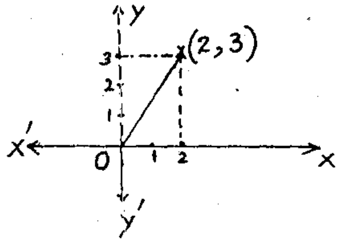
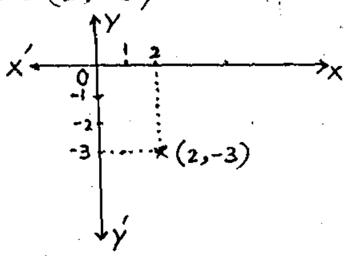
EXERCISE 1.3

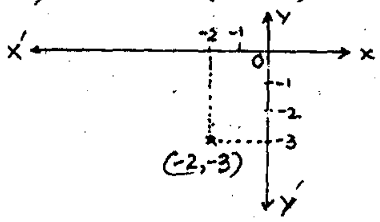
① i) $2+3i=(2,3)$



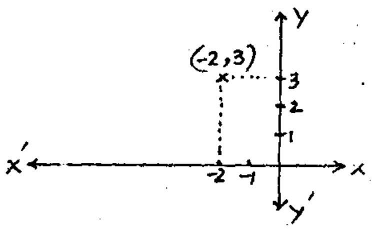
ii) $2-3i=(2,-3)$



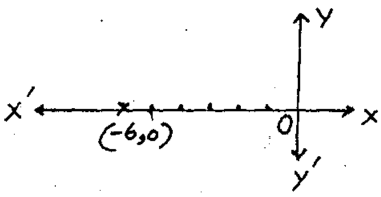
iii) $-2-3i=(-2,-3)$



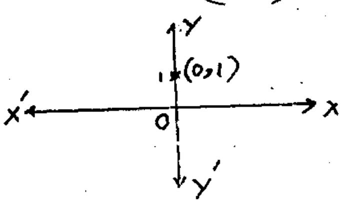
iv) $-2+3i=(-2,3)$



v) $-6=-6+i\cdot 0=(-6,0)$

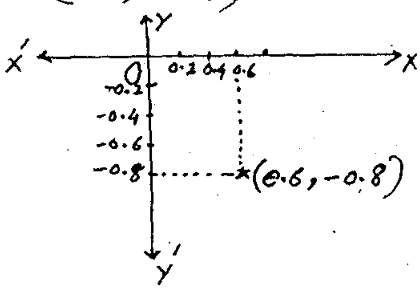


vi) $i=0+i=(0,1)$

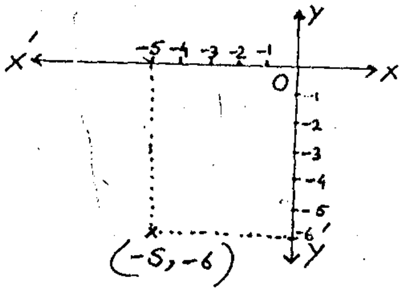


vii) $\frac{3}{5}-\frac{4}{5}i=0.6-0.8i$

$=(0.6,-0.8)$



viii) $-5-6i=(-5,-6)$



---

② i) Let $z=-3i$

Multiplicative inverse of z

$=z^{-1}=\frac{1}{z}=\frac{1}{-3i}=\frac{1}{-3i}\times\frac{i}{i}=\frac{i}{-3i^2}$

$=\frac{i}{-3(-1)}=\frac{i}{3}$ Ans.

