Q.No 1 $a = 7, \quad b = 7, \quad c = 9$

$$s = \frac{a+b+c}{2} = \frac{7+7+9}{2} = \frac{23}{2} = 11.5$$

Now

$$\cos\frac{\alpha}{2} = \sqrt{\frac{s(s-a)}{bc}} = \sqrt{\frac{(11.5)(11.5-7)}{(7)(9)}}$$

$$= \sqrt{\frac{(11.5)(4.5)}{63}} = \sqrt{0.821} = 0.906$$

$$\Rightarrow \frac{\alpha}{2} = \cos^{-1}(0.906) = 24.99 \approx 25$$

$$\Rightarrow \alpha = 2(25) \Rightarrow \boxed{\alpha = 50^\circ}$$

Now

$$\cos\frac{\beta}{2} = \sqrt{\frac{s(s-b)}{ca}} = \sqrt{\frac{11.5(11.5-7)}{(9)(7)}}$$

$$= \sqrt{0.821} = 0.906$$

$$\Rightarrow \frac{\beta}{2} = \cos^{-1}(0.906) = 24.99 \approx 25^\circ$$

$$\Rightarrow \beta = 2(25) \Rightarrow \boxed{\beta = 50^\circ}$$

Now $\alpha + \beta + \gamma = 180^\circ$

$$\Rightarrow \gamma = 180 - \alpha - \beta$$

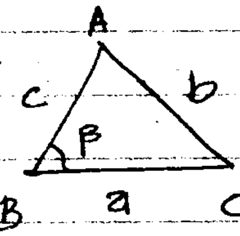
$$= 180 - 50 - 50$$

$$\Rightarrow \boxed{\gamma = 80^\circ}$$

Note: You may use any formula in above Question

Q.No. 2 to 5 Do yourself

Q No 6 $a = 37.34$, $b = 3.24$, $c = 35.06$



Since angle opposite to smallest side is smallest therefore β is the smallest angle

&

$$\cos\beta = \frac{c^2 + a^2 - b^2}{2ca}$$

$$= \frac{(35.06)^2 + (37.34)^2 - (3.24)^2}{2(35.06)(37.34)}$$

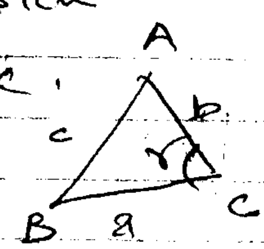
$$= \frac{2612.98}{2618.28} = 0.9979$$

$$\Rightarrow \beta = \cos^{-1}(0.9979)$$

$$= 3.82 \Rightarrow \boxed{\beta = 3^\circ 50'}$$

Q No 7 Let $a = 16$, $b = 20$, $c = 33$

Since angle opposite to the largest side is largest. here $c = 33$ is largest side therefore γ is the largest angle.



$$\cos\gamma = \frac{a^2 + b^2 - c^2}{2ab}$$

x^2+x+1

Do yourself

$\gamma = ?$

Q No 8 Let $a = x^2 + x + 1$, $b = 2x + 1$, $c = x^2 - 1$,

Since $a = x^2 + x + 1$ is the greatest side

Therefore α is the greatest angle.

Now $\cos\alpha = \dfrac{b^2+c^2-a^2}{2bc}$

$$= \frac{(2x+1)^2+(x^2-1)^2-(x^2+x+1)^2}{2(2x+1)(x^2-1)}.$$

$$= \frac{4x^2+4x+1+x^4-2x^2+1-(x^4+x^2+1+2x^3+2x+2x^2)}{2(2x^3+x^2-2x-1)}.$$

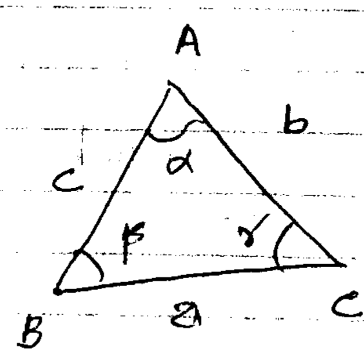
$$= \frac{\cancel{x^4}+\cancel{2x^2}+4x+2-\cancel{x^4}-x^2-1-2x^3-2x-\cancel{2x^2}}{2(2x^3+x^2-2x-1)}.$$

$$= \frac{-2x^3-x^2+2x+1}{2(2x^3+x^2-2x-1)} = \frac{-(2x^3+x^2-2x-1)}{2(2x^3+x^2-2x-1)}$$

$\Rightarrow \cos\alpha = -\dfrac{1}{2} \Rightarrow \alpha = \cos^{-1}\left(-\dfrac{1}{2}\right)$

$\Rightarrow \boxed{\alpha = 120^\circ}$

Q No 9 Sol $a = 413$, $b = 214$, $c = 375$



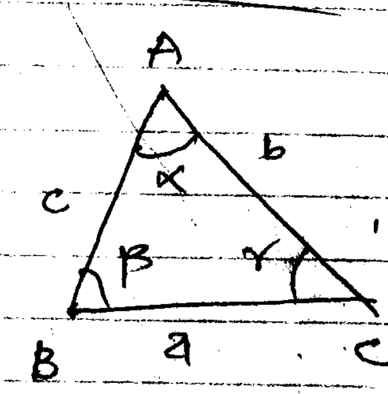
Since

$$\cos\alpha = \frac{b^2+c^2-a^2}{2bc}$$

Do yourself.

Find all angle in this Question.

Q No 10 Sol $a = 6$, $b = 9$, $c = 13$.



Now $\cos\alpha = \dfrac{b^2+c^2-a^2}{2bc}$

$$= \frac{(9)^2+(13)^2-(6)^2}{2(9)(13)}$$

$$= \frac{214}{234} = 0.9145$$

$\Rightarrow \alpha = \cos^{-1}(0.9145) \simeq 23.86^\circ$

$\Rightarrow \boxed{\alpha = 23^\circ 52'}$

Now

$$\cos\beta = \frac{c^2 + a^2 - b^2}{2ca}$$

$$= \frac{(13)^2 + (6)^2 - (9)^2}{2(13)(6)} = \frac{124}{156} = 0.7948$$

$\Rightarrow \beta = \cos^{-1}(0.7948) = 37^\circ 21'$

$\Rightarrow \boxed{\beta = 37^\circ 21'}$

Now

$\alpha + \beta + \gamma = 180$

$\Rightarrow \gamma = 180 - \alpha - \beta$

$= 180 - 23^\circ 52' - 37^\circ 21'$

$\Rightarrow \boxed{\gamma = 118^\circ 47'}$

Thus road of vilages make angles $23^\circ 52'$, $37^\circ 21'$ and $118^\circ 47'$ ~~to~~ to each other