

1. Solve the triangle ABC if: $\beta = 60^\circ$, $\gamma = 15^\circ$, $b = \sqrt{6}$.

Solution. We know that $\alpha + \beta + \gamma = 180^\circ$

$$\text{Now } \alpha = 180 - \beta - \gamma = 180^\circ - 60^\circ - 15^\circ = 105^\circ$$

$$\begin{aligned} \text{By Law of Sines: } \frac{a}{\sin \alpha} &= \frac{b}{\sin \beta} \Rightarrow a = b \cdot \frac{\sin \alpha}{\sin \beta} \\ &= \sqrt{6} \cdot \frac{\sin 105^\circ}{\sin 60^\circ} = \sqrt{6} \cdot \frac{0.9659}{0.8660} = \sqrt{6} \times 1.1153 = 2.731. \end{aligned}$$

$$\begin{aligned} \text{Again, } \frac{c}{\sin \gamma} &= \frac{b}{\sin \beta} \Rightarrow c = b \cdot \frac{\sin \gamma}{\sin \beta} \\ &= \sqrt{6} \cdot \frac{\sin 15^\circ}{\sin 60^\circ} = \sqrt{6} \cdot \frac{0.2588}{0.8660} = \sqrt{6} \times 0.2989 = 0.732 \end{aligned}$$

$$\text{Hence } \boxed{\alpha = 105^\circ, a = 2.73, c = 0.73}$$

2. $\beta = 52^\circ$, $\gamma = 89^\circ 35'$, $a = 89.35$

Solution. We know that $\alpha + \beta + \gamma = 180^\circ$

$$\text{Now } \alpha = 180 - \beta - \gamma = 180^\circ - 52^\circ - 89^\circ 35' = 38^\circ 25'$$

$$\begin{aligned} \text{By Law of Sines: } \frac{a}{\sin \alpha} &= \frac{b}{\sin \beta} \Rightarrow b = a \cdot \frac{\sin \beta}{\sin \alpha} \\ &= 89.35 \cdot \frac{\sin 52^\circ}{\sin 38^\circ 25'} = \sqrt{6} \cdot \frac{0.7880}{0.6213} = 89.35 \times 1.268 = 113.31. \end{aligned}$$

$$\begin{aligned} \text{Again, } \frac{b}{\sin \beta} &= \frac{c}{\sin \gamma} \Rightarrow c = b \cdot \frac{\sin \gamma}{\sin \beta} \\ &= 113.31 \cdot \frac{\sin 89^\circ 35'}{\sin 52^\circ} = 113.31 \cdot \frac{0.999}{0.788} = 113.31 \times 1.269 = 143.79 \end{aligned}$$

$$\text{Hence } \boxed{\alpha = 38^\circ 25', b = 113.31, c = 143.79}$$

3. $b = 125$, $\gamma = 53^\circ$, $\alpha = 47^\circ$

Solution. We know that $\alpha + \beta + \gamma = 180^\circ$

$$\text{Now } \beta = 180 - \alpha - \gamma = 180^\circ - 47^\circ - 53^\circ = 80^\circ$$

$$\begin{aligned} \text{By Law of Sines: } \frac{a}{\sin \alpha} &= \frac{b}{\sin \beta} \\ \Rightarrow a &= b \cdot \frac{\sin \alpha}{\sin \beta} = 125 \cdot \frac{\sin 47^\circ}{\sin 80^\circ} = 125 \cdot \frac{0.731}{0.985} = 125 \times 0.742 = 93 \end{aligned}$$

$$\begin{aligned} \text{Again, } \frac{c}{\sin \gamma} &= \frac{b}{\sin \beta} \\ \Rightarrow c &= b \cdot \frac{\sin \gamma}{\sin \beta} = 125 \cdot \frac{\sin 53^\circ}{\sin 80^\circ} = 125 \cdot \frac{0.7986}{0.985} = 125 \times 0.811 = 101 \end{aligned}$$

$$\text{Hence } \boxed{\beta = 80^\circ, a = 93, c = 101}$$

4. $c = 16.1$, $\alpha = 42^\circ 45'$, $\gamma = 74^\circ 32'$

Solution. We know that $\alpha + \beta + \gamma = 180^\circ$

$$\text{Now } \beta = 180 - \alpha - \gamma = 180^\circ - 42^\circ 45' - 74^\circ 32' = 62^\circ 43'$$

$$\begin{aligned} \text{Now } \frac{b}{\sin \beta} &= \frac{c}{\sin \gamma} \Rightarrow b = c \cdot \frac{\sin \beta}{\sin \gamma} \\ &= 16.1 \times \frac{\sin 62^\circ 43'}{\sin 74^\circ 32'} = 16.1 \times \frac{0.899}{0.964} = 16.1 \times 0.922 = 14.85 \end{aligned}$$

$$\begin{aligned} \text{Again: } \frac{a}{\sin \alpha} &= \frac{b}{\sin \beta} \Rightarrow a = b \cdot \frac{\sin \alpha}{\sin \beta} \\ &= 14.85 \times \frac{\sin 42^\circ 45'}{\sin 62^\circ 43'} = 14.85 \times \frac{0.678}{0.899} = 14.85 \times 0.7635 = 11.34 \end{aligned}$$

$$\text{Hence } \boxed{\beta = 62^\circ 43', a = 11.34, b = 14.85}$$

$$5. \quad \alpha = 53^\circ, \quad \beta = 88^\circ 36', \quad \gamma = 31^\circ 54'$$

Solution. We know that $\alpha + \beta + \gamma = 180^\circ$

$$\text{Now } \alpha = 180 - \beta - \gamma = 180^\circ - 88^\circ 36' - 31^\circ 54' = 59^\circ 30'$$

$$\begin{aligned} \text{By Law of Sines: } \frac{a}{\sin \alpha} &= \frac{b}{\sin \beta} \Rightarrow b = a \cdot \frac{\sin \beta}{\sin \alpha} \\ &= 53 \times \frac{\sin 88^\circ 36'}{\sin 59^\circ 30'} = 53 \times \frac{0.9997}{0.8616} = 53 \times 1.16 = 61.49 \end{aligned}$$

$$\begin{aligned} \text{Again, } \frac{c}{\sin \gamma} &= \frac{b}{\sin \beta} \Rightarrow c = b \cdot \frac{\sin \gamma}{\sin \beta} \\ &= 61.5 \times \frac{\sin 31^\circ 54'}{\sin 88^\circ 36'} = 61.5 \times \frac{0.528}{0.9997} = 61.5 \times 0.5286 = 32.51 \end{aligned}$$

$$\text{Hence } \boxed{\alpha = 59^\circ 30', b = 61.49, c = 32.51}$$

The End

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