---

ii) Let $z=1-2i$

Multiplicative inverse of $z=\frac{1}{z}$

$=\frac{1}{1-2i}=\frac{1}{1-2i}\times\frac{1+2i}{1+2i}$

$=\frac{1+2i}{(1)^2-(2i)^2}=\frac{1+2i}{1-4i^2}=\frac{1+2i}{1-4(-1)}$

$=\frac{1+2i}{1+4}=\frac{1+2i}{5}=\frac{1}{5}+\frac{2}{5}i$ Ans

---

iii) Let $z=-3-5i$

Multiplicative inverse of $z=\frac{1}{z}$

$=\frac{1}{-3-5i}=\frac{1}{-3-5i}\times\frac{-3+5i}{-3+5i}$

$=\frac{-3+5i}{(-3)^2-(5i)^2}$

$$= \frac{\ \ }{9-25i^2} = \frac{\ \ }{9+25}$$

$$= \frac{-3+5i}{34} = \frac{-3}{34} + \frac{5}{34}i \text{ Ans.}$$

iv) Let $z = (1,2) = 1+2i$

Multiplicative inverse of $z = \frac{1}{z}$

$$= \frac{1}{1+2i} = \frac{1}{1+2i} \times \frac{1-2i}{1-2i} = \frac{1-2i}{(1)^2-(2i)^2}$$

$$= \frac{1-2i}{1-4i^2} = \frac{1-2i}{1-4(-1)} = \frac{1-2i}{1+4} = \frac{1-2i}{5}$$

$$= \frac{1}{5} - \frac{2}{5}i = \left(\frac{1}{5}, -\frac{2}{5}\right) \text{ Ans}$$

③ i) $i^{101} = (i^2)^{50} \cdot i = (-1)^{50} \cdot i = 1 \cdot i$

$= i$ Ans.

ii) $(-ai)^4 = (-a)^4 i^4 = a^4 (i^2)^2$

$= a^4 (-1)^2 = a^4 \cdot 1 = a^4$ Ans

iii) $i^{-3} = \frac{1}{i^3} = \frac{1}{i^2 \cdot i} = \frac{1}{(-1)(i)} = \frac{1}{-i}$

$$= \frac{1}{-i} \times \frac{i}{i} = \frac{i}{-i^2} = \frac{i}{-(-1)} = \frac{i}{1}$$

$= i$ Ans.

iv) $i^{-10} = \frac{1}{i^{10}} = \frac{1}{(i^2)^5} = \frac{1}{(-1)^5} = \frac{1}{-1}$

$= -1$ Ans.

④ Suppose that Let $z = a+bi$, $a,b \in \mathbb{R}$ — ①

$\bar{z} = z$

We shall prove that z is real

$\because \bar{z} = z$

$\Rightarrow \overline{a+ib} = a+ib$

$\Rightarrow a-ib = a+ib$

$\Rightarrow a-ib-a-ib = 0$

$\Rightarrow -2ib = 0$

$\Rightarrow b = 0 \quad \because -2i \neq 0$

$\therefore$ ① becomes

$z = a + i(0) = a+0 = a$

$\Rightarrow z$ is real.

Conversely suppose that z is real.

Then we shall prove that

$\bar{z} = z$.

$\because z$ is real

$\Rightarrow$ Imaginary part of $z = 0$

$\Rightarrow b = 0$

$\therefore$ ① becomes

$z = a + i(0) = a+0 = a$

$\Rightarrow z = a$ —— ②

Taking conjugate on both sides

$\bar{z} = \bar{a} \Rightarrow \bar{z} = a$ —③ $\because a \in \mathbb{R}$

$\therefore$ From ② and ③, we get

$\bar{z} = z$ (proved)

⑤ i) $5 + 2\sqrt{-4} = 5 + 2\sqrt{(-1)(4)}$

$= 5 + 2\sqrt{-1}\sqrt{4} = 5 + 2(i)(2) = 5+4i$ Ans.

ii) $(2+\sqrt{-3})(3+\sqrt{-3})$

$= \left(2+\sqrt{(-1)(3)}\right)\left(3+\sqrt{(-1)(3)}\right)$

$= \left(2+\sqrt{-1}\sqrt{3}\right)\left(3+\sqrt{-1}\sqrt{3}\right)$

$= \left(2+i\sqrt{3}\right)\left(3+i\sqrt{3}\right)$

$= 6 + i2\sqrt{3} + i3\sqrt{3} + i^2(\sqrt{3})^2$

$= 6 + 5\sqrt{3}i + (-1)(3)$

$= 6 + 5\sqrt{3}i - 3 = 3 + 5\sqrt{3}i$ Ans.

