

Question # 1

Express the following products as sums or differences:

- (i) $2\sin 3\theta \cos \theta$ (ii) $2\cos 5\theta \cos 3\theta$ (iii) $\sin 5\theta \cos 2\theta$
 (iv) $2\sin 7\theta \sin 2\theta$ (v) $\cos(x+y)\sin(x-y)$ (vi) $\cos(2x+30^\circ)\cos(2x-30^\circ)$
 (vii) $\sin 12^\circ \sin 46^\circ$ (viii) $\sin(x+45^\circ)\sin(x-45^\circ)$

Solution

(i) Since $2\sin \alpha \cos \beta = \sin(\alpha + \beta) + \sin(\alpha - \beta)$

Put $\alpha = 3\theta$ and $\beta = \theta$

$$2\sin 3\theta \cos \theta = \sin(3\theta + \theta) + \sin(3\theta - \theta)$$

$$= \sin 4\theta + \sin 2\theta$$

(ii) Since $2\cos \alpha \cos \beta = \cos(\alpha + \beta) + \cos(\alpha - \beta)$

Put $\alpha = 5\theta$ and $\beta = 3\theta$

$$2\cos 5\theta \cos 3\theta = \cos(5\theta + 3\theta) + \cos(5\theta - 3\theta)$$

$$= \cos 8\theta + \cos 2\theta$$

(iii) Since $2\sin \alpha \cos \beta = \sin(\alpha + \beta) + \sin(\alpha - \beta)$

Put $\alpha = 5\theta$ and $\beta = 2\theta$

$$2\sin 5\theta \cos 2\theta = \sin(5\theta + 2\theta) + \sin(5\theta - 2\theta)$$

$$= \sin 7\theta + \sin 3\theta$$

$$\Rightarrow \sin 5\theta \cos 2\theta = \frac{1}{2}(\sin 7\theta + \sin 3\theta)$$

(iv) Since $-2\sin \alpha \sin \beta = \cos(\alpha + \beta) - \cos(\alpha - \beta)$

Put $\alpha = 7\theta$ and $\beta = 2\theta$

$$-2\sin \alpha \sin \beta = \cos(\alpha + \beta) - \cos(\alpha - \beta)$$

$$-2\sin 7\theta \sin 2\theta = \cos(7\theta + 2\theta) - \cos(7\theta - 2\theta)$$

$$= \cos 9\theta - \cos 5\theta$$

$$2\sin 7\theta \sin 2\theta = \cos 5\theta - \cos 9\theta$$

(v) Since $2\cos \alpha \sin \beta = \sin(\alpha + \beta) - \sin(\alpha - \beta)$

Put $\alpha = x + y$, $\beta = x - y$

$$2\cos(x+y)\sin(x-y) = \sin(x+y+x-y) - \sin(x+y-x-y)$$

$$= \sin 2x - \sin 2y$$

$$\Rightarrow \cos(x+y)\sin(x-y) = \frac{1}{2}(\sin 2x - \sin 2y)$$

(vi) Since $2\cos \alpha \cos \beta = \cos(\alpha + \beta) + \cos(\alpha - \beta)$

Put $\alpha = 2x + 30^\circ$ and $\beta = 2x - 30^\circ$

$$2 \cos(2x + 30^\circ) \cos(2x - 30^\circ) = \cos(2x + 30^\circ + 2x - 30^\circ) + \cos(2x + 30^\circ - 2x + 30^\circ)$$

$$= \cos(4x) + \cos(60^\circ)$$

$$\Rightarrow \cos(2x + 30^\circ) \cos(2x - 30^\circ) = \frac{1}{2}(\cos 4x + \cos 60^\circ)$$

(vii) Since $-2 \sin \alpha \sin \beta = \cos(\alpha + \beta) - \cos(\alpha - \beta)$

Put $\alpha = 12^\circ$ and $\beta = 46^\circ$

$$-2 \sin 12^\circ \sin 46^\circ = \cos(12 + 46) - \cos(12 - 46)$$

$$= \cos 58 - \cos(-34)$$

$$= \cos 58 - \cos 34 \quad \because \cos(-\theta) = \cos \theta$$

$$\Rightarrow \sin 12^\circ \sin 46^\circ = -\frac{1}{2}(\cos 58 - \cos 34)$$

$$= \frac{1}{2}(\cos 34 - \cos 58)$$

(viii) Since $-2 \sin \alpha \sin \beta = \cos(\alpha + \beta) - \cos(\alpha - \beta)$

Put $\alpha = x + 45^\circ$ and $\beta = x - 45^\circ$

$$-2 \sin(x + 45^\circ) \sin(x - 45^\circ) = \cos\{(x + 45^\circ) + (x - 45^\circ)\} - \cos\{(x + 45^\circ) - (x - 45^\circ)\}$$

$$= \cos 2x - \cos 90^\circ$$

$$\Rightarrow \sin(x + 45^\circ) \sin(x - 45^\circ) = \cos 90^\circ - \frac{1}{2} \cos 2x$$

