

1. Evaluate without using tables / calculator:

$$\begin{array}{lll} \text{i)} \quad \sin^{-1}(1) & \text{ii)} \quad \sin^{-1}(-1) & \text{iii)} \quad \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) \\ \text{iv)} \quad \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) & \text{v)} \quad \cos^{-1}\left(\frac{1}{2}\right) & \text{vi)} \quad \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) \\ \text{vii)} \quad \cot^{-1}(-1) & \text{viii)} \quad \operatorname{cosec}^{-1}\left(\frac{-2}{\sqrt{3}}\right) & \text{ix)} \quad \sin^{-1}\left(-\frac{1}{\sqrt{2}}\right) \end{array}$$

2. Without using table/ Calculator show that:

$$\begin{array}{ll} \text{i)} \quad \tan^{-1}\frac{5}{12} = \sin^{-1}\frac{5}{13} & \text{ii)} \quad 2 \cos^{-1}\frac{4}{5} = \sin^{-1}\frac{24}{25} \\ \text{iii)} \quad \cos^{-1}\frac{4}{5} = \cot^{-1}\frac{4}{3} \end{array}$$

3) Find the value of each expression:

$$\begin{array}{lll} \text{i)} \quad \cos\left(\sin^{-1}\frac{1}{\sqrt{2}}\right) & \text{ii)} \quad \sec\left(\cos^{-1}\frac{1}{2}\right) & \text{iii)} \quad \tan\left(\cos^{-1}\frac{\sqrt{3}}{2}\right) \\ \text{iv)} \quad \operatorname{csc}(\tan^{-1}(-1)) & \text{v)} \quad \sec\left(\sin^{-1}\left(-\frac{1}{2}\right)\right) & \text{vi)} \quad \tan(\tan^{-1}(-1)) \\ \text{vii)} \quad \sin\left(\sin^{-1}\left(\frac{1}{2}\right)\right) & \text{viii)} \quad \tan\left(\sin^{-1}\left(-\frac{1}{2}\right)\right) & \text{ix)} \quad \sin(\tan^{-1}(-1)) \end{array}$$

**Solution Are Given Below**

**Solution # 1**

$$\text{(i)} \quad \text{Suppose } y = \sin^{-1}(1) \quad \text{where } y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$\Rightarrow \sin y = 1$$

$$\Rightarrow y = \frac{\pi}{2} \quad \because \sin\left(\frac{\pi}{2}\right) = 1 \text{ Answer}$$

$$\text{(ii)} \quad \text{Suppose } y = \sin^{-1}(-1) \quad \text{where } y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$\Rightarrow \sin y = -1$$

$$\Rightarrow y = -\frac{\pi}{2} \quad \because \sin\left(-\frac{\pi}{2}\right) = -1 \text{ Answer}$$

(iii) Suppose  $y = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$  where  $y \in [0, \pi]$

$$\Rightarrow \cos y = \frac{\sqrt{3}}{2}$$

$$\Rightarrow y = \frac{\pi}{6} \quad \because \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} \text{ Answer}$$

(iv) Suppose  $y = \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$  where  $y \in \left]-\frac{\pi}{2}, \frac{\pi}{2}\right[$

$$\Rightarrow \tan y = -\frac{1}{\sqrt{3}}$$

$$\Rightarrow y = -\frac{\pi}{6} \quad \because \tan\left(-\frac{\pi}{6}\right) = -\frac{1}{\sqrt{3}} \text{ Answer}$$

(v) Suppose  $y = \cos^{-1}\left(\frac{1}{2}\right)$  where  $y \in [0, \pi]$

$$\Rightarrow \cos y = \frac{1}{2}$$

$$\Rightarrow y = \frac{\pi}{3} \quad \because \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}. \text{ Answer}$$

(vi) Suppose  $y = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$  where  $y \in \left]-\frac{\pi}{2}, \frac{\pi}{2}\right[$

$$\Rightarrow \tan y = \frac{1}{\sqrt{3}}$$

$$\Rightarrow y = \frac{\pi}{6} \quad \because \tan\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}}. \text{ Answer}$$