iii) $\frac{2}{\sqrt{5}+\sqrt{-8}} = \frac{2}{\sqrt{5}+\sqrt{(-1)(8)}} = \frac{2}{\sqrt{5}+\sqrt{-1}\sqrt{8}}$

$$= \frac{2}{\sqrt{5} + i(2\sqrt{2})} \quad \left\{ \begin{aligned} \sqrt{8} &= \sqrt{2\times2\times2} \\ &= \sqrt{2^2\times2} \\ &= 2\sqrt{2} \end{aligned} \right.$$

$$=\frac{2}{\sqrt5+2\sqrt2\,i}$$

$$=\frac{2}{\sqrt5+2\sqrt2\,i}\times\frac{\sqrt5-2\sqrt2\,i}{\sqrt5-2\sqrt2\,i}$$

$$=\frac{2(\sqrt5-2\sqrt2\,i)}{(\sqrt5)^2-(2\sqrt2\,i)^2}=\frac{2(\sqrt5-2\sqrt2\,i)}{5-4(2)i^2}$$

$$=\frac{2(\sqrt5-2\sqrt2\,i)}{5+8}=\frac{2\sqrt5-4\sqrt2\,i}{13}$$

$$=\frac{2\sqrt5}{13}-\frac{4\sqrt2}{13}i \quad \text{Ans.}$$

---

iv) $$\frac{3}{\sqrt6-\sqrt{-12}}=\frac{3}{\sqrt6-\sqrt{(-1)(12)}}=\frac{3}{\sqrt6-\sqrt{-1}\sqrt{12}}$$

$$=\frac{3}{\sqrt6-i\sqrt{12}}=\frac{3}{\sqrt6-i(2\sqrt3)}$$

$$\left\{\begin{aligned}\sqrt{12}&=\sqrt{2\times2\times3}\\&=\sqrt{2^2\times3}\\&=2\sqrt3\end{aligned}\right.$$

$$=\frac{3}{\sqrt6-2\sqrt3\,i}\times\frac{\sqrt6+2\sqrt3\,i}{\sqrt6+2\sqrt3\,i}$$

$$=\frac{3(\sqrt6+2\sqrt3\,i)}{(\sqrt6)^2-(2\sqrt3\,i)^2}=\frac{3(\sqrt6+2\sqrt3\,i)}{6-4(3)i^2}$$

$$=\frac{3(\sqrt6+2\sqrt3\,i)}{6+12}=\frac{3(\sqrt6+2\sqrt3\,i)}{18}$$

$$=\frac{\sqrt6+2\sqrt3\,i}{6}=\frac{\sqrt6}{6}+\frac{2\sqrt3}{6}i$$

$$=\frac{\sqrt6}{6}+\frac{\sqrt3}{3}i=\frac{\sqrt6}{\sqrt6\times\sqrt6}+\frac{\sqrt3}{\sqrt3\times\sqrt3}i$$

$$=\frac{1}{\sqrt6}+\frac{1}{\sqrt3}i \quad \text{Ans}$$

---

(6) i) $z^2+\bar z^2$ is a real number.

Let $z=a+ib$, $a,b\in R$

then $\bar z=a-ib$

$$z^2+\bar z^2=(a+ib)^2+(a-ib)^2$$

$$=a^2+i^2b^2+2abi+a^2+i^2b^2-2abi$$

$$=2a^2+2i^2b^2$$

$$=2a^2+2(-1)b^2=2a^2-2b^2$$

which is real.

[24]

ii) $z^2-\bar z^2$ is Imaginary.

Let $z=a+ib$, $a,b\in R$

then $\bar z=a-ib$

Now $z^2-\bar z^2=(a+ib)^2-(a-ib)^2$

$$=(a^2+i^2b^2+2abi)-(a^2+i^2b^2-2abi)$$

$$=a^2+i^2b^2+2abi-a^2-i^2b^2+2abi$$

$=4abi$ which is imaginary number.