Question # 2

Express the following sum or difference as product:

(i) $\sin 5\theta + \sin 3\theta$

(ii) $\sin 8\theta - \sin 4\theta$

(iii) $\cos 6\theta + \cos 3\theta$

(iv) $\cos 7\theta - \cos \theta$

(v) $\cos 12^\circ + \cos 48^\circ$

(vi) $\sin(x + 30^\circ) + \sin(x - 30^\circ)$

Solution

(i) Since $\sin \alpha + \sin \beta = 2 \sin\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha - \beta}{2}\right)$

Put $\alpha = 5\theta$, $\beta = 3\theta$

$$\sin 5\theta + \sin 3\theta = 2 \sin\left(\frac{5\theta + 3\theta}{2}\right) \cos\left(\frac{5\theta - 3\theta}{2}\right)$$

$$= 2 \sin\left(\frac{8\theta}{2}\right) \cos\left(\frac{2\theta}{2}\right) = 2 \sin 4\theta \cos \theta$$

(ii) Since $\sin \alpha - \sin \beta = 2 \cos\left(\frac{\alpha + \beta}{2}\right) \sin\left(\frac{\alpha - \beta}{2}\right)$

Put $\alpha = 8\theta$, $\beta = 4\theta$

$$\sin 8\theta - \sin 4\theta = 2 \cos\left(\frac{8\theta + 4\theta}{2}\right) \sin\left(\frac{8\theta - 4\theta}{2}\right)$$

$$= 2 \cos 6\theta \sin 2\theta$$

(iii) *Do yourself*

(iv) Since $\cos \alpha - \cos \beta = -2 \sin \left(\frac{\alpha + \beta}{2} \right) \sin \left(\frac{\alpha - \beta}{2} \right)$

Put $\alpha = 7\theta$, $\beta = \theta$

$$\begin{aligned} \cos 7\theta - \cos \theta &= -2 \sin \left(\frac{7\theta + \theta}{2} \right) \sin \left(\frac{7\theta - \theta}{2} \right) = -2 \sin \left(\frac{8\theta}{2} \right) \sin \left(\frac{6\theta}{2} \right) \\ &= -2 \sin 4\theta \sin 3\theta \end{aligned}$$

(v) Since $\cos \alpha + \cos \beta = 2 \cos \left(\frac{\alpha + \beta}{2} \right) \cos \left(\frac{\alpha - \beta}{2} \right)$

Put $\alpha = 12$, $\beta = 48$

$$\begin{aligned} \cos 12 + \cos 48 &= 2 \cos \left(\frac{12 + 48}{2} \right) \cos \left(\frac{12 - 48}{2} \right) \\ &= 2 \cos \left(\frac{60}{2} \right) \cos \left(\frac{-36}{2} \right) = 2 \cos 30 \cos(-18) \\ &= 2 \cos 30^\circ \cos 18^\circ \quad \because \cos(-\theta) = \cos \theta \end{aligned}$$

(vi) Since $\sin \alpha + \sin \beta = 2 \sin \left(\frac{\alpha + \beta}{2} \right) \cos \left(\frac{\alpha - \beta}{2} \right)$

Put $\alpha = x + 30$, $\beta = x - 30$

$$\begin{aligned} \sin(x + 30) + \sin(x - 30) &= 2 \sin \left(\frac{x + 30 + x - 30}{2} \right) \cos \left(\frac{x + 30 - x + 30}{2} \right) \\ &= 2 \sin \left(\frac{2x}{2} \right) \cos \left(\frac{60}{2} \right) = 2 \sin x \cos 30 \end{aligned}$$

Question # 3

Prove the following identities:

(i) $\frac{\sin 3x - \sin x}{\cos x - \cos 3x} = \cot 2x$

(ii) $\frac{\sin 8x + \sin 2x}{\cos 8x + \cos 2x} = \tan 5x$

(iii) $\frac{\sin \alpha - \sin \beta}{\sin \alpha + \sin \beta} = \cot \left(\frac{\alpha + \beta}{2} \right) \tan \left(\frac{\alpha - \beta}{2} \right)$