(vii) Suppose  $y = \cot^{-1}(-1)$  where  $y \in ]0, \pi[$

$$\Rightarrow \cot y = -1$$

$$\Rightarrow \frac{1}{\cot y} = \frac{1}{-1}$$

$$\Rightarrow \tan y = -1$$

$$\Rightarrow y = \frac{3\pi}{4} \quad \because \tan\left(\frac{3\pi}{4}\right) = -1 \text{ Answer}$$

(viii) Suppose  $y = \operatorname{cosec}^{-1}\left(-\frac{2}{\sqrt{3}}\right)$  where  $y \in \left]-\frac{\pi}{2}, \frac{\pi}{2}\right[$ ,  $y \neq 0$

$$\Rightarrow \operatorname{cosec} y = -\frac{2}{\sqrt{3}}$$

$$\Rightarrow \frac{1}{\operatorname{cosec} y} = \frac{1}{-\frac{2}{\sqrt{3}}}$$

$$\Rightarrow \sin y = -\frac{\sqrt{3}}{2}$$

$$\Rightarrow y = -\frac{\pi}{3} \quad \because \sin\left(-\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2} \text{ Answer}$$

(ix) Suppose  $y = \sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)$  where  $y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$\Rightarrow \sin y = -\frac{1}{\sqrt{2}}$$

$$\Rightarrow y = -\frac{\pi}{4} \quad \because \sin\left(-\frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}}$$

**Solution # 2**

(i) Suppose  $\alpha = \sin^{-1} \frac{5}{13}$  ..... (i) where  $\alpha \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$\Rightarrow \sin \alpha = \frac{5}{13}$$

Now  $\cos \alpha = \pm \sqrt{1 - \sin^2 \alpha}$

Since  $\alpha \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$  therefore  $\cos$  is +ive.

$$\begin{aligned} \cos \alpha &= +\sqrt{1 - \sin^2 \alpha} \\ &= \sqrt{1 - \left(\frac{5}{13}\right)^2} = \sqrt{1 - \frac{25}{169}} \\ &= \sqrt{\frac{144}{169}} = \frac{12}{13} \end{aligned}$$

Now  $\tan \alpha = \frac{\sin \alpha}{\cos \alpha} = \frac{5/13}{12/13} = \frac{5}{12}$

$$\Rightarrow \alpha = \tan^{-1} \frac{5}{12} \text{ ..... (ii)}$$

From (i) and (ii)

$$\tan^{-1} \frac{5}{12} = \sin^{-1} \frac{5}{13} \quad \text{Proved}$$

(ii) Suppose  $\alpha = 2 \cos^{-1} \frac{4}{5}$  ..... (i) where  $\frac{\alpha}{2} \in [0, \pi]$

$$\Rightarrow \frac{\alpha}{2} = \cos^{-1} \frac{4}{5} \Rightarrow \cos \frac{\alpha}{2} = \frac{4}{5}$$

Now  $\sin \frac{\alpha}{2} = \pm \sqrt{1 - \cos^2 \frac{\alpha}{2}}$

Since  $\frac{\alpha}{2} \in [0, \pi]$  therefore *sin* is +ive.

$$\begin{aligned} \sin \frac{\alpha}{2} &= +\sqrt{1 - \cos^2 \frac{\alpha}{2}} \\ &= \sqrt{1 - \frac{16}{25}} = \sqrt{1 - \frac{16}{25}} \\ &= \sqrt{\frac{9}{25}} = \frac{3}{5} \end{aligned}$$

Now  $\sin \alpha = 2 \sin \frac{\alpha}{2} \cos \frac{\alpha}{2} \Rightarrow \sin \alpha = 2 \left(\frac{3}{5}\right) \left(\frac{4}{5}\right)$

$\Rightarrow \sin \alpha = \frac{24}{25} \Rightarrow \alpha = \sin^{-1} \frac{24}{25} \dots\dots\dots$  (ii)

From (i) and (ii)

$2 \cos^{-1} \frac{4}{5} = \sin^{-1} \frac{24}{25}$  *Prove*

(iii) Suppose  $\alpha = \cos^{-1} \frac{4}{5} \dots\dots\dots$  (i) where  $\alpha \in [0, \pi]$