---

(7) Simplify

i) $$\left(-\frac12+\frac{\sqrt3}{2}i\right)^3$$

$$=\left(-\frac12+\frac{\sqrt3}{2}i\right)^2\left(-\frac12+\frac{\sqrt3}{2}i\right)$$

$$=\left[\left(-\frac12\right)^2+\left(\frac{\sqrt3}{2}i\right)^2+2\left(-\frac12\right)\left(\frac{\sqrt3}{2}i\right)\right]\left(-\frac12+\frac{\sqrt3}{2}i\right)$$

$$=\left(\frac14+\frac34i^2-\frac{\sqrt3}{2}i\right)\left(-\frac12+\frac{\sqrt3}{2}i\right)$$

$$=\left(\frac14-\frac34-\frac{\sqrt3}{2}i\right)\left(-\frac12+\frac{\sqrt3}{2}i\right)$$

$$\left\{\begin{aligned}&\frac14-\frac34\\&=\frac{1-3}{4}\\&=-\frac24=-\frac12\end{aligned}\right.$$

$$=\left(-\frac12-\frac{\sqrt3}{2}i\right)\left(-\frac12+\frac{\sqrt3}{2}i\right)$$

$$=\left(-\frac12\right)^2-\left(\frac{\sqrt3}{2}i\right)^2=\frac14-\frac34i^2$$

$$=\frac14+\frac34=\frac{1+3}{4}=\frac44=1 \quad \text{Ans.}$$

---

ii) $$\left(-\frac12-\frac{\sqrt3}{2}i\right)^3$$

$$=\left(-\frac12-\frac{\sqrt3}{2}i\right)^2\left(-\frac12-\frac{\sqrt3}{2}i\right)$$

$$=\left[\left(-\frac12\right)^2+\left(\frac{\sqrt3}{2}i\right)^2-2\left(-\frac12\right)\left(\frac{\sqrt3}{2}i\right)\right]\left(-\frac12-\frac{\sqrt3}{2}i\right)$$

$$=\left(\frac14-\frac34+\frac{\sqrt3}{2}i\right)\left(-\frac12-\frac{\sqrt3}{2}i\right)$$

$$=\left(-\frac12+\frac{\sqrt3}{2}i\right)\left(-\frac12-\frac{\sqrt3}{2}i\right)$$

$$=\left(-\frac12\right)^2-\left(\frac{\sqrt3}{2}i\right)^2$$

$$=\frac14-\frac34i^2=\frac14+\frac34=\frac{1+3}{4}=\frac44=1$$

Ans

iii) $\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)^{-2}\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)$

$= \dfrac{-\frac{1}{2}-\frac{\sqrt{3}}{2}i}{\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)^2}$

$= \dfrac{-\frac{1}{2}-\frac{\sqrt{3}}{2}i}{\left(-\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}i\right)^2+2\left(-\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}i\right)}$

$= \dfrac{-\frac{1}{2}-\frac{\sqrt{3}}{2}i}{\frac{1}{4}-\frac{3}{4}-\frac{\sqrt{3}}{2}i} = \dfrac{-\frac{1}{2}-\frac{\sqrt{3}}{2}i}{-\frac{1}{2}-\frac{\sqrt{3}}{2}i} = 1$ Ans

(iii) $\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)^{-2}\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)^{1}$ (according to book)

$= \left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)^{-1} = \dfrac{1}{-\frac{1}{2}-\frac{\sqrt{3}}{2}i}$

$= \dfrac{1}{-\frac{1}{2}-\frac{\sqrt{3}}{2}i} \times \dfrac{-\frac{1}{2}+\frac{\sqrt{3}}{2}i}{-\frac{1}{2}+\frac{\sqrt{3}}{2}i}$

