sqrt{4+16}}{8} = \frac{-2 \pm \sqrt{20}}{8}$
$\Rightarrow \sin x = \frac{-2 \pm 2\sqrt{5}}{8} = \cancel{2}\left[\frac{-1 \pm \sqrt{5}}{\cancel{8}4}\right]$
$\Rightarrow \sin x = \frac{-1 \pm \sqrt{5}}{4}$
$\Rightarrow \sin x = \frac{-1+\sqrt{5}}{4}$ & $\sin x = \frac{-1-\sqrt{5}}{4}$
$\Rightarrow \sin x = 0.3090$ & $\sin x = -0.8090$

$\therefore$ Roots of ① are
$\sin x = 1$, $\sin x = 0.3090$, $\sin x = -0.8090$

Now $\sin x = 1$
$\because \sin x$ is +ve in I & II quad.
with ref. angle $= \pi/2$
$\therefore x = \frac{\pi}{2},\ \pi - \frac{\pi}{2} = \frac{\pi}{2}, \frac{\pi}{2}$
$\Rightarrow x = \frac{\pi}{2}$ where $x \in [0, 2\pi]$
$\because 2\pi$ is the period of $\sin x$
$\therefore$ general values of x are
$\frac{\pi}{2} + 2n\pi,\ n \in Z$

For $\sin x = 0.3090$
$\because \sin x$ is +ve in I & II quad.
with ref. angle $= \sin^{-1}(0.3090)$
$= 18° = 18 \times \frac{\pi}{180}$ rad.
$= \frac{\pi}{10}$ radians

$\therefore x = \frac{\pi}{10}$ and $x = \pi - \frac{\pi}{10} = \frac{9\pi}{10}$ where $x \in [0, 2\pi]$
$\because 2\pi$ is the period of $\sin x$
$\therefore$ general values of x are
$\frac{\pi}{10} + 2n\pi$ and $\frac{9\pi}{10} + 2n\pi,\ n \in Z$

For $\sin x = -0.8090$
$\because \sin x$ is −ve in III and IV quad.
with ref. angle $= \sin^{-1}(+0.8090)$
$= 54° = 54 \times \frac{\pi}{180}$ rad.
$= \frac{3\pi}{10}$ radians

$\therefore x = \pi + \frac{3\pi}{10} = \frac{13\pi}{10}$ & $x = 2\pi - \frac{3\pi}{10} = \frac{17\pi}{10}$
where $x \in [0, 2\pi]$
$\because 2\pi$ is the period of $\sin x$
$\therefore$ general values of x are
$\frac{13\pi}{10} + 2n\pi$ and $\frac{17\pi}{10} + 2n\pi,\ n \in Z$

$\therefore S.S. = \{\frac{\pi}{2} + 2n\pi\} \cup \{\frac{\pi}{10} + 2n\pi\} \cup \{\frac{9\pi}{10} + 2n\pi\}$
$\cup \{\frac{13\pi}{10} + 2n\pi\} \cup \{\frac{17\pi}{10} + 2n\pi\},\ n \in Z$

(11) $\sec 3\theta = \sec\theta$

$\Rightarrow \frac{1}{\cos 3\theta} = \frac{1}{\cos\theta}$

$\Rightarrow \cos\theta = \cos 3\theta$

$\Rightarrow \cos\theta - \cos 3\theta = 0$

$\Rightarrow -2\sin\left(\frac{\theta+3\theta}{2}\right)\sin\left(\frac{\theta-3\theta}{2}\right) = 0$