Solution

$$\begin{aligned} \text{(i) L.H.S} &= \frac{\sin 3x - \sin x}{\cos x - \cos 3x} \\ &= \frac{2 \cos \left(\frac{3x + x}{2} \right) \sin \left(\frac{3x - x}{2} \right)}{-2 \sin \left(\frac{x + 3x}{2} \right) \sin \left(\frac{x - 3x}{2} \right)} = \frac{\cos \left(\frac{4x}{2} \right) \sin \left(\frac{2x}{2} \right)}{-\sin \left(\frac{4x}{2} \right) \sin \left(\frac{-2x}{2} \right)} \end{aligned}$$

$$= \frac{\cos(2x)\sin(x)}{+\sin(2x)\sin(x)} = \cot 2x = \text{R.H.S}$$

(ii) *Do yourself*

$$\begin{aligned} \text{(iii) L.H.S} &= \frac{\sin \alpha - \sin \beta}{\sin \alpha + \sin \beta} \\ &= \frac{2 \cos\left(\frac{\alpha + \beta}{2}\right) \sin\left(\frac{\alpha - \beta}{2}\right)}{2 \sin\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha - \beta}{2}\right)} = \cot\left(\frac{\alpha + \beta}{2}\right) \tan\left(\frac{\alpha - \beta}{2}\right) = \text{R.H.S} \end{aligned}$$

Question # 4

Prove that:

(i) $\cos 20^\circ + \cos 100^\circ + \cos 140^\circ = 0$ (ii) $\sin\left(\frac{\pi}{4} - \theta\right) \sin\left(\frac{\pi}{4} + \theta\right) = \frac{1}{2} \cos 2\theta$

(iii) $\frac{\sin \theta + \sin 3\theta + \sin 5\theta + \sin 7\theta}{\cos \theta + \cos 3\theta + \cos 5\theta + \cos 7\theta} = \tan 4\theta$

Solution

(i) L.H.S = $\cos 20^\circ + \cos 100^\circ + \cos 140^\circ$
 $= (\cos 100^\circ + \cos 20^\circ) + \cos 140^\circ$
 $= 2 \cos\left(\frac{100+20}{2}\right) \cos\left(\frac{100-20}{2}\right) + \cos 140^\circ$
 $= 2 \cos 60^\circ \cos 40^\circ + \cos 140^\circ = 2\left(\frac{1}{2}\right) \cos 40^\circ + \cos 140^\circ$
 $= \cos 140^\circ + \cos 40^\circ = 2 \cos\left(\frac{140+40}{2}\right) \cos\left(\frac{140-40}{2}\right)$
 $= 2 \cos 90^\circ \cos 50^\circ = 2(0) \cos 50^\circ = 0 = \text{R.H.S}$

(ii) L.H.S = $\sin\left(\frac{\pi}{4} - \theta\right) \sin\left(\frac{\pi}{4} + \theta\right)$
 $= \left(\sin \frac{\pi}{4} \cos \theta - \cos \frac{\pi}{4} \sin \theta\right) \left(\sin \frac{\pi}{4} \cos \theta + \cos \frac{\pi}{4} \sin \theta\right)$
 $= \left(\frac{1}{\sqrt{2}} \cos \theta - \frac{1}{\sqrt{2}} \sin \theta\right) \left(\frac{1}{\sqrt{2}} \cos \theta + \frac{1}{\sqrt{2}} \sin \theta\right)$
 $= \left(\frac{1}{\sqrt{2}} \cos \theta\right)^2 - \left(\frac{1}{\sqrt{2}} \sin \theta\right)^2 = \frac{1}{2} \cos^2 \theta - \frac{1}{2} \sin^2 \theta$
 $= \frac{1}{2} (\cos^2 \theta - \sin^2 \theta) = \frac{1}{2} \cos 2\theta = \text{R.H.S}$

(iii) L.H.S = $\frac{\sin \theta + \sin 3\theta + \sin 5\theta + \sin 7\theta}{\cos \theta + \cos 3\theta + \cos 5\theta + \cos 7\theta}$

$$\begin{aligned}
&= \frac{(\sin 7\theta + \sin \theta) + (\sin 5\theta + \sin 3\theta)}{(\cos 7\theta + \cos \theta) + (\cos 5\theta + \cos 3\theta)} \\
&= \frac{2\sin\left(\frac{7\theta + \theta}{2}\right)\cos\left(\frac{7\theta - \theta}{2}\right) + 2\sin\left(\frac{5\theta + 3\theta}{2}\right)\cos\left(\frac{5\theta - 3\theta}{2}\right)}{2\cos\left(\frac{7\theta + \theta}{2}\right)\cos\left(\frac{7\theta - \theta}{2}\right) + 2\cos\left(\frac{5\theta + 3\theta}{2}\right)\cos\left(\frac{5\theta - 3\theta}{2}\right)} \\
&= \frac{2\sin 4\theta \cos 3\theta + 2\sin 4\theta \cos \theta}{2\cos 4\theta \cos 3\theta + 2\cos 4\theta \cos \theta} \\
&= \frac{2\sin 4\theta (\cos 3\theta + \cos \theta)}{2\cos 4\theta (\cos 3\theta + \cos \theta)} = \frac{\sin 4\theta}{\cos 4\theta} = \tan 4\theta = \text{R.H.S}
\end{aligned}$$