$\Rightarrow \cos \alpha = \frac{4}{5}$

Now  $\sin \alpha = \pm \sqrt{1 - \cos^2 \alpha}$

Since  $\alpha \in [0, \pi]$  therefore *sin* is +ive.

$$\begin{aligned} \sin \alpha &= +\sqrt{1 - \cos^2 \alpha} \\ &= \sqrt{1 - \left(\frac{4}{5}\right)^2} = \sqrt{1 - \frac{16}{25}} = \sqrt{\frac{9}{25}} = \frac{3}{5}. \end{aligned}$$

Now  $\cot \alpha = \frac{\cos \alpha}{\sin \alpha} = \frac{\frac{4}{5}}{\frac{3}{5}} = \frac{4}{3}$

$\Rightarrow \alpha = \cot^{-1} \frac{4}{3} \dots\dots\dots$  (ii)

From (i) and (ii)

$\cos^{-1} \frac{4}{5} = \cot^{-1} \frac{4}{3}$  *Proved*

**Solution # 3**

(i) Suppose  $y = \sin^{-1} \frac{1}{\sqrt{2}}$  where  $y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\Rightarrow \sin y = \frac{1}{\sqrt{2}}$

$$\Rightarrow y = \frac{\pi}{4} \quad \because \sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

Now  $\cos\left(\sin^{-1}\frac{1}{\sqrt{2}}\right) = \cos y = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$  *Answer* □

(ii) Suppose  $y = \cos^{-1}\frac{1}{2}$  where  $y \in [0, \pi]$

$$\Rightarrow \cos y = \frac{1}{2}$$

$$\Rightarrow y = \frac{\pi}{3} \quad \because \cos\frac{\pi}{3} = \frac{1}{2}$$

Now  $\sec\left(\cos^{-1}\frac{1}{2}\right) = \sec y = \frac{1}{\cos y} = \frac{1}{\cos\frac{\pi}{3}} = \frac{1}{\frac{1}{2}} = 2$  *Answer*

(iii) *Do yourself* □

(iv) Suppose  $y = \tan^{-1}(-1)$  where  $y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$\Rightarrow \tan y = -1$$

$$\Rightarrow y = -\frac{\pi}{4} \quad \because \tan\left(-\frac{\pi}{4}\right) = -1$$

Now  $\operatorname{cosec}\left(\tan^{-1}(-1)\right) = \operatorname{cosec} y = \frac{1}{\sin y} = \frac{1}{\sin\left(-\frac{\pi}{4}\right)} = \frac{1}{-\frac{1}{\sqrt{2}}} = -\sqrt{2}$  *Answer*

(v) *Do yourself*

(vi) Suppose  $y = \tan^{-1}(-1)$  where  $y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$\Rightarrow \tan y = -1$$

$$\Rightarrow y = -\frac{\pi}{4} \quad \because \tan\left(-\frac{\pi}{4}\right) = -1$$

Now  $\tan\left(\tan^{-1}(-1)\right) = \tan y = \tan\left(-\frac{\pi}{4}\right) = -1$  *Answer*

(vii) Suppose  $y = \sin^{-1}\frac{1}{2}$  where  $y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$\Rightarrow \sin y = \frac{1}{2}$$

$$\Rightarrow y = \frac{\pi}{6} \quad \because \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$$

Now  $\sin\left(\sin^{-1}\frac{1}{2}\right) = \sin y = \sin\frac{\pi}{6} = \frac{1}{2}$  *Answer* □

(viii) Suppose  $y = \sin^{-1}\left(-\frac{1}{2}\right)$  where  $y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$\Rightarrow \sin y = -\frac{1}{2}$$

$$\Rightarrow y = -\frac{\pi}{6} \quad \because \sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}$$

Now  $\tan\left(\sin^{-1}\left(-\frac{1}{2}\right)\right) = \tan y = \tan\left(-\frac{\pi}{6}\right) = -\frac{1}{\sqrt{3}}$  Answer □

(ix) *Do yourself*

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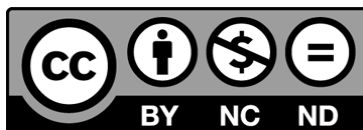
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Book: **Exercise 13.1 (Page 398)**  
 Text Book of Algebra and Trigonometry Class XI  
 Punjab Textbook Board, Lahore.

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