$\Rightarrow -2\sin 2\theta \sin(-\theta) = 0$

$\Rightarrow -2\sin 2\theta(-\sin\theta) = 0$

$\Rightarrow 2\sin 2\theta \sin\theta = 0$

$\Rightarrow \sin 2\theta \sin\theta = 0 \quad \because 2 \neq 0$

$\Rightarrow \sin 2\theta = 0$ | $\sin\theta = 0$

$\Rightarrow 2\theta = n\pi, n\in Z$ | $\Rightarrow \theta = n\pi, n\in Z$

$\Rightarrow \theta = \frac{n\pi}{2}, n\in Z$

$\therefore S.S. = \{\frac{n\pi}{2}\} \cup \{n\pi\}$

(12) $\tan 2\theta + \cot\theta = 0$

$\Rightarrow \frac{\sin 2\theta}{\cos 2\theta} + \frac{\cos\theta}{\sin\theta} = 0$

$\Rightarrow \frac{\sin 2\theta \sin\theta + \cos 2\theta\cos\theta}{\cos 2\theta \sin\theta} = 0$

$\Rightarrow \sin 2\theta \sin\theta + \cos 2\theta \cos\theta = 0$

$\Rightarrow \cos 2\theta \cos\theta + \sin 2\theta \sin\theta = 0$

$\Rightarrow \cos(2\theta - \theta) = 0$

$\Rightarrow \cos\theta = 0$

$\Rightarrow \theta = (2n+1)\frac{\pi}{2}, \quad n\in Z$

$\therefore S.S. = \{(2n+1)\frac{\pi}{2}\}, n\in Z$

Method II

$\tan 2\theta + \cot\theta = 0$

$\Rightarrow \frac{\sin 2\theta}{\cos 2\theta} + \frac{\cos\theta}{\sin\theta} = 0 \Rightarrow \frac{\sin 2\theta\sin\theta + \cos 2\theta\cos\theta}{\cos 2\theta \sin\theta} = 0$

$\Rightarrow \sin 2\theta \sin\theta + \cos 2\theta\cos\theta = 0$

$\Rightarrow \cos 2\theta\cos\theta + \sin 2\theta \sin\theta = 0$

$\Rightarrow \cos(2\theta - \theta) = 0 \quad \Rightarrow \cos\theta = 0$

$\Rightarrow \theta = \frac{\pi}{2}$ and $\theta = \frac{3\pi}{2}$ where $\theta\in[0, 2\pi]$

$\because 2\pi$ is the period of $\cos\theta$

$\therefore$ general values of θ are

$\frac{\pi}{2} + 2n\pi$ and $\frac{3\pi}{2} + 2n\pi, \; n\in Z$

$\therefore S.S. = \{\frac{\pi}{2} + 2n\pi\} \cup \{\frac{3\pi}{2} + 2n\pi\}, n\in Z$

$= \{(2n+1)\frac{\pi}{2}\}, \quad n\in Z$ Ans.

(13) $\sin 2x + \sin x = 0$

$\Rightarrow 2\sin x\cos x + \sin x = 0$

$\Rightarrow \sin x[2\cos x + 1] = 0$

$\Rightarrow \sin x = 0$

$\Rightarrow x = 0$ & $x = \pi$ where $x\in[0, 2\pi]$

$\because 2\pi$ is the period of $\sin x$

$\therefore$ general values of x are $0 + 2n\pi = 2n\pi$ and $\pi + 2n\pi$, $n\in Z$

$2\cos x + 1 = 0$

$2\cos x = -1$

$\Rightarrow \cos x = -\frac{1}{2}$

$\because \cos x$ is $-$ve in II and III quad. with ref. angle $= \frac{\pi}{3}$

$\therefore x = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$ &

$x = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$

where $x\in[0, 2\pi]$

$\because 2\pi$ is the period of $\cos x$

$\therefore$ general values of x are $\frac{2\pi}{3} + 2n\pi$ &

$\frac{4\pi}{3} + 2n\pi, n\in Z$

$\therefore S.S. = \{2n\pi\} \cup \{\pi + 2n\pi\} \cup \{\frac{2\pi}{3} + 2n\pi\}$