Question # 5

Prove that:

$$(i) \cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ = \frac{1}{16} \qquad (ii) \sin \frac{\pi}{9} \sin \frac{2\pi}{9} \sin \frac{\pi}{3} \sin \frac{4\pi}{9} = \frac{3}{16}$$

Solution

$$\begin{aligned}
(i) \quad \text{L.H.S} &= \cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ \\
&= \cos 20^\circ \cos 40^\circ \left(\frac{1}{2}\right) \cos 80^\circ = \frac{1}{2} \cos 80^\circ \cos 40^\circ \cos 20^\circ \\
&= \frac{1}{4} (2\cos 80^\circ \cos 40^\circ) \cos 20^\circ = \frac{1}{4} (\cos(80+40) + \cos(80-40)) \cos 20^\circ \\
&= \frac{1}{4} (\cos 120^\circ + \cos 40^\circ) \cos 20^\circ = \frac{1}{4} \left(-\frac{1}{2} + \cos 40^\circ\right) \cos 20^\circ \\
&= -\frac{1}{8} \cos 20^\circ + \frac{1}{4} \cos 40^\circ \cos 20^\circ = -\frac{1}{8} + \frac{1}{8} (2\cos 40^\circ \cos 20^\circ) \\
&= -\frac{1}{8} \cos 20^\circ + \frac{1}{8} (\cos(40+20) + \cos(40-20)) \\
&= -\frac{1}{8} \cos 20^\circ + \frac{1}{8} (\cos 60 + \cos 20) = -\frac{1}{8} \cos 20^\circ + \frac{1}{8} \left(\frac{1}{2} + \cos 20\right) \\
&= -\frac{1}{8} \cos 20^\circ + \frac{1}{16} + \frac{1}{8} \cos 20^\circ = \frac{1}{16} = \text{R.H.S}
\end{aligned}$$

$$\begin{aligned}
(ii) \quad \text{L.H.S} &= \sin \frac{\pi}{9} \sin \frac{2\pi}{9} \sin \frac{\pi}{3} \sin \frac{4\pi}{9} \\
&= \sin \frac{180^\circ}{9} \sin \frac{2(180^\circ)}{9} \sin \frac{(180^\circ)}{3} \sin \frac{4(180^\circ)}{9} \qquad \because \pi = 180^\circ \\
&= \sin 20^\circ \sin 40^\circ \sin 60^\circ \sin 80^\circ = \sin 20^\circ \sin 40^\circ \frac{\sqrt{3}}{2} \sin 80^\circ \\
&= \frac{\sqrt{3}}{2} \sin 80^\circ \sin 40^\circ \sin 20^\circ = -\frac{\sqrt{3}}{4} (-2\sin 80^\circ \sin 40^\circ) \sin 20^\circ
\end{aligned}$$

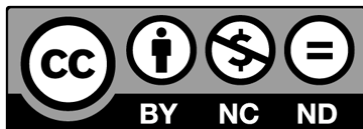
$$\begin{aligned}
 &= -\frac{\sqrt{3}}{4}(\cos(80 + 40) - \cos(80 - 40))\sin 20^\circ \\
 &= -\frac{\sqrt{3}}{4}(\cos 120^\circ - \cos 40^\circ)\sin 20^\circ = -\frac{\sqrt{3}}{4}\left(-\frac{1}{2} - \cos 40^\circ\right)\sin 20^\circ \\
 &= \frac{\sqrt{3}}{8}\sin 20^\circ + \frac{\sqrt{3}}{4}\cos 40^\circ \sin 20^\circ = \frac{\sqrt{3}}{8}\sin 20^\circ + \frac{\sqrt{3}}{8}(2\cos 40^\circ \sin 20^\circ) \\
 &= \frac{\sqrt{3}}{8}\sin 20^\circ + \frac{\sqrt{3}}{8}(\sin(40 + 20) - \sin(40 - 20)) \\
 &= \frac{\sqrt{3}}{8}\sin 20^\circ + \frac{\sqrt{3}}{8}(\sin 60^\circ - \sin 20^\circ) = \frac{\sqrt{3}}{8}\sin 20^\circ + \frac{\sqrt{3}}{8}\left(\frac{\sqrt{3}}{2} - \sin 20^\circ\right) \\
 &= \frac{\sqrt{3}}{8}\sin 20^\circ + \frac{3}{16} - \frac{\sqrt{3}}{8}\sin 20^\circ = \frac{3}{16} = \text{R.H.S}
 \end{aligned}$$

(iii) *Do yourself as above*

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Book: *Exercise 10.4*
Text Book of Algebra and Trigonometry Class XI
Punjab Textbook Board, Lahore.

Available online at <http://www.MathCity.org> in PDF Format
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