$= \dfrac{-\frac{1}{2}+\frac{\sqrt{3}}{2}i}{\left(-\frac{1}{2}\right)^2-\left(\frac{\sqrt{3}}{2}i\right)^2} = \dfrac{-\frac{1}{2}+\frac{\sqrt{3}}{2}i}{\frac{1}{4}+\frac{3}{4}}$

$= \dfrac{-\frac{1}{2}+\frac{\sqrt{3}}{2}i}{\frac{4}{4}} = -\frac{1}{2}+\frac{\sqrt{3}}{2}i$

$= \dfrac{-1+\sqrt{3}i}{2}$ Ans

iv) $(a+bi)^2 = a^2+b^2i^2+2abi$

$= a^2+b^2(-1)+2abi$

$= a^2-b^2+2abi$ Ans

v) $(a+bi)^{-2} = \dfrac{1}{(a+bi)^2} = \dfrac{1}{a^2+b^2i^2+2abi}$

$= \dfrac{1}{a^2-b^2+2abi}$

$= \dfrac{1}{(a^2-b^2)+2abi} \times \dfrac{(a^2-b^2)-2abi}{(a^2-b^2)-2abi}$

$= \dfrac{(a^2-b^2)-2abi}{(a^2-b^2)^2-(2abi)^2}$

$= \dfrac{(a^2-b^2)-2abi}{a^4+b^4-2a^2b^2-4a^2b^2i^2}$

$= \dfrac{(a^2-b^2)-2abi}{a^4+b^4-2a^2b^2+4a^2b^2}$

$= \dfrac{(a^2-b^2)-2abi}{a^4+b^4+2a^2b^2} = \dfrac{(a^2-b^2)-2abi}{(a^2+b^2)^2}$

$= \dfrac{a^2-b^2}{(a^2+b^2)^2} - \dfrac{2ab}{(a^2+b^2)^2}i$ Ans

vi) $(a+bi)^3 = a^3+(bi)^3+3(a)(bi)(a+bi)$

$= a^3+b^3i^3+3abi(a+bi)$ $\quad \left\{\begin{aligned} i^3 &= i^2\cdot i \\ &= -1\cdot i \\ &= -i \end{aligned}\right.$

$= a^3+b^3(-i)+3a^2bi+3ab^2i^2$

$= a^3-b^3i+3a^2bi-3ab^2$

$= a^3-3ab^2+3a^2bi-b^3i$

$= a^3-3ab^2+i(3a^2b-b^3)$ Ans

vii) $(a-bi)^3$

$= (a)^3-(bi)^3-3(a)(bi)(a-bi)$

$= a^3-b^3i^3-3abi(a-bi)$

$= a^3-b^3(-i)-3a^2bi+3ab^2i^2$

$= a^3+b^3i-3a^2bi-3ab^2$

$= a^3-3ab^2-3a^2bi+b^3i$

$= a^3-3ab^2-i(3a^2b-b^3)$ Ans

viii) $(3-\sqrt{-4})^{-3} = \dfrac{1}{[3-\sqrt{-4}]^3} = \dfrac{1}{[3-\sqrt{(-1)(4)}]^3}$

$= \dfrac{1}{(3-\sqrt{-1}\sqrt{4})^3} = \dfrac{1}{[3-i(2)]^3} = \dfrac{1}{(3-2i)^3}$

$= \dfrac{1}{(3)^3-(2i)^3-3(3)(2i)(3-2i)}$

$= \dfrac{1}{27-8i^3-18i(3-2i)}$

$$= \frac{1}{27-8(-i)-54i+36i^2}$$

$$= \frac{1}{27+8i-54i-36}$$

$$= \frac{1}{-9-46i}$$

$$= \frac{1}{-9-46i} \times \frac{-9+46i}{-9+46i}$$

$$= \frac{-9+46i}{(-9)^2-(46i)^2} = \frac{-9+46i}{81-2116i^2}$$

$$= \frac{-9+46i}{81+2116} = \frac{-9+46i}{2197}$$

$$= \frac{-9}{2197} + \frac{46}{2197}i \quad \text{Ans.}$$

End of Chapter No. 1.