$\cup \{\frac{4\pi}{3} + 2n\pi\}, \; n\in Z$

$= \{n\pi\} \cup \{\frac{2\pi}{3} + 2n\pi\} \cup \{\frac{4\pi}{3} + 2n\pi\}$

$n\in Z$

(14) $\sin 4x - \sin 2x = \cos 3x$

$\Rightarrow 2\cos\left(\frac{4x+2x}{2}\right)\sin\left(\frac{4x-2x}{2}\right) = \cos 3x$

$\Rightarrow 2\cos 3x \sin x = \cos 3x$

$\Rightarrow 2\cos 3x \sin x - \cos 3x = 0$

$\Rightarrow \cos 3x(2\sin x - 1) = 0$

$\Rightarrow \cos 3x = 0$

$\Rightarrow 3x = \frac{\pi}{2}$ &

$3x = \frac{3\pi}{2}$

$\because 2\pi$ is the period of $\cos$

$\therefore$ general values of $3x$ are

$3x = \frac{\pi}{2} + 2n\pi$

& $3x = \frac{3\pi}{2} + 2n\pi$

$\Rightarrow x = \frac{\pi}{6} + \frac{2n\pi}{3}$ &

$x = \frac{\pi}{2} + \frac{2n\pi}{3}$

$n\in Z$

$2\sin x - 1 = 0$

$\Rightarrow 2\sin x = 1$

$\Rightarrow \sin x = \frac{1}{2}$

$\because \sin x$ is +ve in I & II quad. with ref. angle $= \frac{\pi}{6}$

$\therefore x = \frac{\pi}{6}$ & $x = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

where $x\in[0, 2\pi]$

$\because 2\pi$ is the period of $\sin x$

$\therefore x = \frac{\pi}{6} + 2n\pi$ &

$x = \frac{5\pi}{6} + 2n\pi$,

$n\in Z$

$\therefore S.S. = \{\frac{\pi}{6} + \frac{2n\pi}{3}\} \cup \{\frac{\pi}{2} + 2n\pi\}$

$\cup \{\frac{\pi}{6} + 2n\pi\} \cup \{\frac{5\pi}{6} + 2n\pi\}, n\in Z$

(15) $\sin x + \cos 3x = \cos 5x$

$\Rightarrow \sin x = \cos 5x - \cos 3x$

$\Rightarrow \sin x = -2\sin\left(\frac{5x+3x}{2}\right)\sin\left(\frac{5x-3x}{2}\right)$

$\Rightarrow \sin x = -2\sin 4x \sin x$

$\Rightarrow \sin x + 2\sin 4x \sin x = 0$

$\Rightarrow \sin x\,[1 + 2\sin 4x] = 0$

$\Rightarrow \sin x = 0$

$\Rightarrow x = 0$ & $x = \pi$ where $x \in [0, 2\pi]$

$\because 2\pi$ is the period of $\sin x$

$\therefore x = 0 + 2n\pi = 2n\pi$

& $x = \pi + 2n\pi$

$n \in \mathbb{Z}$

$1 + 2\sin 4x = 0$

$\Rightarrow 2\sin 4x = -1$

$\Rightarrow \sin 4x = -\frac{1}{2}$

$\because \sin 4x$ is $-ve$ in III & IV quad. with ref. angle $= \frac{\pi}{6}$

$\therefore 4x = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$ &

$4x = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$

$\because 2\pi$ is the period of sin

$\therefore 4x = \frac{7\pi}{6} + 2n\pi$ &

$4x = \frac{11\pi}{6} + 2n\pi$

$\Rightarrow x = \frac{7\pi}{24} + \frac{n\pi}{2}$ &

$x = \frac{11\pi}{24} + \frac{n\pi}{2}$, $n \in \mathbb{Z}$

$\therefore S.S. = \{2n\pi\} \cup \{\pi + 2n\pi\} \cup \{\frac{7\pi}{24} + \frac{n\pi}{2}\} \cup \{\frac{11\pi}{24} + \frac{n\pi}{2}\}$, $n \in \mathbb{Z}$.

or $S.S = \{n\pi\} \cup \{\frac{7\pi}{24} + \frac{n\pi}{2}\} \cup \{\frac{11\pi}{24} + \frac{n\pi}{2}\}$, $n \in \mathbb{Z}$

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(16) $\sin 3x + \sin 2x + \sin x = 0$

$\Rightarrow (\sin 3x + \sin x) + \sin 2x = 0$

$\Rightarrow 2\sin\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right) + \sin 2x = 0$

$\Rightarrow 2\sin 2x \cos x + \sin 2x = 0$

$\Rightarrow \sin 2x\,(2\cos x + 1) = 0$

$\Rightarrow \sin 2x = 0$

$\Rightarrow 2x = 0$ & $2x = \pi$

$\because 2\pi$ is the period of sin

$\therefore 2x = 0 + 2n\pi$

& $2x = \pi + 2n\pi$

$\Rightarrow x = n\pi$ &

$x = \frac{\pi}{2} + n\pi$

$n \in \mathbb{Z}$.

$2\cos x + 1 = 0$

$\Rightarrow 2\cos x = -1$

$\Rightarrow \cos x = -\frac{1}{2}$

$\because \cos x$ is $-ve$ in II & III quad. with ref. angle $= \frac{\pi}{3}$

$\therefore x = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$ and

$x = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$

$\because 2\pi$ is the period of cos

$\therefore$ general values of x are

$\frac{2\pi}{3} + 2n\pi$ & $\frac{4\pi}{3} + 2n\pi$, $n \in \mathbb{Z}$.

$\therefore S.S = \{n\pi\} \cup \{\frac{\pi}{2} + n\pi\} \cup \{\frac{2\pi}{3} + 2n\pi\} \cup \{\frac{4\pi}{3} + 2n\pi\}$, $n \in \mathbb{Z}$

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(17) $\sin 7x - \sin x = \sin 3x$

$\Rightarrow 2\cos\left(\frac{7x+x}{2}\right)\sin\left(\frac{7x-x}{2}\right) = \sin 3x$

$\Rightarrow 2\cos 4x \sin 3x = \sin 3x$

$\Rightarrow 2\cos 4x \sin 3x - \sin 3x = 0$

$\Rightarrow \sin 3x\,[2\cos 4x - 1] = 0$

$\Rightarrow \sin 3x = 0$

$\Rightarrow 3x = 0$ &

$3x = \pi$

$\because 2\pi$ is the period of sin

$\therefore 3x = 0 + 2n\pi$

& $3x = \pi + 2n\pi$

$\Rightarrow x = \frac{2n\pi}{3}$ &

$x = \frac{\pi}{3} + \frac{2n\pi}{3}$

$n \in \mathbb{Z}$

$2\cos 4x - 1 = 0$

$\Rightarrow \cos 4x = 1/2$

$\because \cos 4x$ is $+ve$ in I & IV quad. with ref. angle $= \frac{\pi}{3}$

$\therefore 4x = \frac{\pi}{3}$ and $4x = 2\pi - \frac{\pi}{3}$

$\Rightarrow 4x = \frac{\pi}{3}$ & $4x = \frac{5\pi}{3}$

$\because 2\pi$ is the period of cos

$\therefore 4x = \frac{\pi}{3} + 2n\pi$ &

$4x = \frac{5\pi}{3} + 2n\pi$

$\Rightarrow x = \frac{\pi}{12} + \frac{n\pi}{2}$ &

$x = \frac{5\pi}{12} + \frac{n\pi}{2}$, $n \in \mathbb{Z}$

$\therefore S.S. = \{\frac{2}{3}n\pi\} \cup \{\frac{\pi}{3} + \frac{2}{3}n\pi\} \cup \{\frac{\pi}{12} + \frac{n\pi}{2}\} \cup \{\frac{5\pi}{12} + \frac{n\pi}{2}\}$, $n \in \mathbb{Z}$

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(18) $\sin x + \sin 3x + \sin 5x = 0$

$\Rightarrow (\sin 5x + \sin x) + \sin 3x = 0$

$\Rightarrow 2\sin\left(\frac{5x+x}{2}\right)\cos\left(\frac{5x-x}{2}\right) + \sin 3x = 0$

$\Rightarrow 2\sin 3x \cos 2x + \sin 3x = 0$

$\Rightarrow \sin 3x\,[2\cos 2x + 1] = 0$

$\Rightarrow \sin 3x = 0$

$\Rightarrow 3x = 0$ &

$3x = \pi$

$\because 2\pi$ is the period of sin

$\therefore 3x = 0 + 2n\pi = 2n\pi$

& $3x = \pi + 2n\pi$

$\Rightarrow x = \frac{2n\pi}{3}$ &

$x = \frac{\pi}{3} + \frac{2n\pi}{3}$

$n \in \mathbb{Z}$

$2\cos 2x + 1 = 0$

$\Rightarrow 2\cos 2x = -1 \Rightarrow \cos 2x = -\frac{1}{2}$

$\because \cos 2x$ is $-ve$ in II & III quad. with ref. angle $= \frac{\pi}{3}$

$\therefore 2x = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$ and

$2x = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$

$\because 2\pi$ is the period of cos

$\therefore 2x = \frac{2\pi}{3} + 2n\pi$ &

$2x = \frac{4\pi}{3} + 2n\pi$

$\Rightarrow x = \frac{\pi}{3} + n\pi$ &

$x = \frac{2\pi}{3} + n\pi$, $n \in \mathbb{Z}$

$\therefore S.S. = \{\frac{2}{3}n\pi\} \cup \{\frac{\pi}{3} + \frac{2}{3}n\pi\} \cup \{\frac{\pi}{3} + n\pi\} \cup \{\frac{2\pi}{3} + n\pi\}$, $n \in \mathbb{Z}$

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(19) $\sin\theta + \sin 3\theta + \sin 5\theta + \sin 7\theta = 0$

$\Rightarrow (\sin 7\theta + \sin\theta) + (\sin 5\theta + \sin 3\theta) = 0$

$\Rightarrow 2\sin\left(\frac{7\theta+\theta}{2}\right)\cos\left(\frac{7\theta-\theta}{2}\right) + 2\sin\left(\frac{5\theta+3\theta}{2}\right)\cos\left(\frac{5\theta-3\theta}{2}\right) = 0$

$\Rightarrow 2\sin 4\theta \cos 3\theta + 2\sin 4\theta \cos\theta = 0$

$\Rightarrow 2\sin 4\theta\,(\cos 3\theta + \cos\theta) = 0$

$$\Rightarrow 2\sin 4\theta\left[2\cos\left(\frac{3\theta+\theta}{2}\right)\cos\left(\frac{3\theta-\theta}{2}\right)\right]=0$$

$$\Rightarrow 4\sin 4\theta\cos 2\theta\cos\theta=0$$

$$\Rightarrow \sin 4\theta\cos 2\theta\cos\theta=0 \quad \because 4\neq 0$$

$\Rightarrow \sin 4\theta=0$

$\Rightarrow 4\theta=0$ & $4\theta=\pi$

$\because 2\pi$ is the period of sin

$\therefore 4\theta=0+2n\pi=2n\pi$ & $4\theta=\pi+2n\pi$

$\Rightarrow \theta=\frac{n\pi}{2}$ & $\theta=\frac{\pi}{4}+\frac{n\pi}{2}$, $n\in Z$

$\cos 2\theta=0$

$\Rightarrow 2\theta=\frac{\pi}{2}$ & $2\theta=\frac{3\pi}{2}$

$\because 2\pi$ is the period of cos

$\therefore 2\theta=\frac{\pi}{2}+2n\pi$ & $2\theta=\frac{3\pi}{2}+2n\pi$

$\Rightarrow \theta=\frac{\pi}{4}+n\pi$ & $\theta=\frac{3\pi}{4}+n\pi$, $n\in Z$

$\cos\theta=0$

$\Rightarrow \theta=\frac{\pi}{2}$ & $\theta=\frac{3\pi}{2}$

$\because 2\pi$ is the period of cos

$\therefore \theta=\frac{\pi}{2}+2n\pi$ & $\theta=\frac{3\pi}{2}+2n\pi$, $n\in Z$

$$\therefore S.S.=\left\{\frac{n\pi}{2}\right\}\cup\left\{\frac{\pi}{4}+\frac{n\pi}{2}\right\}\cup\left\{\frac{\pi}{4}+n\pi\right\}\cup\left\{\frac{3\pi}{4}+n\pi\right\}\cup\left\{\frac{\pi}{2}+2n\pi\right\}\cup\left\{\frac{3\pi}{2}+2n\pi\right\},\ n\in Z$$

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(20) $\cos\theta+\cos 3\theta+\cos 5\theta+\cos 7\theta=0$

$$\Rightarrow(\cos 7\theta+\cos\theta)+(\cos 5\theta+\cos 3\theta)=0$$

$$\Rightarrow 2\cos\left(\frac{7\theta+\theta}{2}\right)\cos\left(\frac{7\theta-\theta}{2}\right)+2\cos\left(\frac{5\theta+3\theta}{2}\right)\cos\left(\frac{5\theta-3\theta}{2}\right)=0$$

$$\Rightarrow 2\cos 4\theta\cos 3\theta+2\cos 4\theta\cos\theta=0$$

$$\Rightarrow 2\cos 4\theta[\cos 3\theta+\cos\theta]=0$$

$$\Rightarrow 2\cos 4\theta\left[2\cos\left(\frac{3\theta+\theta}{2}\right)\cos\left(\frac{3\theta-\theta}{2}\right)\right]=0$$

$$\Rightarrow 2\cos 4\theta[2\cos 2\theta\cos\theta]=0$$

$$\Rightarrow 4\cos 4\theta\cos 2\theta\cos\theta=0$$

$$\Rightarrow \cos 4\theta\cos 2\theta\cos\theta=0$$

$\Rightarrow \cos 4\theta=0$

$\Rightarrow 4\theta=\frac{\pi}{2}$ & $4\theta=\frac{3\pi}{2}$

$\because 2\pi$ is period of cos

$\therefore 4\theta=\frac{\pi}{2}+2n\pi$ & $4\theta=\frac{3\pi}{2}+2n\pi$

$\Rightarrow \theta=\frac{\pi}{8}+\frac{n\pi}{2}$ & $\theta=\frac{3\pi}{8}+\frac{n\pi}{2}$, $n\in Z$

$\cos 2\theta=0$

$\Rightarrow 2\theta=\frac{\pi}{2}$ & $2\theta=\frac{3\pi}{2}$

$\because 2\pi$ is the period of cos

$\therefore 2\theta=\frac{\pi}{2}+2n\pi$ & $2\theta=\frac{3\pi}{2}+2n\pi$

$\Rightarrow \theta=\frac{\pi}{4}+n\pi$, $\theta=\frac{3\pi}{4}+n\pi$, $n\in Z$

$\cos\theta=0$

$\Rightarrow \theta=\frac{\pi}{2}$ & $\theta=\frac{3\pi}{2}$

$\because 2\pi$ is the period of cos

$\therefore \theta=\frac{\pi}{2}+2n\pi$ & $\theta=\frac{3\pi}{2}+2n\pi$, $n\in Z$

$$\therefore S.S.=\left\{\frac{\pi}{8}+\frac{n\pi}{2}\right\}\cup\left\{\frac{3\pi}{8}+\frac{n\pi}{2}\right\}\cup\left\{\frac{\pi}{4}+n\pi\right\}\cup\left\{\frac{3\pi}{4}+n\pi\right\}\cup\left\{\frac{\pi}{2}+2n\pi\right\}\cup\left\{\frac{3\pi}{2}+2n\pi\right\}.\ n\in Z$$
